Poincare Journal of Analysis & Applications
Vol. 8, No. 1(I) (2021), Special issue, 41-50
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DOI: 10.33786/pjaa.2021.v08i01(i).005

CONNECTIVITY BETWEEN DOOR AND IDEAL SPACES VIA INTUITIONISTIC PERSPECTIVE

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(Dedicated to Professor Maximilian Ganster on the occasion of his retirement)

Date of Receiving	:	10. 08. 2020
Date of Revision	:	02. 09. 2020
Date of Acceptance	:	08. 09. 2020

Abstract. In this paper we present a new concept of intuitionistic door space. We prove that Intuitionistic α-open sets does not form intuitionistic topology. We first discuss several properties of intuitionistic door space. Next, we give some interesting characterizations of intuitionistic continuity. Finally, we obtain the connections between intuitionistic door space and intuitionistic ideal door space.

1. Introduction

The concept of intuitionistic fuzzy sets was introduced by Krassimir T. Atanassov [9]. The classical version of this concept was introduced by Coker [4]. He studied the topology on intuitionistic sets [5]. Several characterizations of door space were presented by Dontchev [8]. The submaximal intuitionistic topology was defined by Ozcelik and Narli [1].

In section 1, we studied door space, continuity, ideal in the context of intuitionistic topological space(ITS). In section 2, we have given preliminaries for ITS. In section 3, we have proved that intuitionistic α-open sets not necessarily form ITS. Intuitionistic continuous function in intuitionistic door space is given in section 4. We have given an example for the case which an intuitionistic door space is intuitionistic semi door space but not for the converse case. Intuitionistic continuous image of intuitionistic door space is not necessarily an intuitionistic door space. In section 5, intuitionistic ideals in intuitionistic door space is discussed.

2010 *Mathematics Subject Classification.* 54D15.
Key words and phrases. Intuitionistic sets, Intuitionistic topology, Intuitionistic submaximal spaces, Intuitionistic extremally disconnected space.
Communicated by. W. Al Omari and N. Rajesh
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2. Preliminaries

Definition 2.1. [4]. Let X be a nonempty fixed set. An intuitionistic set(IS) A is an object having the form A = $< A_1, A_2 >$, where A_1 and A_2 are disjoint subsets of X. The set A_1 is called the set of members of A, while A_2 is called the set of nonmembers of A.

Definition 2.2. [4]. Let X be a nonempty set, and the IS's A and B be in the form A = $< A_1, A_2 >$ and B = $< B_1, B_2 >$, respectively and let $\{A_i$:i $\in$ J$\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = < A_i^1, A_i^2 >$. Then

(i) A $\subseteq$ B iff $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$.
(ii) A = B iff A $\subseteq$ B and B $\subseteq$ A.
(iii) A $\subseteq$ B iff $A_1 \cup A_2 \supseteq B_1 \cup B_2$.
(iv) $A^c = < A_2, A_1 >$,here A^c is the complementary of A.
(v) $\cup\ A_i = < \cup A_i^1, \cap A_i^2 >$.
(vi) $\cap\ A_i = < \cap A_i^1, \cup A_i^2 >$.
(vii) A-B = A $\cap$ B^c.
(viii) $\tilde{\emptyset} = < \emptyset, X >$.
(ix) $\tilde{X} = < X, \emptyset >$.

Definition 2.3. [4]. Let X be a nonempty set and p $\in$ X a fixed element in X. Then the IS $\tilde{p}$= $< p, p^c >$ is called an intuitionistic point(IP) in X.

Definition 2.4. [4]. Let X be a nonempty set and p $\in$ X a fixed element in X. Then the IS $\tilde{p_*} = < \emptyset, p^c >$ is called the vanishing intuitionistic point.

Definition 2.5. [5]. An intuitionistic topology (IT) on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following conditions.

(i) $\tilde{\emptyset}$,$\tilde{X} \in \tau$.
(ii) $G_1 \cap G_2 \in \tau$ for any G_1,$G_2 \in \tau$.
(iii) $\cup\ G_i \in \tau$ for any arbitrary family $\{G_i$:i$\in$J$\} \subset \tau$.

In this case, the pair (X,τ) is called an Intuitionistic Topological Space(ITS). Every member of τ is known as Intuitionistic Open Sets(IOS) in X. The complement of such an IOS in X is called an Intuitionistic Closed Sets(ICS) in X.

