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VAGUE BINARY SOFT CONTINUITY

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Abstract. Vague binary soft continuity over a binary universe with a fixed parameter set is developed in this paper. Pasting Lemma & and some basic theorems are also discussed

1. Introduction

Two objects are equivalent in topology if they are homeomorphic. Continuity acts like a backbone to homeomorphism. Molodtsov [9] introduced soft sets in 1999. It was a novel approach in set theory, which helped a lot to overcome all the existing difficulties of classical set theory. In 1997, E.L.Atik.A.A [4] studied some types of mappings on topological spaces. In 2010, Athar Kharal and Ahmad.B [3] gave mappings on soft classes. In 2012, Banashree Bora redefined the same and reached to the same result of Kharal et al.,. In 2013, Cigdem Gundur Aras et at., [6] defined soft continuity using the definition of soft point. In 2009, Ahmad and kharal [1] discussed on fuzzy soft sets and gave several ideas. In 2012, Fuzzy soft continuity was developed by Banashree Bora [5]. In 2015, Jeyaraman et al., [7] gave a decomposition of fuzzy continuity. In 2010, Pyung Ki Lim, So Ra Kim and Kul Hur [10] developed vague continuity. In 2016, Mary Margaret and Arockiarani.I. [8] studied vague generalized pre-continuous mapping. In 2016, Ahu Acikgöz et. al., [2] developed Binary soft set theory. Remya. P. B and Francina Shalini. A [11] developed vague binary soft sets in 2018. In this paper authors further extended their work to continuity of vague binary soft sets.

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2. Preliminaries

In this section, some preliminaries are given.

Definition 2.1. [11] Refined Definition : **(Vague Binary Soft Set)**

Let (U_1, U_2) be a binary universe with
$U_1 = \{u_1, u_2, --, u_r, --, u_i\}, U_2 = \{u_1^*, u_2^*, --, u_r^*, --, u_j^*\}$.
E $= \{e_1, e_2, --, e_r, --, e_k\}$ be a fixed parameter set with $A \subseteq E$.
Let $V(U_1)$, $V(U_2)$ denote power set of vague sets on U_1, U_2 respectively.

A pair $\left(\ddot{F}, A\right)$ is said to be a Vague Binary Soft Set (in short, VBSS) over (U_1, U_2) where $\ddot{F}$ is a mapping given by, $\ddot{F} : A \longrightarrow V(U_1) \times V(U_2)$.

Here, $\left(\ddot{F}, A\right) = \left\{e_r \in A / \left(e_r, \ddot{F}(e_r)\right)\right\}$
where $\ddot{F}(e_r) = \left(\left\langle \frac{V_{\ddot{F}(e_r)}(u_r)}{u_r}; \forall e_r \in A, \forall u_r \in U_1 \right\rangle, \left\langle \frac{V_{\ddot{F}(e_r)}(u_r^*)}{u_r^*}; \forall e_r \in A, \forall u_r^* \in U_2 \right\rangle\right)$
$= \left(\left\langle \frac{\left[t_{\ddot{F}(e_r)}(u_r), 1-f_{\ddot{F}(e_r)}(u_r)\right]}{u_r}; \forall e_r \in A, \forall u_r \in U_1 \right\rangle, \left\langle \frac{\left[t_{\ddot{F}(e_r)}(u_r^*), 1-f_{\ddot{F}(e_r)}(u_r^*)\right]}{u_r^*}; \forall e_r \in A, \forall u_r^* \in U_2 \right\rangle\right)$

Truth Membership function $t_{\ddot{F}(e_r)}(u_r)$ is a lower bound on the grade of membership of u_r derived for the evidence for $u_r \in U_1$ for the parameter e_r under the mapping $\ddot{F}$, False Membership function $f_{\ddot{F}(e_r)}(u_r)$ is a lower bound on the grade of membership of u_r derived for the evidence for $u_r \in U_1$ for the parameter e_r under the mapping $\ddot{F}$

&

Truth Membership function $t_{\ddot{F}(e_r)}(u_r^*)$ is a lower bound on the grade of membership of u_r^* derived for the evidence for $u_r^* \in U_2$ for the parameter e_r under the mapping $\ddot{F}$, False Membership function $f_{\ddot{F}(e_r)}(u_r^*)$ is a lower bound on the grade of membership of u_r^* derived for the evidence for $u_r^* \in U_2$ for the parameter e_r under the mapping $\ddot{F}$

&

$t_{\ddot{F}(e_r)}(u_r), f_{\ddot{F}(e_r)}(u_r) : U_1 \to [0,1] \qquad t_{\ddot{F}(e_r)}(u_r) + f_{\ddot{F}(e_r)}(u_r) \leq 1;$

$t_{\ddot{F}(e_r)}(u_r^*), f_{\ddot{F}(e_r)}(u_r^*) : U_2 \to [0,1] \; ; \qquad t_{\ddot{F}(e_r)}(u_r^*) + f_{\ddot{F}(e_r)}(u_r^*) \leq 1$

The grade of membership of $u_r \in U_1$ in the vague binary soft set $\left(\ddot{F}, A\right)$ is bounded to a sub-interval $\left[t_{\ddot{F}(e_r)}(u_r), 1 - f_{\ddot{F}(e_r)}(u_r)\right]$ of [0, 1]

The grade of membership of $u_r^* \in U_2$ in the vague binary soft set $\left(\ddot{F}, A\right)$ is bounded to a sub-interval $\left[t_{\ddot{F}(e_r)}(u_r^*), 1 - f_{\ddot{F}(e_r)}(u_r^*)\right]$ of [0, 1]

The *vague binary soft value* $V_{\ddot{F}(e_r)}(u_r) = \left[t_{\ddot{F}(e_r)}(u_r), 1 - f_{\ddot{F}(e_r)}(u_r)\right]$ indicates that the exact grade of membership $\mu_{\ddot{F}(e_r)}(u_r)$ of $u_r \in U_1$ may be unknown, but it is bounded by $t_{\ddot{F}(e_r)}(u_r) \leq \mu_{\ddot{F}(e_r)}(u_r) \leq 1 - f_{\ddot{F}(e_r)}(u_r)$.

The *vague binary soft value* $V_{\ddot{F}(e_r)}(u_r^*) = \left[t_{\ddot{F}(e_r)}(u_r^*), 1 - f_{\ddot{F}(e_r)}(u_r^*)\right]$ indicates that the exact grade of membership $\mu_{\ddot{F}(e_r)}(u_r^*)$ of $u_r^* \in U_2$ may be unknown, but it is bounded by $t_{\ddot{F}(e_r)}(u_r^*) \leq \mu_{\ddot{F}(e_r)}(u_r^*) \leq 1 - f_{\ddot{F}(e_r)}(u_r^*)$

Definition 2.2. [11] Refined Definition : **(Null Vague Binary Soft Set)**
A Vague Binary Soft Set $\left(\ddot{F}, A\right)$ over a binary universe (U_1, U_2) with a fixed parameter set E having $A \subseteq E$ is said to be a Null Vague Binary Soft Set (in short, Null $-$ VBSS) denoted by $\left(\ddot{\Phi}, A\right)$ if

$[t_{\ddot{F}(e_r)}(u_r), \quad 1 - f_{\ddot{F}(e_r)}(u_r)] \quad = [0,0]; \qquad \forall \quad u_r \in U_1, \quad \forall e_r \in A$

$[t_{\ddot{F}(e_r)}(u_r^*)), \quad 1 - f_{\ddot{F}(e_r)}(u_r^*)] \quad = [0,0]; \quad \forall \quad u_r^* \in U_2, \quad \forall e_r \in A$

$\left(\ddot{\Phi}, A\right) = \left\{\left(e_r, \left(\left\langle \frac{[0,0]}{u_r}; \forall e_r \in A, \forall u_r \in U_1\right\rangle, \left\langle \frac{[0,0]}{u_r^*}; \forall e_r \in A, \forall u_r^* \in U_2\right\rangle\right)\right)\right\}$

Definition 2.3. [11] Refined Definition : **(Absolute Vague Binary Soft Set)**
A Vague Binary Soft Set $\left(\ddot{F}, A\right)$ over a binary universe (U_1, U_2) with a fixed parameter set E having $A \subseteq E$ is said to be an Absolute Vague Binary Soft Set (in short, Absolute-VBSS) denoted by $\left(\ddot{U}, A\right)$ if

$[t_{\ddot{F}(e_r)}(u_r), \quad 1 - f_{\ddot{F}(e_r)}(u_r)] \quad = [1,1]; \qquad \forall \quad u_r \in U_1, \quad \forall \quad e_r \in A$

$[t_{\ddot{F}(e_r)}(u_r^*), \quad 1 - f_{\ddot{F}(e_r)}(u_r^*)] \quad = [1,1]; \qquad \forall \quad u_r^* \in U_2, \quad \forall \quad e_r \in A$

$\left(\ddot{U}, A\right) = \left\{\left(e_r, \left(\left\langle \frac{[1,1]}{u_r}; \forall e_r \in A, \forall u_r \in U_1\right\rangle, \left\langle \frac{[1,1]}{u_r^*}; \forall e_r \in A, \forall u_r^* \in U_2\right\rangle\right)\right)\right\}$

Definition 2.4. [11] Refined Definition : **(Vague Binary Soft Complement)**
Let (U_1, U_2) be a binary universe. E be a fixed set of parameters with A $\subseteq$ E.
Vague Binary Soft Complement of a Vague Binary Soft Set $\left(\ddot{F}, A\right)$ with respect to Absolute Vague Binary Soft Set $\left(\ddot{U}, A\right)$ is a mapping given by $\ddot{F^c} : A \to V(U_1) \times V(U_2)$

Here, $\left(\ddot{F^c}, A\right) = \left\{e_r \in A / \left(e_r, \ddot{F^c}(e_r)\right)\right\}$
where $\ddot{F^c}(e_r) = \left(\left\langle \frac{V_{\ddot{F^c}(e_r)}(u_r)}{u_r}; \forall e_r \in A, \forall u_r \in U_1\right\rangle, \left\langle \frac{V_{\ddot{F^c}(e_r)}(u_r^*)}{u_r^*}; \forall e_r \in A, \forall u_r^* \in U_2\right\rangle\right)$
$= \left(\left\langle \frac{\left[t_{\ddot{F^c}(e_r)}(u_r), 1 - f_{\ddot{F^c}(e_r)}(u_r)\right]}{u_r}; \forall e_r \in A, \forall u_r \in U_1\right\rangle, \left\langle \frac{\left[t_{\ddot{F^c}(e_r)}(u_r^*), 1 - f_{\ddot{F^c}(e_r)}(u_r^*)\right]}{u_r^*}; \forall e_r \in A, \forall u_r^* \in U_2\right\rangle\right)$

$$\text{where} \begin{cases} t_{\ddot{F^c}(e_r)}(u_r) = f_{\ddot{F}(e_r)}(u_r); & \forall e_r \in A; \quad \forall u_r \in U_1 \\ 1 - f_{\ddot{F^c}(e_r)}(u_r) = 1 - t_{\ddot{F}(e_r)}(u_r); & \forall e_r \in A; \quad \forall u_r \in U_1 \\ \\ t_{\ddot{F^c}(e_r)}(u_r^*) = f_{\ddot{F}(e_r)}(u_r^*); & \forall e_r \in A; \quad \forall u_r^* \in U_2 \\ 1 - f_{\ddot{F^c}(e_r)}(u_r^*) = 1 - t_{\ddot{F}(e_r)}(u_r^*); & \forall e_r \in A; \quad \forall u_r^* \in U_2 \end{cases} \tag{2.1}$$

It is denoted by $\left(\ddot{F}, A\right)^c = \left(\ddot{F^c}, A\right)$

Definition 2.5. [11] Refined Definition : (**Vague Binary Soft Intersection**)
Let (U_1, U_2) be a binary universe. E be a fixed set of parameters with A, B $\subseteq$ E. Vague

Binary Soft Intersection of two Vague Binary Soft Sets $\left(\ddot{F}, A\right)$ and $\left(\ddot{G}, B\right)$ over (U_1, U_2) is denoted as, $(\ddot{F}, A)\ddot{\cap}(\ddot{G}, B)$. Let the output be $(\ddot{K}, D)$, where D $= (A \cup B)$; $\cup$ represents the usual set theoretic union.

Then $\forall e_r \in D$,

$$\ddot{K}(e_r) = \begin{cases} \ddot{F}(e_r) & ; e_r \in B^c;\ c\ is\ Cantor\ Set\ Complement\ operation\ with\ respect\ to\ D \\ \ddot{G}(e_r) & ; e_r \in A^c;\ c\ is\ is\ Cantor\ Set\ Complement\ operation\ with\ respect\ to\ D \\ \ddot{F}(e_r)\ddot{\cap}\ddot{G}(e_r) & ; e_r \in (A \cap B); \cap\ is\ Cantor\ Set\ Intersection\ Operation \end{cases} \tag{2.2}$$

$$t_{\ddot{K}(e_r)}(u_r) = \begin{cases} t_{\ddot{F}(e_r)}(u_r); & \forall e_r \in B^c; \quad \forall u_r \in U_1 \\ t_{\ddot{G}(e_r)}(u_r); & \forall e_r \in A^c; \quad \forall u_r \in U_1 \\ min\left\{t_{\ddot{F}(e_r)}(r), t_{\ddot{G}(e_r)}(u_r)\right\}; & \forall e_r \in (A \cap B), \quad \forall u_r \in U_1 \end{cases} \tag{2.3}$$

$$1 - f_{\ddot{K}(e_r)}(u_r) = \begin{cases} 1 - f_{\ddot{F}(e_r)}(u_r); & \forall e_r \in B^c; \quad \forall u_r \in U_1 \\ 1 - f_{\ddot{G}(e_r)}(u_r); & \forall e_r \in A^c; \quad \forall u_r \in U_1 \\ min\left\{1 - f_{\ddot{F}(e_r)}(u_r), 1 - f_{\ddot{G}(e_r)}(u_r)\right\}; & \forall e_r \in (A \cap B), \quad \forall u_r \in U_1 \end{cases} \tag{2.4}$$

$$t_{\ddot{K}(e_r)}(u_r^*) = \begin{cases} t_{\ddot{F}(e_r)}(u_r^*); & \forall e_r \in B^c; \quad \forall u_r^* \in U_2 \\ t_{\ddot{G}(e_r)}(u_r^*); & \forall e_r \in A^c, \quad \forall u_r^* \in U_2 \\ min\left\{t_{\ddot{F}(e_r)}(u_r^*), t_{\ddot{G}(e_r)}(u_r^*)\right\}; & \forall e_r \in (A \cap B), \quad \forall u_r^* \in U_2 \end{cases} \tag{2.5}$$

$$1 - f_{\ddot{K}(e_r)}(u_r^*) = \begin{cases} 1 - f_{\ddot{F}(e_r)}(u_r^*); & \forall e_r \in B^c; \quad \forall u_r^* \in U_2 \\ 1 - f_{\ddot{G}(e_r)}(u_r^*); & \forall e_r \in A^c, \quad \forall u_r^* \in U_2 \\ min\left\{1 - f_{\ddot{F}(e_r)}(u_r^*), 1 - f_{\ddot{G}(e_r)}(u_r^*)\right\}; & \forall e_r \in (A \cap B), \quad \forall u_r^* \in U_2 \end{cases} \tag{2.6}$$

Definition 2.6. [11] Refined Definition : (**Vague Binary Soft Union**)
Let (U_1, U_2) be a binary universe. E be a fixed set of parameters with A, B $\subseteq$ E. Vague Binary Soft Union of two Vague Binary Soft Sets $\left(\ddot{F}, A\right)$ and $\left(\ddot{G}, B\right)$ over (U_1, U_2) is denoted as, $(\ddot{F}, A)\ddot{\cup}(\ddot{G}, B)$. Let the output be $(\ddot{H}, D)$, where D $= (A \cup B)$; $\cup$ represents the Cantor set union.

Then $\forall e_r \in D$,

$$\ddot{H}(e_r) = \begin{cases} \ddot{F}(e_r); e_r \in B^c; c\ is\ Cantor\ Set\ Complement\ operation\ with\ respect\ to\ D \\ \ddot{G}(e_r); e_r \in A^c; c\ is\ Cantor\ Set\ Complement\ operation\ with\ respect\ to\ D \\ \ddot{F}(e_r)\ddot{\cup}\ddot{G}(e_r); e_r \in (A \cap B);\ \cap\ is\ Cantor\ Set\ Intersection\ Operation \end{cases} \tag{2.7}$$

$$t_{\ddot{H}(e_r)}(u_r) = \begin{cases} t_{\ddot{F}(e_r)}(u_r) & ;\forall e_r \in B^c; \quad \forall u_r \in U_1 \\ t_{\ddot{G}(e_r)}(u_r) & ;\forall e_r \in A^c; \quad \forall u_r \in U_1 \\ max\left\{t_{\ddot{F}(e_r)}(r), t_{\ddot{G}(e_r)}(u_r)\right\} & ;\forall e_r \in (A \cap B), \quad \forall u_r \in U_1 \end{cases} \tag{2.8}$$

$$1 - f_{\ddot{H}(e_r)}(u_r) = \begin{cases} 1 - f_{\ddot{F}(e_r)}(u_r) & ;\forall e_r \in B^c; \quad \forall u_r \in U_1 \\ 1 - f_{\ddot{G}(e_r)}(u_r) & ;\forall e_r \in A^c; \quad \forall u_r \in U_1 \\ max\left\{1 - f_{\ddot{F}(e_r)}(u_r), 1 - f_{\ddot{G}(e_r)}(u_r)\right\}; & \forall e_r \in (A \cap B), \quad \forall u_r \in U_1 \end{cases} \tag{2.9}$$

$$t_{\ddot{H}(e_r)}(u_r^*) = \begin{cases} t_{\ddot{F}(e_r)}(u_r^*) & ;\forall e_r \in B^c; \quad \forall u_r^* \in U_2 \\ t_{\ddot{G}(e_r)}(u_r^*) & ;\forall e_r \in A^c, \quad \forall u_r^* \in U_2 \\ max\left\{t_{\ddot{F}(e_r)}(u_r^*), t_{\ddot{G}(e_r)}(u_r^*)\right\} & ;\forall e_r \in (A \cap B), \quad \forall u_r^* \in U_2 \end{cases} \tag{2.10}$$

$$1 - f_{\ddot{H}(e_r)}(u_r^*) = \begin{cases} 1 - f_{\ddot{F}(e_r)}(u_r^*); & \forall e_r \in B^c; \quad \forall u_r^* \in U_2 \\ 1 - f_{\ddot{G}(e_r)}(u_r^*); & \forall e_r \in A^c, \quad \forall u_r^* \in U_2 \\ max\left\{1 - f_{\ddot{F}(e_r)}(u_r^*), 1 - f_{\ddot{G}(e_r)}(u_r^*)\right\} & ;\forall e_r \in (A \cap B), \quad \forall u_r^* \in U_2 \end{cases} \tag{2.11}$$

Definition 2.7. [11] Refined Definition : (**Vague Binary Soft Subset**)
Let (U_1, U_2) be a binary universe. E be a fixed set of parameters with A, B $\subseteq$ E. A Vague Binary Soft Set $\left(\ddot{F}, A\right)$ is contained in another Vague Binary Soft Set $(\ddot{G}, B)$ denoted as $\left(\ddot{F}, A\right) \ddot{\subseteq} \left(\ddot{G}, B\right)$ if
(1) A $\subseteq$ B ; $\subseteq$ denotes usual set theoretical subset operation
(2) $\forall \quad e_r \in A, \quad \ddot{F}(e_r)$ and $\ddot{G}(e_r)$ are identical approximations.