Definition 2.6. : Let (X,τ) be an ITS and A = $< A_1, A_2 >$ be an IS in X. Then an intuitionistic point $\tilde{x} \in$ X is called an intuitionistic limit point of A iff every intuitionistic open set G containing $\tilde{x}$ where $[G - \tilde{x}] \cap$ A $\neq \tilde{\emptyset}$. The set of all limit points of A is denoted by $\tilde{D}$(A).

Definition 2.7. [5]. Let (X,τ) be an ITS and A = $< A_1, A_2 >$ an IS in X. Then the interior of A is defined by int(A) = $\bigcup$\{G:G is an IOS X,G $\subseteq$ A\}

Definition 2.8. [5]. Let (X,τ) be an ITS and A = $< A_1, A_2 >$ an IS in X. Then the closure of A is defined by cl(A) = $\bigcap$\{K:K is an ICS, A $\subseteq$ K\}

It can be shown that cl(A) is an ICS, int(A) is an IOS in X, A is an ICS in X iff cl(A) = A and A is an IOS in X iff int(A) = A.

Proposition 2.9. [5]. Let (X,τ) be an ITS and A,B be IS's in X. Then the following properties hold:

(i) int(A)$\subseteq$ A.
(ii) A $\subseteq$ B $\Rightarrow$ int(A) $\subseteq$ int(B).
(iii) int(int(A))=int(A).
(iv) int(A $\cap$ B)=int(A) $\cap$ int(B).
(v) int($\tilde{X}$)=$\tilde{X}$.
(vi) A $\subseteq$ B $\Rightarrow$ cl(A) $\subseteq$ cl(B).
(vii) A $\subseteq$ cl(A).
(viii) cl(cl(A))=cl(A).
(ix) cl(A $\cup$ B)=cl(A) $\cup$ cl(B).
(x) cl($\tilde{\emptyset}$)=$\tilde{\emptyset}$.

Proposition 2.10. : Let (X,τ) be an ITS and A,B be IS's in X.Then the following properties hold:

(i) int(A) $\cup$ int(B) $\subseteq$ int(A $\cup$ B)
(ii) cl(A $\cap$ B) $\subseteq$ cl(A) $\cap$ cl(B)

Example 2.11. : Let X={a,b}, $\tau = \{ < X, \emptyset >, < \emptyset, X >, < \{a\}, \{b\} >, < \{a\}, \emptyset >\}.\tau$ closed sets are $\{ < X, \emptyset >, < \emptyset, X >, < \{b\}, \{a\} >, \ < \emptyset, \{a\} >\}$, A = $< \{a\}, \{b\} >$ and B = $< \{a\}, \emptyset >$, int(A) = $< \{a\}, \{b\} >$ and int(B) = $< \{a\}, \emptyset >$. Here int(A) $\cup$ int(B) $\subseteq$ int(A $\cup$ B).cl(A)= $< X, \emptyset >$, cl(B)= $< X, \emptyset >$. Here cl(A $\cap$ B) $\subseteq$ cl(A) $\cap$ cl(B).

Definition 2.12. [14]. Let (X,τ) be an ITS and let A = $< A_1, A_2 >$ be an IS in X is called:

(i) Intuitionistic Preopen Set (IPOS) if A $\subseteq$ int(cl(A)) and Intuitionistic Preclosed Set (IPCS) if cl(int(A)) $\subseteq$ A.
(ii) Intuitionistic Semiopen Sets (ISOS) if A $\subseteq$ cl(int(A) and Intuitionistic Semiclosed Sets (ISCS for short) if int(cl(A)) $\subseteq$ A.
(iii) Intuitionistic α-open Sets (Iα-OS) if A $\subseteq$ int(cl(int(A))) and Intuitionistic α-closed Set (Iα-CS) if cl (int(cl(A))) $\subseteq$ A.
(iv) Intuitionistic Regular Open Set (IROS) if A=int(cl(A)) and Intuitionistic Regular Closed Set (IRCS) if A=cl(int(A)).
(v) Intuitionistic Nowhere Dense Set int(cl(A))=$\tilde{\emptyset}$.

Definition 2.13. [10]. An intuitionistic topological space is said to be an intuitionistic extremally disconnected(IED) if the closure of every IOS is intuitionistic open.