In this case, $\left(\ddot{F}, A\right)$ is called Vague Binary Soft SubSet of $\left(\ddot{G}, B\right)$(in short, VBSSS) denoted as $\left(\ddot{F}, A\right) \ddot{\supseteq} \left(\ddot{G}, B\right)$ and $\left(\ddot{G}, B\right)$ is called Vague Binary Soft Superset of $\left(\ddot{F}, A\right)$ denoted as $\left(\ddot{G}, B\right) \ddot{\supseteq} \left(\ddot{F}, A\right)$

Definition 2.8. Refined Definition : (**Vague Binary Soft Equal Operation**)
Let $\left(\ddot{F}, A\right)$ and $\left(\ddot{G}, B\right)$ are two VBSS's. These sets are said to be vague binary soft equal if $\left(\ddot{F}, A\right)$ is a vague binary soft subset of $(\ddot{G}, B)$ and $\left(\ddot{G}, B\right)$ is a vague binary soft superset of $\left(\ddot{F}, A\right)$. It is denoted by $\left(\ddot{F}, A\right) \ddot{=} \left(\ddot{G}, B\right)$
i.e., $\left(\ddot{F}, A\right) \ddot{\supseteq} \left(\ddot{G}, B\right)$ and $\left(\ddot{G}, B\right) \ddot{\supseteq} \left(\ddot{F}, A\right) \Rightarrow \left(\ddot{F}, A\right) \ddot{=} \left(\ddot{G}, B\right)$

Definition 2.9. [11]
Refined Definition:(**Vague Binary Soft Cartesian Product Operation**)

Let (U_1, U_2) be a binary universe. E be a fixed set of parameters with A, B $\subseteq$ E.

Vague Binary Soft Cartesian Product of $\left(\ddot{F}, A\right)$ and $\left(\ddot{G}, B\right)$ denoted as $\left(\ddot{F}, A\right) \ddot{\times} \left(\ddot{G}, B\right)$ is a mapping given by $\ddot{M} : A \times B \to V\left(U_1 \times U_1\right) \times V\left(U_2 \times U_2\right)$; with

$\ddot{M}(u_r, u_r^*) = \ddot{F}(u_r) \times \ddot{G}(u_r^*); \quad \forall \quad (u_r, u_r^*) \in (A \times B) = P.$

Let the output is denoted as $\left(\ddot{M}, P\right)$.

$\ddot{M}(u_r, u_r^*) =$

$$\left\{ \left(\begin{array}{c} \left\langle \dfrac{\left[min\left(t_{\ddot{F}(e_r,e_r^*)}\left(u_r,u_r\right), t_{\ddot{G}(e_r,e_r^*)}\left(u_r,u_r\right)\right), max\left(1-t_{\ddot{F}(e_r,e_r^*)}\left(u_r,u_r\right), 1-t_{\ddot{G}(e_r,e_r^*)}\left(u_r,u_r\right)\right)\right]}{\left(u_r,u_r\right)} \right\rangle \\ \left\langle \dfrac{\left[min\left(t_{\ddot{F}(e_r,e_r^*)}\left(u_r^*,u_r^*\right), t_{\ddot{G}(e_r,e_r^*)}\left(u_r^*,u_r^*\right)\right), max\left(1-t_{\ddot{F}(e_r,e_r^*)}\left(u_r^*,u_r^*\right), 1-t_{\ddot{G}(e_r,e_r^*)}\left(u_r^*,u_r^*\right)\right)\right]}{\left(u_r^*,u_r^*\right)} \right\rangle \end{array} \right) \right\} \tag{2.12}$$

$;\forall\left(e_r, e_r^*\right) \in P; \forall\left(u_r, u_r\right) \in U_1 \times U_1 \qquad ;\forall\left(e_r, e_r^*\right) \in P; \forall\left(u_r^*, u_r^*\right) \in U_2 \times U_2$

Definition 2.10. [7] **(Fuzzy Continuous Map)**
A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be fuzzy continuous if $\mathrm{f}^{-1}(\lambda)$ is fuzzy open in X for each fuzzy open set λ in Y.

Definition 2.11. [1, 5] **(Fuzzy Soft Continuous Map)**
Let (U, E, τ) and (U^*, E^*, τ^*) be fuzzy soft topological spaces. Let $\rho : U \to U^*$ and $\Psi : E \to E^*$ be mappings and $f = (\rho, \Psi)$ is said to be fuzzy soft continous if the inverse image under $f = (\rho, \Psi)$ of any $(G, B) \in \tau^*$ is a fuzzy soft set $(F, A) \in \tau$. i.e., $f^{-1}(G, B) \in \tau$ whenever $(G, B) \in \tau^*$

Definition 2.12. [3, 4, 6] **(Soft Continuous Map)**
Let (U, E, τ) and $\left(V, E, \tau^{'}\right)$ be two soft topological spaces. A mapping $f : (U, E, \tau) \to \left(V, E, \tau^{'}\right)$ is said to be soft continous if for each soft open set (G, E) over Y, $f^{-1}(G, E)$ is a soft open set over X

Definition 2.13. [8] **(Vague Continuous Map)**
Let (X, τ) and (Y, υ) be any topological spaces. A map $f : (X, \tau) \to (Y, \upsilon)$ is said to be vague continous (V continous in short) if $f^{-1}(V)$ is vague soft closed set in (X, τ) for every vague soft closed set V of (Y, υ)

Remark 2.14. [3]

(1) Soft classes are collections of soft sets.
(2) Let X be a universe and E a set of attributes. Then the collection of all soft sets over X with attributes from E is called a soft class and is denoted as (X, E)

3. Vague Binary Soft Continuity

In this section continuity is developed for vague soft sets. Based on this, continuity for vague binary soft sets are developed. Before entering to continuity concept some topological aspects for VBSS's are furnished.

Definition 3.1. (Vague Binary Soft Topological Space)
A vague binary soft topology over binary universe (U_1, U_2) with a fixed set of parameter set $A \subseteq E$ is a family $\ddot{\tau}_\Delta$ of vague binary soft sets in (U_1, U_2) satisfying the following axioms:
(i) $\left(\ddot{\phi}, A\right), \left(\ddot{U}, A\right) \in \ddot{\tau}_\Delta$
(ii)$(G_1, A) \ddot{\cap} (G_2, A) \in \ddot{\tau}_\Delta$ for any $(G_1, A), (G_2, A) \in \ddot{\tau}_\Delta$
(iii)$\ddot{\cup}_{i\in I}(G_i, A) \in \ddot{\tau}_\Delta$, for any family $\{(G_i, A)/i \in I\} \ddot{\subseteq} \ddot{\tau}_\Delta$

In this case the 4-tuple $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ is called a vague binary soft topological space (VBSTS in short) and any vague binary soft set in $\ddot{\tau}_\Delta$ is known as a vague binary soft open set (VBSOS in short) in (U_1, U_2). Any Vague Binary Soft Compliment of a Vague Binary Soft Open Set with respect to Absolute Vague Binary SoftSet is known as Vague Binary Soft Closed Set (in short, VBSCS). Vague Binary Soft Open Subset and Vague Binary Soft Closed Subset are denoted as VBSOSS, VBSCSS respectively.

Definition 3.2. (Vague Binary Soft Hausdorff Space) A VBSTS $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ over $V(U_1) \times V(U_2)$ is said to be a vague binary soft T_2 space or vague binary soft hausdorff space if for every two distinct VBSP's in $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ there exists disjoint VBSOS containing each of them, where $V(U_1), V(U_2)$ denotes power set of vague sets on U_1, U_2 respectively.

Definition 3.3. (Vague Binary Soft Normal Space) A VBSTS $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ over $V(U_1) \times V(U_2)$ is said to be a vague binary soft normal if for every two distinct VBSCS's $\left(\ddot{C}, A\right)$ and $\left(\ddot{D}, A\right)$ of $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ there exists two disjoint VBSOS's $\left(\ddot{S}, A\right)$ and $\left(\ddot{V}, A\right)$ such that $\left(\ddot{C}, A\right) \ddot{\subset} \left(\ddot{S}, A\right)$ and $\left(\ddot{D}, A\right) \ddot{\subset} \left(\ddot{S}, A\right)$, where $V(U_1), V(U_2)$ denotes power set of vague sets on U_1, U_2 respectively.

Remark 3.4.

(1) (a) Let $\{W\}$ be a common universe with attribute set $A \subseteq E$. Corresponding vague soft class is denoted by $\left(\overset{\rtimes}{W}, A\right)$
(b) vague soft class is a collection of vague soft sets.

(2) (a) Let $\{W_1, W_2\}$ be a common universe with attribute set $A \subseteq E$. Corresponding vague binary soft class is denoted by $\left(\overset{\bowtie}{W}, A\right)$
(b) Vague binary soft classes are collections of vague binary soft sets.

Definition 3.5. (Vague Soft Continuous Mapping) Let $(U, E, \tau_1)_A$ and $(W, E^*, \tau_2)_{A^*}$ be two vague soft topological spaces. Let $\zeta : U \to W$ and $\Psi : A \to A^*$ be mappings and $h = (\zeta, \Psi) : (U, A) \to (W, A^*)$ be a vague soft mapping, where $A \subseteq E$ and $A^* \subseteq E^*$ be the fixed parameter sets. Then h is said to be vague soft continous if the inverse image under h of any vague soft open set $\left(\hat{G}, A^*\right) \in \tau_2$ with the mapping $\hat{G} : A^* \to V(W)$ is a vague soft open set $\left(\hat{F}, A\right) \in \tau_1$ with $\hat{F} : A \to V(U)$; where $V(U)$ and $V(W)$ denotes set of all vague sets over U and W respectively. i.e., $h^{-1}\left(\hat{G}, A^*\right) \in \tau_1$ whenever $\left(\hat{G}, A^*\right) \in \tau_2$

Example 3.6. Let $(U, E, \tau_1)_A$ and $(W, E^*, \tau_2)_{A^*}$ be two vague soft topological spaces with $U = \{p, q, r\}$ and $W = \{m, n, d\}$, $A = \{e_1, e_2, e_3\} \subseteq E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A^* = \left\{e_2^{'}, e_3^{'}, e_4^{'}\right\} \subseteq E^* = \left\{e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, e_5^{'}, e_6^{'}\right\}$ and $(\overset{\rtimes}{U}$, A)and $(\overset{\rtimes}{W}, A^*)$ be vague soft classes over U and W with fixed parameter sets A and A^* respectively. Let $\zeta : U \to W$ and $\Psi : A \to A^*$ be mappings defined as $\zeta(p) = d, \zeta(q) = m, \zeta(r) = n$ and $\Psi(e_1) = e_3^{'}, \Psi(e_2) = e_4^{'}, \Psi(e_3) = e_2^{'}$
Let $\tau_1 = \left\{\left(\widehat{\Phi}, A\right), \left(\widehat{F}_1, A\right), \left(\widehat{F}_2, A\right), \left(\widehat{F}_3, A\right), \left(\widehat{F}_4, A\right), \left(\widehat{U}, A\right)\right\}$ be a vague soft topology.

$$(\widehat{F}_1, A) = \left\{ \begin{array}{l} \left(e_1, \left(\left\langle \frac{[0.2, 0.7]}{p}, \frac{[0.6, 0.7]}{q}, \frac{[0.1, 0.8]}{r} \right\rangle\right)\right) \\ \left(e_2, \left(\left\langle \frac{[0.6, 0.7]}{p}, \frac{[0.1, 0.7]}{q}, \frac{[0.3, 0.5]}{r} \right\rangle\right)\right) \\ \left(e_3, \left(\left\langle \frac{[0.3, 0.4]}{p}, \frac{[0.1, 0.8]}{q}, \frac{[0.2, 0.8]}{r} \right\rangle\right)\right) \end{array} \right\} \tag{3.13}$$

$$(\widehat{F}_2, A) = \left\{ \begin{array}{l} \left(e_1, \left(\left\langle \frac{[0.2, 0.5]}{p}, \frac{[0.5, 0.7]}{q}, \frac{[0.1, 0.7]}{r} \right\rangle\right)\right) \\ \left(e_2, \left(\left\langle \frac{[0.5, 0.7]}{p}, \frac{[0.1, 0.6]}{q}, \frac{[0.2, 0.5]}{r} \right\rangle\right)\right) \\ \left(e_3, \left(\left\langle \frac{[0.2, 0.4]}{p}, \frac{[0.1, 0.6]}{q}, \frac{[0.1, 0.3]}{r} \right\rangle\right)\right) \end{array} \right\} \tag{3.14}$$

$$(\widehat{F}_3, A) = \left\{ \begin{array}{l} \left(e_1, \left(\left\langle \frac{[0.3, 0.5]}{p}, \frac{[0.5, 0.9]}{q}, \frac{[0.3, 0.7]}{r} \right\rangle\right)\right) \\ \left(e_2, \left(\left\langle \frac{[0.5, 0.8]}{p}, \frac{[0.6, 0.6]}{q}, \frac{[0.2, 0.9]}{r} \right\rangle\right)\right) \\ \left(e_3, \left(\left\langle \frac{[0.2, 0.7]}{p}, \frac{[0.2, 0.6]}{q}, \frac{[0.1, 0.6]}{r} \right\rangle\right)\right) \end{array} \right\} \tag{3.15}$$

$$(\widehat{F}_4, A) = \left\{ \begin{array}{l} \left(e_1, \left(\left\langle \frac{[0.3, 0.7]}{p}, \frac{[0.6, 0.9]}{q}, \frac{[0.3, 0.8]}{r} \right\rangle\right)\right) \\ \left(e_2, \left(\left\langle \frac{[0.6, 0.8]}{p}, \frac{[0.6, 0.7]}{q}, \frac{[0.3, 0.9]}{r} \right\rangle\right)\right) \\ \left(e_3, \left(\left\langle \frac{[0.3, 0.7]}{p}, \frac{[0.2, 0.8]}{q}, \frac{[0.2, 0.6]}{r} \right\rangle\right)\right) \end{array} \right\} \tag{3.16}$$

Let $\tau_2 = \left\{\left(\widehat{\Phi}, A^*\right), \left(\widehat{G}_1, A^*\right), \left(\widehat{G}_2, A^*\right), \left(\widehat{G}_3, A^*\right), \left(\widehat{G}_4, A^*\right), \left(\widehat{U}, A^*\right)\right\}$ be a vague soft topology.

$$\left(\widehat{G}_1, A^*\right) = \left\{ \begin{array}{l} \left(e_2^{'}, \left(\left\langle \frac{[0.2,0.7]}{m}, \frac{[0.6,0.7]}{n}, \frac{[0.1,0.8]}{d} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.6,0.7]}{m}, \frac{[0.1,0.7]}{n}, \frac{[0.3,0.5]}{d} \right\rangle\right)\right) \\ \left(e_4^{'}, \left(\left\langle \frac{[0.3,0.4]}{m}, \frac{[0.1,0.8]}{n}, \frac{[0.2,0.8]}{d} \right\rangle\right)\right) \end{array} \right\} \tag{3.17}$$

$$\left(\widehat{G}_2, A^*\right) = \left\{ \begin{array}{l} \left(e_2^{'}, \left(\left\langle \frac{[0.2,0.5]}{m}, \frac{[0.5,0.7]}{n}, \frac{[0.1,0.7]}{d} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.5,0.7]}{m}, \frac{[0.1,0.6]}{n}, \frac{[0.2,0.5]}{d} \right\rangle\right)\right) \\ \left(e_4^{'}, \left(\left\langle \frac{[0.2,0.4]}{m}, \frac{[0.1,0.6]}{n}, \frac{[0.1,0.3]}{d} \right\rangle\right)\right) \end{array} \right\} \tag{3.18}$$

$$\left(\widehat{G}_3, A^*\right) = \left\{ \begin{array}{l} \left(e_2^{'}, \left(\left\langle \frac{[0.3,0.5]}{m}, \frac{[0.5,0.9]}{n}, \frac{[0.3,0.7]}{d} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.5,0.8]}{m}, \frac{[0.6,0.6]}{n}, \frac{[0.2,0.9]}{d} \right\rangle\right)\right) \\ \left(e_4^{'}, \left(\left\langle \frac{[0.2,0.7]}{m}, \frac{[0.2,0.6]}{n}, \frac{[0.1,0.6]}{d} \right\rangle\right)\right) \end{array} \right\} \tag{3.19}$$

$$\left(\widehat{G}_4, A^*\right) = \left\{ \begin{array}{l} \left(e_2^{'}, \left(\left\langle \frac{[0.3,0.7]}{m}, \frac{[0.6,0.9]}{n}, \frac{[0.3,0.8]}{d} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.6,0.8]}{m}, \frac{[0.6,0.7]}{n}, \frac{[0.3,0.9]}{d} \right\rangle\right)\right) \\ \left(e_4^{'}, \left(\left\langle \frac{[0.3,0.7]}{m}, \frac{[0.2,0.8]}{n}, \frac{[0.2,0.6]}{d} \right\rangle\right)\right) \end{array} \right\} \tag{3.20}$$

$g^{-1}\left(\widehat{\phi}, A^*\right) = \left(\widehat{\phi}, A\right), g^{-1}\left(\widehat{G_1}, A^*\right) = \left(\widehat{F_1}, A\right), g^{-1}\left(\widehat{G_2}, A^*\right) = \left(\widehat{F_3}, A\right),$
$g^{-1}\left(\widehat{G_3}, A^*\right) = \left(\widehat{F_2}, A\right), g^{-1}\left(\widehat{G_4}, A^*\right) = \left(\widehat{F_4}, A\right), g^{-1}\left(\widehat{U}, A^*\right) = \left(\widehat{U}, A\right).$
It is obvious that $g = (\zeta, \Psi) : (U, A) \to (W, A^*)$ is vague soft continous.