Definition 2.14. [1]. Let (X,τ) be an ITS and A = $< A_1, A_2 >$ an IS in X. Then An intuitionistic dense set is a set A such that cl(A)= $\tilde{X}$.

Definition 2.15. [1]. An ITS is said to be an intuitionistic submaximal if every intuitionistic dense set in ITS is intuitionistic open.

Definition 2.16. [5]. Let X,Y be two nonempty sets and f:X $\longrightarrow$ Y a function:
(a)If B = $< B_1, B_2 >$ is an IS's in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is an IS in X defined by $f^{-1}(B) = \ < f^{-1}(B_1), f^{-1}(B_2) >$.
(b)If A = $< A_1, A_2 >$ is an IS's in X, then the image of A under f, denoted by f(A), is the IS in Y defined by f(A)= $< f(A_1), (f({A_2}^c))^c >$.

Definition 2.17. [5]. Let (X, τ) and (Y, σ) be two ITS then a mapping $f : X \to Y$ is an intuitionistic continuous mapping on X if the inverse image of every IOS in Y is IOS in X.

3. Weaker forms in ITS

Here, in this section the family of all Iα-OS does not necessarily form an intuitionistic topology. Example 3.1 shows that an IS's in (X,τ) is an Iα-OS iff it is ISO and IPO. Another theorem shows that an ISO(X) is an IT on X iff (X,τ) is IED.

Example 3.1. :Let X= {a,b,c}, $\tau = \{\tilde{X}, \tilde{\emptyset}, <\{a\}, \emptyset>, <\{a, c\}, \{b\}>, <\{a\}, \{b\}>, <\{a, c\}, \emptyset>, <\{c\}, \{a, b\}>, <\emptyset, \{a, b\}>\}$. I$\alpha$-OS are $\{\tilde{X}, \tilde{\emptyset}, <\{a\}, \emptyset>, <\{a, c\}, \{b\}>, <\{a\}, \{b\}>, <\{a, c\}, \emptyset>, <\{c\}, \{a, b\}>, <\emptyset, \{a, b\}>, <\{c\}, \{a\}>, <\{c\}, \{b\}>, <\{b, c\}, \{a\}>, <\{c\}, \emptyset>, <\{b, c\}, \emptyset>\}$. Here I$\alpha$OS does not necessarily form an intuitionistic topology since $<\{a\}, \emptyset> \cap <\{c\}, \{a\}> = <\emptyset, \{a\}>$ which is not IαOS.

Theorem 3.2. *: An IS A = $<A_1, A_2>$ in an ITS (X,τ) is an Iα-OS if and only if it is both an ISOS and an IPOS.*

Proof. : If A = $<A_1, A_2>$ is an Iα-OS then A $\subseteq$ int(cl(int(A))) and hence A is ISOS and A is IPOS. Conversely, A is both an ISOS and an IPOS. Then A $\subseteq$ cl(int(A)) and so cl(A) $\subseteq$ cl(cl(int(A))) = cl(int(A)).It follows that A $\subseteq$ int(cl(A)) $\subseteq$ int(cl(int(A))) and hence A is an Iα-OS. $\square$

Note: Every IOS in ITS is ISOS but clearly ISOS may not be IOS.

Example 3.3. : Let X ={a,b},τ ={ $<X, \emptyset>$, $<\emptyset, X>$, $<\{a\}, \{b\}>$},τ closed sets are { $<X, \emptyset>$, $<\emptyset, X>$, $<\{b\}, \{a\}>$}. Intuitionistic Semiopen sets = { $<X, \emptyset>$, $<\emptyset, X>$, $<\{a\}, \emptyset>$, $<\{a\}, \{b\}>$}

Theorem 3.4. *: An ISOS(X) is an IT on X iff X is IED.*

Proof. : If ISOS(X) is an IT on X then A $\subseteq$ cl(int(A)).Now cl(A) $\subseteq$ cl(cl(int(A)) = cl(int(A)) and hence the closure of IOS in X is intuitionistic open. Conversely, let A and B are two intuitionistic semiopen sets. Then A $\subseteq$ cl(int(A)) and B $\subseteq$ cl(int(B)).Now A $\cup$ B $\subseteq$ cl(int(A))$\cup$ cl(int(B)) = cl[int(A) $\cup$ int(B)] $\subseteq$ cl(int(A) $\cup$ int(B)). Hence A $\cup$ B is ISO. Also A $\cap$ B = cl(int(A)) $\cap$ cl(int(B)) $\subseteq$ cl(int(A) $\cap$ int(B)).Since X is extremally disconnected, then cl (int(A $\cap$ B)) and A $\cap$ B $\subseteq$ cl(int(A $\cap$ B)) is intuitionistic open.Hence A $\cap$ B is ISOS. $\square$