Definition 3.7. (Vague Binary Soft Continuous Mapping) Let $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ and $(W_1, W_2, E^*, \ddot{\sigma}_\Delta)_{A^*}$ be two VBSTS's over binary universe's (U_1, U_2) and (W_1, W_2) respectively, where $A \subseteq E$, $A^* \subseteq E^*$. Let $\zeta_1 : U_1 \to W_1$, $\zeta_2 : U_2 \to W_2$ and $\Psi : A \to A^*$ be mappings. Then vague binary soft mapping $g = ((\zeta_1, \zeta_2), \Psi) : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (W_1, W_2, E^*, \ddot{\sigma}_\Delta)_{A^*}$ is said to be vague binary soft continous if the inverse image under g of any vague binary soft open set $\left(\ddot{H}, A^*\right) \in \ddot{\sigma}_\Delta$ with $H : A^* \to V(W_1) \times V(W_2)$, there exists some vague binary soft open set $\left(\ddot{G}, A\right) \in \ddot{\tau}_\Delta$ with $G : A \to V(U_1) \times V(U_2)$

Example 3.8. Let $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ and $(W_1, W_2, E^*, \ddot{\sigma}_\Delta)_{A^*}$ be two VBSTS's.
Let $(U_1 = \{p, q, r\}, U_2 = \{x, y\})$ be a binary universe with a fixed parameter set $A \subseteq E$ where $\{e_2, e_4\} \subseteq \{e_1, e_2, e_3, e_4, e_5\}$. Let $(W_1 = \{m, n, d\}, W_2 = \{k, l\})$ be another binary

universe with fixed parameter set $A^* \subseteq E^*$ where $\left\{e_1^{'}, e_3^{'}\right\} \subseteq \left\{e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, e_5^{'}, e_6^{'}\right\}$. Let $\zeta_1 : U_1 \to W_1$ is defined as $\zeta_1(p) = d, \zeta_1(q) = n, \zeta_1(r) = m, \zeta_2 : U_2 \to W_2$ is defined as $\zeta_2(x) = l, \zeta_2(y) = k$ and $\Psi : E \to E^*$ is defined as $\Psi(e_2) = e_3^{'}, \Psi(e_4) = e_1^{'}$

Let $\ddot{\tau}_\Delta = \left\{\left(\ddot{\Phi}, A\right), \left(\ddot{F}_1, A\right), \left(\ddot{F}_2, A\right), \left(\ddot{F}_3, A\right), \left(\ddot{F}_4, A\right), \left(\ddot{U}, A\right)\right\}$ be a VBST,

$$(\widehat{F}_1, A) = \left\{\begin{array}{l} \left(e_2, \left(\left\langle \frac{[0.3,0.4]}{p}, \frac{[0.1,0.5]}{q}, \frac{[0.6,0.7]}{r}\right\rangle, \left\langle \frac{[0.3,0.6]}{x}, \frac{[0.7,0.8]}{y}\right\rangle\right)\right), \\ \left(e_4, \left(\left\langle \frac{[0.1,0.8]}{p}, \frac{[0.2,0.4]}{q}, \frac{[0.1,0.7]}{r}\right\rangle, \left\langle \frac{[0.2,0.7]}{x}, \frac{[0.5,0.7]}{y}\right\rangle\right)\right) \end{array}\right\} \tag{3.21}$$

$$(\widehat{F}_2, A) = \left\{\begin{array}{l} \left(e_2, \left(\left\langle \frac{[0.4,0.5]}{p}, \frac{[0.1,0.3]}{q}, \frac{[0.2,0.6]}{r}\right\rangle, \left\langle \frac{[0.1,0.7]}{x}, \frac{[0.2,0.9]}{y}\right\rangle\right)\right), \\ \left(e_4, \left(\left\langle \frac{[0.1,0.8]}{p}, \frac{[0.2,0.4]}{q}, \frac{[0.1,0.7]}{r}\right\rangle, \left\langle \frac{[0.2,0.7]}{x}, \frac{[0.5,0.7]}{y}\right\rangle\right)\right) \end{array}\right\} \tag{3.22}$$

$$(\widehat{F}_3, A) = \left\{\begin{array}{l} \left(e_2, \left(\left\langle \frac{[0.3,0.4]}{p}, \frac{[0.1,0.3]}{q}, \frac{[0.2,0.6]}{r}\right\rangle, \left\langle \frac{[0.1,0.6]}{x}, \frac{[0.2,0.8]}{y}\right\rangle\right)\right), \\ \left(e_4, \left(\left\langle \frac{[0.1,0.6]}{p}, \frac{[0.2,0.3]}{q}, \frac{[0.1,0.6]}{r}\right\rangle, \left\langle \frac{[0.2,0.6]}{x}, \frac{[0.5,0.7]}{y}\right\rangle\right)\right) \end{array}\right\} \tag{3.23}$$

$$(\widehat{F}_4, A) = \left\{\begin{array}{l} \left(e_2, \left(\left\langle \frac{[0.4,0.5]}{p}, \frac{[0.1,0.5]}{q}, \frac{[0.6,0.7]}{r}\right\rangle, \left\langle \frac{[0.3,0.7]}{x}, \frac{[0.7,0.9]}{y}\right\rangle\right)\right), \\ \left(e_4, \left(\left\langle \frac{[0.1,0.8]}{p}, \frac{[0.2,0.4]}{q}, \frac{[0.4,0.7]}{r}\right\rangle, \left\langle \frac{[0.5,0.7]}{x}, \frac{[0.6,0.7]}{y}\right\rangle\right)\right) \end{array}\right\} \tag{3.24}$$

Let $\ddot{\sigma}_\Delta = \left\{\left(\ddot{\Phi}, A^*\right), \left(\ddot{G}_1, A^*\right), \left(\ddot{G}_2, A^*\right), \left(\ddot{G}_3, A^*\right), \left(\ddot{G}_4, A^*\right), \left(\ddot{U}, A^*\right)\right\}$ be a VBST,

$$\left(\ddot{G}_1, A^*\right) = \left\{\begin{array}{l} \left(e_1^{'}, \left(\left\langle \frac{[0.1,0.7]}{m}, \frac{[0.2,0.4]}{n}, \frac{[0.1,0.8]}{d}\right\rangle, \left\langle \frac{[0.5,0.7]}{k}, \frac{[0.2,0.7]}{l}\right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.2,0.6]}{m}, \frac{[0.1,0.3]}{n}, \frac{[0.4,0.5]}{d}\right\rangle, \left\langle \frac{[0.2,0.9]}{k}, \frac{[0.1,0.7]}{l}\right\rangle\right)\right) \end{array}\right\} \tag{3.25}$$

$$\left(\ddot{G}_2, A^*\right) = \left\{ \begin{array}{l} \left(e_1^{'}, \left(\left\langle \frac{[0.4, 0.6]}{m}, \frac{[0.2, 0.3]}{n}, \frac{[0.1, 0.6]}{d} \right\rangle, \left\langle \frac{[0.6, 0.7]}{k}, \frac{[0.5, 0.6]}{l} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.6, 0.7]}{m}, \frac{[0.1, 0.5]}{n}, \frac{[0.3, 0.4]}{d} \right\rangle, \left\langle \frac{[0.7, 0.8]}{k}, \frac{[0.3, 0.6]}{l} \right\rangle\right)\right) \end{array} \right\} \tag{3.26}$$

$$\left(\ddot{G}_3, A^*\right) = \left\{ \begin{array}{l} \left(e_1^{'}, \left(\left\langle \frac{[0.1, 0.6]}{m}, \frac{[0.2, 0.3]}{n}, \frac{[0.1, 0.6]}{d} \right\rangle, \left\langle \frac{[0.5, 0.7]}{k}, \frac{[0.2, 0.6]}{l} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.2, 0.6]}{m}, \frac{[0.1, 0.3]}{n}, \frac{[0.3, 0.4]}{d} \right\rangle, \left\langle \frac{[0.2, 0.8]}{k}, \frac{[0.1, 0.6]}{l} \right\rangle\right)\right) \end{array} \right\} \tag{3.27}$$

$$\left(\ddot{G}_4, A^*\right) = \left\{ \begin{array}{l} \left(e_1^{'}, \left(\left\langle \frac{[0.4, 0.7]}{m}, \frac{[0.2, 0.4]}{n}, \frac{[0.1, 0.8]}{d} \right\rangle, \left\langle \frac{[0.6, 0.7]}{k}, \frac{[0.5, 0.7]}{l} \right\rangle\right)\right) \\ \left(e_3^{'}, \left(\left\langle \frac{[0.6, 0.7]}{m}, \frac{[0.1, 0.5]}{n}, \frac{[0.4, 0.5]}{d} \right\rangle, \left\langle \frac{[0.7, 0.9]}{k}, \frac{[0.3, 0.7]}{l} \right\rangle\right)\right) \end{array} \right\} \tag{3.28}$$

$h^{-1}\left(\ddot{\phi}, A^*\right) = \left(\ddot{\phi}, A\right), h^{-1}\left(\ddot{G}_1, A^*\right) = \left(\ddot{F}_2, A\right),$
$h^{-1}\left(\ddot{G}_2, A^*\right) = \left(\ddot{F}_1, A\right), h^{-1}\left(\ddot{G}_3, A^*\right) = \left(\ddot{F}_3, A\right)$
$h^{-1}\left(\ddot{G}_4, A^*\right) = \left(\ddot{F}_4, A\right), h^{-1}\left(\ddot{U}, A^*\right) = \left(\ddot{U}, A\right)$

So h is vague binary soft continous mapping.

Definition 3.9. (Vague Binary Soft Open Map) Let $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ and $(V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$ be two VBSTS's. A vague binary soft map $g : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$ is called a vague binary soft open map if $g\left(\ddot{k}, A\right)$ is vague binary soft open in $(V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$ for each vague binary soft open set $\left(\ddot{K}, A\right)$ in $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$.