Theorem 3.5. *: Let (X,τ) be an intuitionistic submaximal, then IPO(X,τ) = τ.*

Proof. : $\tau \subset$ IPO(X,τ) where A = $<A_1, A_2>$ $\in$ IPO(X,τ).This implies that A = U $\cap$ D where U = $<U_1, U_2>$ $\in \tau$ and D $\subset$ X intuitionistic dense set. Since (X,τ) is intuitionistic submaximal and hence D $\in \tau$,then A $\in \tau$. $\square$

Theorem 3.6. *: Every intuitionistic dense set in X is intuitionistic preopen.*

Proof. : Let A = $<A_1, A_2>$ $\subset$ X be intuitionistic dense. Then cl(A) = X and cl(A) is intuitionistic open. Hence A $\subseteq$ cl(A) = int(cl(A)). Therefore A is intuitionistic preopen. $\square$

4. Door space via Intuitionistic perspective

Here we generalize the concept of intuitionistic door space.

Definition 4.1. : An ITS is said to be an intuitionistic door space(IDS) if every IS's in X is either IOS or ICS.

Example 4.2. : Let X ={a,b,c}, $\tau = \{ < X,\emptyset >, < \emptyset, X >, < \emptyset,\emptyset >, < \{a\},\emptyset >, < \emptyset,\{a\} >, < \{b\},\emptyset >, < \emptyset,\{c\} >, < \{ab\},\emptyset >, < \{bc\},\emptyset >, < \emptyset,\{ac\} >, < \{b\},\{a\} >, < \{bc\},\{a\} >, < \{b\},\{ac\} >, < \{a\},\{c\} >, < \{b\},\{c\} >, < \{ab\},\{c\} >\}$, τ closed sets are $\{ < X,\emptyset >, < \emptyset, X >, < \emptyset,\emptyset >, < \{a\},\emptyset >, < \emptyset,\{a\} >, < \emptyset,\{b\} >, < \{c\},\emptyset >, < \emptyset,\{ab\} >, < \emptyset,\{bc\} >, < \{ac\},\emptyset >, < \{a\},\{b\} >, < \{a\},\{bc\} >, < \{ac\},\{b\} >, < \{c\},\{a\} >, < \{c\},\{b\} >, < \{c\},\{ab\} >\}$

Hence (X,τ) is an IDS.

Theorem 4.3. *:Every IDS X is an intuitionistic submaximal.*

Proof. : Let A = $< A_1, A_2 > \subset$ X be τ is an intuitionistic dense set. If A is not IOS, then A is ICS, since X is an IDS. Then A = cl(A) = X and A is an intuitionistic open. Then X is an intuitionistic submaximal. □

Definition 4.4. : An intuitionistic topological space (X,τ) is said to be an intuitionistic irreducible if every two nonempty intuitionistic open set in X is nonempty.

Example 4.5. : Let X = {a,b,c}, $\tau = \{\tilde{X},\tilde{\emptyset}, < \emptyset,\{c\} >, < \{ac\},\emptyset >, < \{bc\},\emptyset >, < \{c\},\emptyset >\}$. Here every two nonempty IOS intersects and thus X is intuitionistic irreducible.

Theorem 4.6. *:Every intuitionistic irreducible submaximal space X is an IDS.*

Proof. : Let A = $< A_1, A_2 > \subseteq$ X. If A is dense in X, then by submaximality A is an intuitionistic open. If A is not dense, then we can find an intuitionistic open set C = $< C_1, C_2 >$ and D = $< D_1, D_2 >$ in X/A where $C \cap D \neq \tilde{\emptyset}$, since X is an intuitionistic irreducible. Hence X/A is intuitionistic open or equivalently A is intuitionistic closed. Therefore A is either IOS or ICS. This shows that X is an IDS. □

Definition 4.7. : An intuitionistic topological space (X,τ) is an intuitionistic resolvable iff it has two disjoint intuitionistic dense set. Otherwise it is intuitionistic irresolvable.