Theorem 3.10. *Let $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ and $(V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$ be two VBSTS's. $g : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$ be a mapping. Then the following conditions are equivalent:*
(1) $g : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$ is a vague binary soft continous mapping
(2) $g^{-1}\left(\ddot{H}, A\right)$ is VBSCS in $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ whenever $\left(\ddot{H}, A\right)$ is VBSCS in $(V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$
***Proof** (1) Let g be a vague binary soft continous mapping. $\left(\ddot{H}, A\right)$ be any VBSCS of $(V_1, V_2, E, \ddot{\sigma}_\Delta)_{A^*}$. Then $\left(\ddot{H}, A\right)^c$ is VBSOS in $(V_1, V_2, E, \ddot{\sigma}_\Delta)_B$. Since g is vague binary soft continous, $g^{-1}\left(\ddot{H}, A\right)^c$ is VBSOS in $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$. Therefore $g^{-1}\left(\ddot{H}, A\right)$ is VBSCS in $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$*

(2) Let $\left(\ddot{H}, A\right)$ *be a VBSCS of* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_B$*. Then using assumption* $g^{-1}\left(\ddot{H}, A\right)$ *is VBSCS in* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$*. Therefore g is vague binary soft continous mapping.*

Theorem 3.11. *A vague binary soft mapping* $g : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_B$ *is vague binary soft continous if and only if* $g\overline{\left(\ddot{H}, A\right)}\ddot{\subset}\overline{g\left(\ddot{H}, A\right)}$ *for every* $\left(\ddot{H}, A\right) \ddot{\subset} (U_1, U_2, E, \ddot{\tau}_\Delta)_A$

Proof *Let g be a vague binary soft continous and* $\left(\ddot{H}, A\right)$ *be any VBSSS of* $\left(\overset{\bowtie}{U}, A\right)$*. Let* $g\left(\ddot{H}, A\right) \ddot{=} \left(\ddot{N}, A\right)$*. Therefore* $\left(\ddot{N}, A\right) \ddot{\subset} \left(\overset{\bowtie}{V}, A\right)$*. But* $\overline{\left(\ddot{N}, A\right)}$ *is VBSCS of* $\left(\overset{\bowtie}{U}, A\right) \Rightarrow$ $g^{-1}\overline{g\left(\ddot{H}, A\right)}$ *is a VBSCS of* $\left(\overset{\bowtie}{U}, A\right) \Rightarrow \overline{g^{-1}\overline{g\left(\ddot{H}, A\right)}}\ddot{=}g^{-1}\overline{g\left(\ddot{H}, A\right)}$.

Now $g\left(\ddot{H}, A\right) \ddot{\subseteq}\overline{g\left(\ddot{H}, A\right)} \Rightarrow \left(\ddot{H}, A\right) \ddot{\subseteq}g^{-1}\overline{g\left(\ddot{H}, A\right)} \Rightarrow \overline{\left(\ddot{H}, A\right)}\ddot{\subseteq}\overline{g^{-1}\overline{g\left(\ddot{H}, A\right)}}\ddot{=}g^{-1}\overline{g\left(\ddot{H}, A\right)} \Rightarrow$ $g\overline{\left(\ddot{H}, A\right)}\ddot{\subseteq}\overline{g\left(\ddot{H}, A\right)}$*. Conversely, let* $g : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *is vague binary soft mapping with* $g\left(\overline{\ddot{H}, A}\right) \ddot{\subseteq}\overline{g\left(\ddot{H}, A\right)}$ *for any VBSS* $\left(\ddot{H}, A\right)$ *of* $\left(\overset{\bowtie}{U}, A\right)$*.*

Theorem 3.12. *Let* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ & $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *be VBSTS's.* $f : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$*. Then the following are equivalent:*
(i) f is vague binary soft continous
(ii) For every VBSSS $\left(\ddot{J}, A\right)$ *of* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$*.* $f\overline{\left(\ddot{J}, A\right)}\ddot{\subset}\overline{f\left(\ddot{J}, A\right)}$
(iii) For every VBSCS $\left(\ddot{T}, A\right)$ *of* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$*, the VBSS* $f^{-1}\left(\ddot{T}, A\right)$ *is VBSCS in* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$
(iv) For each $\ddot{F}_e \in \left(\overset{\bowtie}{U}, A\right)$ *and each* $\ddot{vbs}$ *nbd* $\left(\ddot{T}, A\right)$ *of* $f\left(\ddot{F}_e\right)$ *there is a* $\ddot{vbs}$ *nbd* $\left(\ddot{J}, A\right)$ *of* $\ddot{F}_e$ *such that* $f\left(\ddot{J}, A\right) \ddot{\subset} \left(\ddot{T}, A\right)$*. In this case f is vague binary soft continous at the point* $\ddot{F}_e \in \left(\overset{\bowtie}{U}, A\right)$

Proof *Method of proof is (i)* $\Longrightarrow$ *(ii)*$\Longrightarrow$ *(i) and (i)*$\Longrightarrow$ *(iv)*$\Longrightarrow$ *(i).*
From definition 3.9 and Theorem 3.10, (i)$\Longrightarrow$ *(ii) and (iii)*$\Longrightarrow$ *(i)are clear. To prove (ii)*$\Rightarrow$ *(iii). Let* $\left(\ddot{J}, A\right)$ *be VBSCS in* $\left(\overset{\bowtie}{V}, A\right)$ *and let* $\left(\ddot{T}, A\right) \ddot{=}f^{-1}\left(\ddot{J}, A\right)$*. To prove* $\left(\ddot{T}, A\right)$ *is VBSCS* $\left(\overset{\bowtie}{U}, A\right)$*. By elementary set theory,* $f\left(\ddot{T}, A\right) = f\left(f^{-1}\left(\ddot{J}, A\right)\right) \subset \left(\ddot{V}, A\right)$*. Hence if* $\ddot{F}_e \in \overline{\left(\ddot{T}, A\right)}$*,* $f\left(\ddot{F}_e\right) \in f\overline{\left(\ddot{T}, A\right)}\ddot{\subset}\overline{f\left(\ddot{T}, A\right)}\ddot{\subset}\overline{\left(\ddot{J}, A\right)}\ddot{=}\left(\ddot{J}, A\right)$*. Therefore* $\ddot{F}_e \in f^{-1}\left(\ddot{J}, A\right) \ddot{=} \left(\ddot{T}, A\right)$*. Thus* $\overline{\left(\ddot{T}, A\right)}\ddot{\subset}\left(\ddot{T}, A\right)$ *and* $\left(\ddot{T}, A\right)\ddot{\subset}\overline{\left(\ddot{T}, A\right)}$*. Combining* $\overline{\left(\ddot{T}, A\right)}\ddot{=}\left(\ddot{T}, A\right) \Rightarrow \overline{\left(\ddot{T}, A\right)}$ *is VBSCS in* $\left(\overset{\bowtie}{U}, A\right) \Rightarrow f^{-1}\left(\ddot{J}, A\right)$ *is VBSCS in* $\left(\overset{\bowtie}{U}, A\right)$ *as required. To prove (i)*$\Rightarrow$ *(iv), let* $\ddot{F}_e \in \left(\overset{\bowtie}{U}, A\right)$ *and* $\left(\ddot{T}, A\right)$ *be a* $\ddot{vbs}$ *nbd of* $f\left(\ddot{F}_e\right)$*. Then the set* $\left(\ddot{J}, A\right) \ddot{=}f^{-1}\left(\ddot{T}, A\right)$ *is a* $\ddot{vbs}$ *nbd of* $\ddot{F}_e$ *such that*

$f\left(\ddot{J},A\right)\ddot{\subset}\left(\ddot{T},A\right)$. To prove (iv) $\Rightarrow$ (i), let $\left(\ddot{T},A\right)$ be a VBSOS of $\left(\overset{\bowtie}{V},A\right)$. Let $\ddot{F}_e$ be a VBSP of $f^{-1}\left(\ddot{T},A\right) \Rightarrow f(\ddot{F}_e) \in \left(\ddot{T},A\right)$. Therefore by hypotheses there exists a $v\ddot{b}s$ nbd $\left(\ddot{R},A\right)$ of $\ddot{F}_e$ such that $f(\ddot{R},A) \subset \left(\ddot{T},A\right)$. It follows that $f^{-1}\left(\ddot{T},A\right)$ can be written as the vague binary soft union of VBSOS's , hence it is VBSOS $\Rightarrow f$ is a vague binary soft continuous mapping.