Example 4.8. :Let X ={a,b,c}, $\tau = \{\tilde{X}, \tilde{\emptyset}, < \emptyset,\emptyset >, < \emptyset,\{a\} >, < \{a\},\emptyset >, < \{b\},\emptyset >, < \emptyset,\{c\} >, < \{ab\},\emptyset >, < \{bc\},\emptyset >, < \emptyset,\{ac\} >, < \{b\},\{a\} >, < \{bc\},\{a\} >, < \{b\},\{ac\} >, < \{a\},\{c\} >, < \{b\},\{c\} >, < \{ab\},\{c\} >\}$. τ closed sets are $\{ < X,\emptyset >, < \emptyset, X >, < \emptyset,\{a\} >, < \{a\},\emptyset >, < \emptyset,\emptyset >, < \{c\},\emptyset >, < \emptyset,\{b\} >, < \{a\},\{b\} >, < \emptyset,\{ab\} >, < \emptyset,\{bc\} >, < \{ac\},\emptyset >, < \{a\},\{bc\} >, < \{ac\},\{b\} >, < \{c\},\{a\} >, < \{c\},\{b\} >, < \{c\},\{ab\} >$. Intuitionistic dense sets are $=\{\tilde{X}, < \{b\},\emptyset >, < \{a,b\},\emptyset >, < \{b,c\},\emptyset >, < \{b\},\{a\} >, < \{b,c\},\{a\} >, < \{b\},\{a,c\} >, < \{b\},\{c\} >\}$. When A = $< \{b\},\emptyset >$ and B = $< \{a,b\},\emptyset >$ then both A and B intersects. Therefore X is an intuitionistic irresolvable.

Definition 4.9. : An ITS is intuitionistic S-space if every intuitionistic set which contains an nonempty intuitionistic open set is intuitionistic open.

Example 4.10. :Let X = {a,b}, $\tau = \{ <X,\emptyset>, <\emptyset,X>, <\{a\},\emptyset>, <\{a\},\{b\}>, <\{b\},\{a\}>, <\{b\},\emptyset>, <\emptyset,\{a\}>, <\emptyset,\{b\}>, <\emptyset,\emptyset>\}$. Here (X,$\tau$) is an intuitionistic S-space.

Definition 4.11. : An ITS(X,τ) is said to be an intuitionistic presemiopen space(PS) if every intuitionistic preopen set is intuitionistic semiopen.

Theorem 4.12. *: Let X be an intuitionistic S-space, Then the following conditions are equivalent.*

(i) X is an IDS.
(ii) X is intuitionistic submaximal.
(iii) X is intuitionistic PS-space.
(iv) X = $\tilde{\emptyset}$ or X is intuitionistic irresolvable.
(v) X is discrete or intuitionistic irreducible door space.

Proof. : $(i) \Rightarrow (ii)$ is trivial by Theorem 4.3.
$(ii) \Rightarrow (iii)$ Let A = $<A_1, A_2> \subset$ X be an intuitionistic preopen set of X. Since X is intuitionistic submaximal, then A is an IOS and hence it is intuitionistic semiopen.
$(iii) \Rightarrow (iv)$ Let X $\neq \tilde{\emptyset}$. Assume that X has two disjoint intuitionistic dense subsets $\tilde{U}$ and $\tilde{V}$.Since X $\neq \tilde{\emptyset}$,then both $\tilde{U}$ and $\tilde{V}$ are non empty. Clearly $\tilde{U}$ and $\tilde{V}$ are intuitionistic preopen and by (iii) it is intuitionistic semiopen. Thus for some nonempty intuitionistic open sets $\tilde{A}$ and $\tilde{B}$ we have $\tilde{A} \subset \tilde{U}$ and $\tilde{B} \subset \tilde{V}$. Hence we have a nonempty intuitionistic open set $\tilde{A}$, disjoint from the dense set $\tilde{V}$. By contradiction X is intuitionistic irresolvable.
$(iv) \Rightarrow (v)$ Let A = $<A_1, A_2> \subset$ X. Since X is intuitionistic irresolvable, then A and X/A are not both dense. Thus one of them is intuitionistic closed. Since X is an intuitionistic S-space, then X is either discrete or intuitionistic irreducible. Hence A and X/A are both intuitionistic open or intuitionistic closed.
$(v) \Rightarrow (i)$ is trivial, since every discrete space is an intuitionistic door space. □

Definition 4.13. : An intuitionistic topological space in X is a pre $T_{1/2}$-space if every IP in X is either intuitionistic preopen or intuitionistic preclosed sets.

Lemma 4.14. *:Every IP in an ITS X is either intuitionistic preopen or intuitionistic nowhere dense sets.*

Proof. : Suppose $\tilde{x}$ is intuitionistic preopen in X. By Definition 2.12 $\tilde{x} \subset$ int(cl($\tilde{x}$)). Since int(cl($\tilde{x}$)) cannot be empty thus $\tilde{x}$ is not intuitionistic nowhere dense set. If $\tilde{x}$ is intuitionistic nowhere dense set then int(cl($\tilde{x}$))= $\emptyset$. Since int(cl($\tilde{x}$)) is empty then $\tilde{x}$ is not intuitionistic preopen set and hence the proof. □

Remark 4.15. :Every intuitionistic nowhere dense set is intuitionistic α- closed.

Example 4.16. : Let X ={a,b}, $\tau =\{\tilde{X}, \tilde{\emptyset}, <\{a\},\emptyset>, <\{a\},\{b\}>, <\emptyset,\{b\}>\}$, τ closed sets are $\{\tilde{X}, \tilde{\emptyset}, <\emptyset,\{a\}>, <\{b\},\{a\}>, <\{b\},\emptyset>\}$, Intuitionistic α-open sets = $\{\tilde{X},\tilde{\emptyset}, <\{a\},\emptyset>, <\{a\},\{b\}>, <\emptyset,\{b\}>\}$, Intuitionistic α-closed sets are $\{\tilde{X}, \tilde{\emptyset}, <\emptyset,\{a\}>, <\{b\},\{a\}>, <\{b\},\emptyset>\}$, Intuitionistic nowheredense sets are =$\{\tilde{\emptyset}, <\emptyset,\{a\}>, <\{b\},\{a\}>\}$. Here intuitionistic nowhere dense are intuitionistic α-closed.

Theorem 4.17. *:Every ITS in X is intuitionistic pre* $T_{1/2}$*-space.*

Proof. : Let $\tilde{x} \in$ X. By Lemma 4.14 $\{\tilde{x}\}$ is either preopen or nowhere dense set in intuitionistic topology. By Remark 4.15 every intuitionistic nowhere dense set is intuitionistic α-closed and hence intuitionistic preclosed. Thus every IP of X is either preopen or preclosed. □

Definition 4.18. : An ITS X is said to be an intuitionistic regular door space(IRDS) if every IS in X is either intuitionistic regular open or intuitionistic regular closed.

Theorem 4.19. *:For an intuitionistic topological space X, the following conditions are equivalent.*

(i) *X is discrete.*
(ii) *X is an IRDS.*

Proof. : $(i) \Rightarrow (ii)$ is trivial.
$(ii) \Rightarrow (i)$ Let $\tilde{x} \in$ X. If $\tilde{x}$ is not intuitionistic regular open, then by (ii) $\tilde{x}$ is intuitionistic regular closed that is $\tilde{x}$ = cl(int($\tilde{x}$)). Since int($\tilde{x}$) cannot be empty, then $\tilde{x}$ = int($\tilde{x}$), hence in both cases $\tilde{x}$ is intuitionistic open. This shows that X is discrete. □

5. Continuity via Intuitionistic perspective

In this section, we examine the relationship between ICF and IDS.

Definition 5.1. :A function f:X $\longrightarrow$ Y between two ITS X and Y is called intuitionistic homeomorphism if it has the following properties:

(i) f is intuitionistic bijection.
(ii) f is intuitionistic continuous.
(iii) f is an intuitionistic open mapping.