Theorem 3.13. *Let $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ and $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ and $\left(W_1,W_2,E,\ddot{\vartheta}_\Delta\right)_A$ be three VBSTS's. If $g:(U_1,U_2,E,\ddot{\tau}_\Delta)\rightarrow(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ and $h:(V_1,V_2,E,\ddot{\sigma}_\Delta)_A \rightarrow \left(W_1,W_2,E,\ddot{\vartheta}_\Delta\right)_A$ are both vague binary soft continous then their composition $h\circ g:(U_1,U_2,E,\ddot{\tau}_\Delta)_A \rightarrow \left(W_1,W_2,E,\ddot{\vartheta}_\Delta\right)_A$ are both vague binary soft continuous*

Proof *Let $\left(\ddot{V},A\right)$ be a VBSS in $\left(\overset{\bowtie}{V},A\right)$. Then $(h\circ g)^{-1}\left(\ddot{V},A\right)\ddot{=}g^{-1}\left(h^{-1}\left(\ddot{V},A\right)\right)$. Since g is vague binary soft continous $g^{-1}\left(h^{-1}\left(\ddot{V},A\right)\right)$ is VBSOS in $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$*

Theorem 3.14. *A vague binary soft mapping $k:(U_1,U_2,E,\ddot{\tau}_\Delta)_A \rightarrow (V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ is vague binary soft continuous if and only if $k^{-1}\left[v\ddot{b}s\ int\left(\ddot{P},A\right)\ddot{\subset}v\ddot{b}s\ int\left[k^{-1}\left(\ddot{P},A\right)\right]\right]$ for every $\left(\ddot{P},A\right)\ddot{\subset}\left(\overset{\bowtie}{V},A\right)$*

Proof *Let $\left(\ddot{P},A\right)$ be a VBSS in $\left(\overset{\bowtie}{V},A\right)$ and also let k be vague soft continuous mapping. Clearly $v\ddot{b}s\ int\left(\ddot{P},A\right)$ is VBSOS of $\left(\overset{\bowtie}{U},A\right)$.*
But $v\ddot{b}s\ int\left(\ddot{P},A\right)\ddot{\subset}\left(\ddot{P},A\right) \Longrightarrow k^{-1}\left[\left(\ddot{P},A\right)\right]\ddot{\subset}k^{-1}\left(\ddot{P},A\right)$
$\Longrightarrow$ $v\ddot{b}s\ int\left[k^{-1}\left[v\ddot{b}s\ int\left(\ddot{P},A\right)\right]\right]\ddot{=}v\ddot{b}s\ int\left[k^{-1}\left(\ddot{P},A\right)\right]\ddot{=}k^{-1}\left(\ddot{P},A\right)$. Conversely, let $\left(\ddot{P},A\right)$ be an arbitrary VBSOS in $\left(\overset{\bowtie}{V},A\right)$ so that $v\ddot{b}s\ int\left(\ddot{P},A\right)\ddot{=}\left(\ddot{P},A\right)$. By hypothesis, $k^{-1}\left[v\ddot{b}s\ int\left(\ddot{P},A\right)\right]\ddot{\subset}v\ddot{b}s\ int\left[k^{-1}\left(\ddot{P},A\right)\right]$
$\Longrightarrow$ $k^{-1}\left(\ddot{P},A\right)\ddot{\subset}v\ddot{b}s\ int\left[k^{-1}\left(\ddot{P},A\right)\right]$. But $v\ddot{b}s\ int$ $\left[k^{-1}\left(\ddot{P},A\right)\right]\ddot{\subset}k^{-1}\left(\ddot{P},A\right)$ always $\Longrightarrow v\ddot{b}s\ int\left[k^{-1}\left(\ddot{P},A\right)\right]\ddot{=}k^{-1}\left(\ddot{P},A\right)\Longrightarrow k^{-1}\left(\ddot{P},A\right)$ is VBSOS of $\left(\overset{\bowtie}{U},A\right)$
$\Longrightarrow$. k is a vague binary soft continuous mapping

Theorem 3.15. *If f and g are vague binary soft continuous functions on a VBSTS $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ with values in a vague binary soft hausdorff space $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ then $\left\{\ddot{G}_e\in(U_1,U_2,E,\ddot{\tau}_\Delta)_A : f\left(\ddot{G}_e\right)=g\left(\ddot{G}_e\right)\right\}$ is a VBSCS*

Proof *Let $T=\left\{\ddot{G}_e\in(U_1,U_2,E,\ddot{\tau}_\Delta)_A : f\left(\ddot{G}_e\right)=g\left(\ddot{G}_e\right)\right\}$. To prove T is vague binary soft closed, it is enough to prove that $T^c=\left\{\ddot{G}_e\in(U_1,U_2,E,\ddot{\tau}_\Delta)_A : f\left(\ddot{G}_e\right)\neq g\left(\ddot{G}_e\right)\right\}$ is VBSOS. Let $\ddot{R}_e$ be a VBSP of T^c. Then $f\left(\ddot{R}_e\right)\neq g\left(\ddot{R}_e\right)$ with $f\left(\ddot{R}_e\right)$.*

$g\left(\ddot{R}_e\right) \in (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$. *Since* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *is vague binary soft hausdorff, there exists VBSOS's* $\left(\ddot{M}, A\right)$ *and* $\left(\ddot{N}, A\right) \in (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *such that* $f\left(\ddot{R}_e\right) \in \left(\ddot{M}, A\right), g\left(\ddot{R}_e\right) \in \left(\ddot{N}, A\right)$ *and* $\left(\ddot{M}, A\right) \ddot{\cap} \left(\ddot{N}, A\right) \ddot{=} \left(\ddot{\phi}, A\right)$. *Since f and g are vague binary soft continuous,* f^{-1}, g^{-1} *are VBSOS's in* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *such that* $\ddot{R}_e \in f^{-1}\left(\ddot{M}, A\right)$ *and* $\ddot{R}_e \in g^{-1}\left(\ddot{N}, A\right)$ *and so* $\ddot{R}_e \in \left(\ddot{M}, A\right) \ddot{\cap} \left(\ddot{N}, A\right) \ddot{=} \left(\ddot{I}, A\right)$ *also. To prove that* $\left(\ddot{I}, A\right) \subset T^c$. *Assume* $\left(\ddot{I}, A\right) \not\subseteq T^c$. *Then there exists at least one point in* $\left(\ddot{I}, A\right)$, *say* $\ddot{D}_e$ *such that* $\ddot{D}_e \neq T^c$ *with respect to* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$. *Now if* $\ddot{D}_e \in \left(\ddot{I}, A\right) \implies \ddot{D}_e \in f^{-1}\left(\ddot{M}, A\right)$ *and* $\ddot{D}_e \in g^{-1}\left(\ddot{N}, A\right) \implies f\left(\ddot{D}_e\right) \in \left(\ddot{M}, A\right)$ *and* $g(\ddot{D}_e) \in (\ddot{N}, A)$. *Also* $\ddot{D}_e \notin T^c \implies \ddot{D}_e \in T \implies f(\ddot{D}_e) \ddot{=} g\left(\ddot{D}_e\right)$ *so* $f\left(\ddot{D}_e\right) \in \left(\ddot{M}, A\right) \ddot{\cap} \left(\ddot{N}, A\right)$ *is a contradiction. Thus* $\ddot{D}_e \in \left(\ddot{I}, A\right) \ddot{\subset} T^c$. *Since* $\ddot{D}_e$ *is taken as arbitrary* T^c *contains a* $\ddot{vbs}$ *nbd of each of its points. Therefore* T^c *is VBSOS* $\implies$ *T is VBSCS* $\implies \left\{\ddot{G}_e \in (U_1, U_2, E, \ddot{\tau}_\Delta)_A : f\left(\ddot{G}_e\right) = g\left(\ddot{G}_e\right)\right\}$ *is VBSCS.*

Theorem 3.16. ***(Pasting Theorem in VBSTS's)*** *Let* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *and* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *be two VBSTS's with* $\left(\overset{\bowtie}{U}, A\right) \ddot{=} \left(\ddot{F}, A\right) \ddot{\cup} \left(\ddot{G}, A\right)$ *where* $\left(\ddot{F}, A\right)$ *and* $\left(\ddot{G}, A\right)$ *are VBSCS in* $\left(\overset{\bowtie}{U}, A\right)$. *Let* $f : \left(\ddot{F}, A\right) \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *and* $g : \left(\ddot{G}, A\right) \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *be vague binary soft continous. If* $f\left(\ddot{N}_e\right) = g\left(\ddot{N}_e\right)$ *for every* $\ddot{N}_e \in \left(\ddot{N}, A\right) \ddot{\cap} \left(\ddot{G}, A\right)$ *then f and g combine to give a vague binary soft continous function* $h : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *defined by setting* $h\left(\ddot{F}_e\right) = f\left(\ddot{N}_e\right)$ *if* $\ddot{N}_e \in \left(\ddot{F}, A\right)$ *and* $h\left(\ddot{N}_e\right) = g\left(\ddot{N}_e\right)$ *if* $\ddot{N}_e \in \left(\ddot{G}, A\right)$

Proof *Let* $\left(\ddot{C}, A\right)$ *be a VBSCS in* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$, *f is vague binary soft continuous* $\implies f^{-1}\left(\ddot{C}, A\right)$ *is VBSCS in* $\left(\ddot{F}, A\right)$ *in turn which is VBSCS in* $\left(\overset{\bowtie}{U}, A\right)$. *g is vague binary soft continuous* $\implies g^{-1}\left(\ddot{C}, A\right)$ *is VBSCS* $\left(\ddot{G}, A\right)$, *in turn which is VBSCS in* $\left(\overset{\bowtie}{U}, A\right)$. *Hence their vague binary soft union* $f^{-1}\left(\ddot{C}, A\right) \ddot{\cup} g^{-1}\left(\ddot{C}, A\right)$ *is VBSCS in* $(\overset{\bowtie}{U}, A)$. *Let* $h^{-1}\left(\ddot{C}, A\right) \ddot{=} f^{-1}\left(\ddot{C}, A\right) \ddot{\cup} g^{-1}\left(\ddot{C}, A\right) \implies h : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *is vague binary soft continuous.*

Theorem 3.17. *If* $\left(\ddot{D}, A\right)$ *is a VBSCSS of a VBSTS* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *and if any vague binary soft continuous extension* $k : \left(\ddot{D}, A\right) \to [-1, 1]$ *has a vague binary soft continuous extension to* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *then* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *is vague binary soft normal space*

Proof *Let* $\left(\ddot{R}, A\right)$ *and* $\left(\ddot{T}, A\right)$ *be disjoint VBSCS's in a VBSTS* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$.