Example 5.2. Let X ={a,b,c} and consider the intuitionistic topology, $\tau = \{\tilde{X}, \tilde{\emptyset}, < \{a\}, \{b, c\} >, < \{a, b\}, \{c\} >\}$. Define f:(X,τ) $\longrightarrow$ (X,τ) by f(a) = a, f(b) = b, f(c) = c. Here (i)f is one to one and onto.
(ii) if $f^{-1}(\tilde{X}) = \tilde{X}$, $f^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$, $f^{-1}(< \{a\}, \{b, c\} >) = < \{a\}, \{b, c\} >$,
$f^{-1}(< \{a, b\}, \{c\} >) = < \{a, b\}, \{c\} >$ which is IOS in (X,τ). Therefore it is ICS.
(iii) $f(\tilde{X}) = \tilde{X}$, $f(\tilde{\emptyset}) = \tilde{\emptyset}$, $f(< \{a\}, \{b, c\} >) = < \{a\}, \{b, c\} >$, $f(< \{a, b\}, \{c\} >) = < \{a, b\}, \{c\} >$. Now f is intuitionistic open mapping and therefore f is an intuitionistic homeomorphism.

Definition 5.3. A property possessed by ITS(X,τ) is said to be intuitionistic topological property if each intuitionistic homeomorphic image of X also possesses that property.

Example 5.4. :Intuitionistic continuous image of IDS is not neessarily an IDS as shown by the following example.
Let X = {a,b} and Y = {1,2} and consider the intuitionistic topology τ =$\{\tilde{X}, \tilde{\emptyset}, < \emptyset, \emptyset >, < \{a\}, \emptyset >, < \{a\}, \{b\} >, < \emptyset, \{b\} >\}$ and $\mu = \{\tilde{Y}, \tilde{\emptyset}, < \emptyset, \emptyset >, < \{1\}, \emptyset >\}$. Define f:(X,τ) $\longrightarrow$ (Y,μ) by f(a) = 1, f(b) = 2. Here $f^{-1}(\tilde{Y}) = \tilde{X}$, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(< \emptyset, \emptyset >) = < \emptyset, \emptyset >$, $f^{-1}(< \{1\}, \emptyset >) = < \{a\}, \emptyset >$. So every inverse image of IOS in Y is IOS in X and therefore f is ICF but the intuitionistic continuous image is not IDS.

Theorem 5.5. *:The property of being IDS is a intuitionistic topological property.*

Proof. :Let X be an IDS and f:X $\longrightarrow$ Y an homeomorphism. Let A = $< A_1, A_2 >$ $\subseteq$ Y. Consider $f^{-1}(A) \subseteq$ X. Since X is an IDS, then $f^{-1}(A)$ is either IOS or ICS in X. Now $f(f^{-1}(A))$ = A. Then A is either IOS or ICS in Y. This implies that Y is an IDS. □

6. Ideal via intuitionistic perspective

In this section, we define and study intuitionistic ideal door space and its properties.

Definition 6.1. : An Intuitionistic Ideal I on an intuitionistic topological space (X,τ) is a nonempty collection of IS's in X which satisfies the following two properties:

(1) A $\in$ I and B $\subset$ A $\Rightarrow$ B $\in$ I.
(2) A $\in$ I and B $\in$ I $\Rightarrow$ A $\cup$ B $\in$ I.

If I is an intuitionistic ideal on X, then (X,τ,I) is an intuitionistic ideal topological space.

Notation. : (i) I_f -intuitionistic ideal of finite sets in (X,τ).
(ii) I_c -intuitionistic ideal of countable sets in (X,τ).
(iii) I_{cd} - intuitionistic ideal of closed discrete sets in (X,τ).

Definition 6.2. : An intuitionistic topological space (X,τ,I) is said to be an intuitionistic ideal door space if every intuitionistic set in X is either IOS or ICS or belongs to intuitionistic ideal.

Theorem 6.3. *:For an intuitionistic topological space (X, τ, I_{cd}) the following conditions are equivalent.*

(i) *(X,τ) is an IED door space.*
(ii) *If A = $< A_1, A_2 > \in \tau$ then A $\in$ I_{cd}.*