Then $\left(\ddot{R},A\right)\ddot{\cup}\left(\ddot{T},A\right)$ *is a VBSCS in* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$. *Define a vague binary soft continous function* $k:\left(\ddot{R},A\right)\ddot{\cup}\left(\ddot{T},A\right)\to[-1,1]$ *by* $k\left(\ddot{M}_e\right)=1$ *if* $\ddot{M}_e\in\left(\ddot{T},A\right)$. *By hypothesis, k has a vague binary soft extension* $h:(U_1,U_2,E,\ddot{\tau}_\Delta)_A\to[-1,1]$. *Since k is a vague binary soft continuous, [-1, 0) and (0, 1]. Also* $\left(\ddot{R},A\right)\ddot{\subseteq}\left(\ddot{S},A\right)$, *VBSOS's in* $[-1,1]$. *Therefore* $\left(\ddot{S},A\right)=k^{-1}[-1,0)$ *and* $\left(\ddot{J},A\right)=k^{-1}(0,-1]$. *Also* $\left(\ddot{R},A\right)\ddot{\subseteq}\left(\ddot{S},A\right),\left(\ddot{T},A\right)\ddot{\subseteq}\left(\ddot{J},A\right)$, $\left(\ddot{S},A\right)\ddot{\cap}\left(\ddot{J},A\right)\ddot{=}\left(\ddot{\phi},A\right)$. *Hence* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ *is vague binary soft normal space.*

Theorem 3.18. *Homeomorphic image of a vague binary soft normal space is vague binary soft normal space*
Proof *Let* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ *and* $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ *be two VBSTS's and also let* $f:(U_1,U_2,E,\ddot{\tau}_\Delta)_A\to(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ *be a vague binary soft homeomorphism. Let* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ *be vague binary soft normal space. Let* $\left(\ddot{E},A\right)$ *and* $\left(\ddot{F},A\right)$ *be any pair of disjoint VBSCS's in* $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$. $f^{-1}\left(\ddot{E},A\right)$ *and* $f^{-1}\left(\ddot{F},A\right)$ *are VBSCS's in* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ *using vague binary soft continuity of f. Now* $f^{-1}\left(\ddot{E},A\right)\ddot{\cap}f^{-1}\left(\ddot{F},A\right)\ddot{=}f^{-1}\left[\left(\ddot{E},A\right)\ddot{\cap}\left(\ddot{F},A\right)\right]$
$\ddot{=}f^{-1}\left(\ddot{\phi},A\right)\ddot{=}\left(\ddot{\phi},A\right)$. *Since* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ *is vague binary soft normal space, there exists VBSOS's* $\left(\ddot{G},A\right)$ *and* $\left(\ddot{H},A\right)$ *such that* $f^{-1}\left(\ddot{E},A\right)\ddot{\subseteq}(\ddot{G},A)$,
$f^{-1}(\ddot{F},A)\ddot{\subseteq}\left(\ddot{H},A\right)$ *and* $\left(\ddot{G},A\right)\ddot{\cap}\left(\ddot{H},A\right)\ddot{=}\left(\ddot{\phi},A\right)$. *Thus* $f\left[f^{-1}\left(\ddot{F},A\right)\right]\ddot{\subseteq}f\left(\ddot{G},A\right)$, $f\left[f^{-1}\left(\ddot{F},A\right)\right]\ddot{\subseteq}f\left(\ddot{H},A\right)$ *and* $f\left[\left(\ddot{G},A\right)\ddot{\cap}\left(\ddot{H},A\right)\right]\ddot{=}\left(\ddot{\phi},A\right)$. *Using vague binary soft homeomorphism, f is vague binary soft open mapping* $\Longrightarrow f\left(\ddot{G},A\right)$ *and* $f\left(\ddot{H},A\right)$ *are VBSOS's in* $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$. *Therefore there exists disjoint VBSOS's* $f\left(\ddot{G},A\right)$ *and* $f\left(\ddot{H},A\right)$ *such that* $\left(\ddot{E},A\right)\ddot{\subseteq}f\left(\ddot{G},A\right),\left(\ddot{F},A\right)\ddot{\subseteq}f\left(\ddot{H},A\right)$. *Hence* $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ *is vague binary soft normal space.*

Theorem 3.19. *Let* $h:(U_1,U_2,E,\ddot{\tau}_\Delta)_A\to(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ *be a one-one, vague binary soft continous mapping with* $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ *is a vague binary soft hausdorff space and then* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$ *is also a vague binary soft hausdorff space*
Proof *Let* $\ddot{B}_e$ *and* $\ddot{K}_e$ *be two distinct VBSP's of* $(U_1,U_2,E,\ddot{\tau}_\Delta)_A$. *Since h is one-one,* $h\left(\ddot{B}_e\right)=\ddot{T}_e$ *and* $h\left(\ddot{K}_e\right)=\ddot{P}_e$ *with* $\ddot{T}_e\neq\ddot{P}_e$. *Using vague binary soft hausdorff property of* $(V_1,V_2,E,\ddot{\sigma}_\Delta)_A$ *there exists disjoint VBSOS's* $\left(\ddot{G},A\right)$ *and* $\left(\ddot{H},A\right)$ *such that* $\ddot{T}_e\in\left(\ddot{G},A\right)$ *and* $\ddot{P}_e\in\left(\ddot{H},A\right)$. *Therefore* $\ddot{B}_e\in h^{-1}\left(\ddot{G},A\right),\ddot{K}_e\in h^{-1}\left(\ddot{H},A\right)$. *Also* $\left(\ddot{G},A\right)\ddot{\cap}\left(\ddot{H},A\right)\ddot{=}\left(\ddot{\phi},A\right)\Longrightarrow h\left[(\ddot{G},A)\ddot{\cap}(\ddot{H},A)\right]\ddot{=}h\left(\ddot{\phi},A\right)\Longrightarrow h\left(\ddot{G},A\right)\ddot{\cap}h\left(\ddot{H},A\right)\ddot{=}\left(\ddot{\phi},A\right)$. *Therefore there exists distinct VBSP's* $\ddot{B}_e$ *and* $\ddot{K}_e$

contained in disjoint VBSOS's $(\ddot{G}, A)$ *and* $\left(\ddot{H}, A\right)$ *in* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$. *Hence* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *is also vague binary soft hausdorff space.*

Theorem 3.20. *The property of being a vague binary soft hausdorff space is preserved by bijective, vague binary soft open mappings and hence is a vague binary soft topological property*

Proof *Let* $k : (U_1, U_2, E, \ddot{\tau}_\Delta)_A \to (V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *be a bijective, vague binary soft open mapping where* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *is a vague binary soft hausdorff space. It will be established that* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *is also a vague binary soft hausdorff space. Let* $\ddot{P}_{e_1}, \ddot{P}_{e_2} \in (U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *are two VBSP's with* $\ddot{P}_{e_1} \neq \ddot{P}_{e_2}$. *Using vague binary soft hausdorff property of* $(U_1, U_2, E, \ddot{\tau}_\Delta)_A$ *there exists disjoint VBSOS's* $\left(\ddot{P}_1, A\right)$ *and* $\left(\ddot{P}_2, A\right)$ *with* $\ddot{P}_{e_1} \in \left(\ddot{P}_1, A\right)$ *and* $\ddot{P}_{e_2} \in \left(\ddot{P}_2, A\right)$. *Now using bijectiveness of* k *there exists VBSP's* $\ddot{T}_{e_1}$ *and* $\ddot{T}_{e_2}$ *such that* $k\left(\ddot{P}_{e_1}\right) = \ddot{T}_{e_1} \in k\left(\ddot{P}_1, A\right)$ *and* $k\left(\ddot{P}_{e_2}\right) = \ddot{T}_{e_2} \in k\left(\ddot{P}_2, A\right)$. *Also* $\left(\ddot{P}_1, A\right) \ddot{\cap} \left(\ddot{P}_2, A\right) \ddot{=} \left(\ddot{\phi}, A\right) \Longrightarrow k[\left(\ddot{P}_1, A\right) \ddot{\cap} \left(\ddot{P}_2, A\right)] \ddot{=} k\left[\left(\ddot{\phi}, A\right)\right] \Longrightarrow k\left[\left(\ddot{P}_1, A\right) \ddot{\cap} \left(\ddot{P}_2, A\right)\right] \ddot{=} \left(\ddot{\phi}, A\right)$. *Since* k *is vague binary soft open mapping,* $k\left(\ddot{P}_1, A\right)$ *and* $k\left(\ddot{P}_2, A\right)$ *are VBSOS's. Therefore* $k\left(\ddot{P}_1, A\right)$ *and* $k\left(\ddot{P}_2, A\right)$ *are disjoint VBSOS's in* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *containing* $\ddot{T}_{e_1}$ *and* $\ddot{T}_{e_2}$ *respectively, with* $\ddot{T}_{e_1} \neq \ddot{T}_{e_2}$. *Hence* $(V_1, V_2, E, \ddot{\sigma}_\Delta)_A$ *is a vague binary soft hausdorff space. So property of being a vague binary soft hausdorff space is preserved under bijective, vague binary soft open mapping* $\Longrightarrow$ *it is also preserved under vague binary soft homeomorphism* $\Longrightarrow$ *vague binary soft hausdorff property is a vague binary soft topological property.*

4. Main Results

Vague Binary Soft Continuity
Vague Soft Continuity

Pasting Theorem

5. Conclusion

Vague binary soft continuity is developed in this paper. Some of it properties are verified. Pasting Theorem holds hood for vague binary soft sets also. Topological properties for vague binary soft sets are found invariant under vague binary soft homeomorphisms in vague binary soft hausdorff spaces. Vague binary soft continuity can be further extended to find more useful results.

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