Proof. :$(i) \Rightarrow (ii)$ A = $< A_1, A_2 > \in$ I_{cd} iff A has no intuitionistic limit point. Assume A $\notin \tau$ and D(A) $\notin \tilde{\emptyset}$. Since X is an IDS, then A is ICS. Let $\tilde{x} \in$ D(A) $\subset$ A and S =(X/A) $\cup\ \tilde{x}$. Since $\tilde{x}$ is a limit point of A, then S cannot be IOS, for this case S would be an open neighbourhood of $\tilde{x}$ containing a point of A is different from $\tilde{x}$. It is not possible, since S $\cap$ A = $\tilde{x}$. We know that S is not IOS and X is an IDS, then S is ICS. Since S is ICS iff cl(S) = S, (X/A) $\cup\ \tilde{x}$ = cl(X/A) $\cup$ cl($\tilde{x}$) and hence (X/A) $\subset$ cl(X/A) $\subset$ S. Assume X is IED, cl(X/A) is IOS. S is ICS, then (X/A) $\subseteq$ cl(X/A) $\subset$ S. So X/A is ICS iff X/A = cl(X/A) or A is IOS. This contradicts the assumption A $\notin \tau$.
$(ii) \Rightarrow (i)$ X is trivially an IDS. Therefore to show that X is an IED, let A = $< A_1, A_2 >$ $\subset$ X be intuitionistic open. Then by Proposition 2.9, where A $\subseteq$ cl(A) and therefore cl(A) is IOS. □

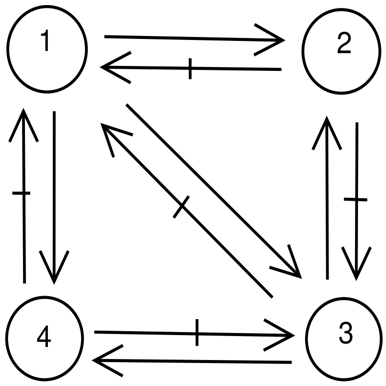
Definition 6.4. : An intuitionistic topological space (X,τ) is said to be be an intuitionistic T_{EF}-space if every intuitionistic finite set in X is either IOS or ICS.

Theorem 6.5. *: For an intuitionistic T_{EF}-space (X,τ,I_f) the following conditions are equivalent.*

(i) *(X,τ) is an IDS.*
(ii) *(X,τ, I_f)is an intuitionistic I_f door space.*

Proof. : $(i) \Rightarrow (ii)$ is trivial.
$(ii) \Rightarrow (i)$ If A = $< A_1, A_2 > \subset$ X is neither IOS nor ICS,then by assumption, A $\in$ I_f. Since X is an intuitionistic T_{EF} space, then A is either IOS or ICS. $\square$

Remark 6.6. :Here are the connections between IDS, intuitionistic ideal door space, intuitionistic T_{EF}-space, intuitionistic submaximal are given.



1.IDS 2.Intuitionistic ideal door space 3.Intuitionistic T_{EF} space 4.Intuitionistic submaximal.

Example 6.7. :Let X ={a,b}, τ ={ $< X, \emptyset >$, $< \emptyset, X >$, $< \{a\}, \emptyset >$, $< \{a\}, \{b\} >$, $< \emptyset, \{b\} >$}, τ closed sets are { $< X, \emptyset >$, $< \emptyset, X >$, $< \{b\}, \{a\} >$, $< b, \emptyset >$, $< \emptyset, \{a\} >$}. Here X is an intuitionistic submaximal but not intuitionistic T_{EF} space since $< \emptyset, \emptyset >$ is neither IOS nor ICS.

Example 6.8. : Let X ={a,b}, τ = { $< X, \emptyset >$, $< \emptyset, X >$, $< \{a\}, \emptyset >$, $< \{a\}, \{b\} >$, $< \emptyset, \{b\} >$}}, τ closed sets are { $< X, \emptyset >$, $< \emptyset, X >$, $< \emptyset, \{a\} >$, $< \{b\}, \{a\} >$, $< \{b\}, \emptyset >$ }, $\tilde{I}$ ={ $< \emptyset, X >$, $< \emptyset, \{a\} >$, $< \emptyset, \{b\} >$, $< \emptyset, \emptyset >$, $< \{b\}, \emptyset >$, $< \{b\}, \{a\} >$}. Here X is an intuitionistic ideal door space but not IDS.

7. Conclusion

Intuitionistic sets are applied in various areas of Mathematics. It has many applications in Operation Research for Linear Programming problems, Psychology for comparison test, Coordinate Geometry and Algebra. In that way we can obtain some real life applications in intuitionistic doorspace. We introduce the concepts of intuitionistic door space and intuitionistic ideal door space and investigated their behaviour. Further our conceps can be extended to generalized intuitionistic topology.

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