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SOME TYPES OF NORMALITY AND REGULARITY BASED ON NEAR SIMPLY OPEN SETS

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Abstract. In the this study we present a new class of near-open sets, in a topological space namely simply α-open and simply pre-open sets, and we used these classes to introduce new types of continuous functions, and we also introduced a number of types of normality and regularity based upon these concepts. We are studying the relation between these types and the others types eventually we 're investigating some of their properties

1. Introduction

In 1975 A.Neubrunnova [8] introduce the notation of simply open sets. In 1965 O. Njasted [10] had introduced the concept of α-open sets. In 1982 A. S. Mashhour and M. E. Abd El-Monsef [[5] - [6]] had introduced the concept of pre open sets, Where these concepts play a major role in topology introduced the concept of soft simply open soft [3], also we introduce modification of simply open sets in [[1] - [2]]. In this paper we introduce new concepts based on simply open sets, simply α-open and simply pre open sets based on open set, α-open, and pre open sets and we use these notions to define the other concepts of normality and regularity during this paper we assume that (X, τ) and (Y, σ) are topological spaces with no separation axioms, $cl(A), int(A)$ denote respectively the closure and the relation of subset $A \subseteq X$.pre open and pre closed) sets in (X, τ) is denoted by $O(X)$ (resp. $C(X)$, $PO(X)$ and $PC(X)$). Where these concepts play a major role in topology We introduced the concept of simply open soft sets.

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2. Preliminaries

Definition 2.1. A subset V of a topological space (X, τ) is said to be:

1. α-open sets [10] if $V \subseteq int(cl(int(V)))$ and α-closed sets if $cl(int(cl(V))) \subseteq V$.
2. Regular-open sets [11] if $V = int(cl(V))$ and regular-closed if $V = cl(int(V))$.
3. Semi open [4] sets if $V \subseteq cl(int(V))$ and α-closed sets if $int(cl(V)) \subseteq V$.
4. simply open [3] sets if $V = G \cup N$ such that G is open and N is nowehere dense.
6. pre open [6] sets if $V \subseteq int(cl(V))$ and pre closed if $cl(int(V)) \subseteq V$.

The classes of simply open sets (resp. simply α-open, simply pre open, and pre-open) sets of a universe set W is indicated for $\overset{M}{S}O(W)$ (resp. $\overset{M}{S}\alpha O(W)$, $\overset{M}{S}PO(W)$ and $PO(W)$).

The supplement of all simply open sets is referred to as simply closed sets. We indicated for any, α-open sets (resp. α-closed, regular closed, regular open, semi open, semi closed, pre open, pre closed, simply open and simply closed) sets of W by $\alpha O(W)$, (resp., $\alpha C(W)$, $RC(W), RO(W)$, $SO(W), SC(W), PO(W)$, $PC(W), \overset{M}{S}O(W)$ and $\overset{M}{S}C(W)$) sets.

3. Some kinds of continuous functions

In this section we introduce two new classes of sets depends on preopen and α-open sets, namely, simply pre open and simply α-open sets And also in this part we use the sorts of sets to introduce some new types of continuity

Definition 3.1. A topological space (X, τ) is called simply pre open (resp. simply α-open) set if if $A = G \cup N$ such that G is pre open (resp. α-open) and N is nowehere dense. We denote the class of all simply preopen (resp. simply α-open) sets by $\overset{M}{S}PO(X)$ (resp. $\overset{M}{S}\alpha O(X)$). The complement of simply preopen (resp. simply α-open) sets is called simply preclosed (resp. simply α-closed) sets which denoted by $\overset{M}{S}PC(X)$ (resp. $\overset{M}{S}\alpha C(X)$).

Remark 3.2. For any topological space (X, τ) the class of any simply-open sets in X is the class of all simply closed sets in X i.e. $\overset{M}{S}O(X) = \overset{M}{S}C(X)$.

Proposition 3.3. A subset A of a space X is simply closed (resp. simply pre closed,simply α-closed) iff $A = F \cap W$, whenever F is closed (resp.pre closed, α-closed) and $int(W)$ is dense in X.

Proof. Obvious. $\square$

The following figure illustrates the relation between the previous types of sets.

Example 3.4. $X = \{a^1, b^2, c^3, d^4, e^5\}, \tau = \{X, \phi, \{d^4\}, \{a^1, b^2\}, \{a^1, b^2, d^4\}, \{c^3, d^4, e^5\}\}$

(implication 1) $\{c^3\} \in \overset{M}{S}O(X)$ but $\{c^3\} \notin \tau$.

(Implication 2) $\{a^1\} \in \overset{M}{S}PO(X)$ but $\{a^1\} \notin \overset{M}{S}\alpha O(X)$

$$\begin{array}{ccccc}
\overset{M}{S}O(X) & \xrightarrow{3} & \overset{M}{S}\alpha O(X) & \xrightarrow{2} & \overset{M}{S}PO(X) \\
1\uparrow & & 4\uparrow & & 5\uparrow \\
\tau = O(X) & \rightarrow & \alpha O(X) & \rightarrow & PO(X)
\end{array}$$

FIGURE 1. shows the relation between the previous types of sets

(Implication 4) $\{e^5\} \in \overset{M}{S}\alpha O(X)$ but $\{e^5\} \notin \alpha O(X)$

(Implication 5) $\{a^1, e^5\} \in \overset{M}{S}\alpha O(X)$ but $\{a^1, c^3\} \notin PO(X)$

Definition 3.5. mapping $f : (X, \tau) \to (Y, \sigma)$ it's called :

1. α-continuous [5] if $f^{-1}(G) \in \alpha O(X)$ for each $G \in \sigma$.
2. Pre-continuous [5] if $f^{-1}(G) \in PO(X)$ for every $G \in \sigma$.
3. α-irresolute [7] if $f^{-1}(G) \in \alpha O(X)$ for every $G \in \alpha O(Y)$.
4. Pre-irresolute [9] if $f^{-1}(G) \in PO(X)$ for every $G \in PO(Y)$.

Definition 3.6. A function $f : (X, \tau) \to (Y, \sigma)$ it's called:

1. Simply irresolute (for short, $\overset{M}{S}$- irresolute) if $f^{-1}(G) \in \overset{M}{S}O(X)$ for every $G \in \overset{M}{S}O(Y)$.

(1) Simply α-continuous (for short, $\overset{M}{S}\alpha$- continuous)if $f^{-1}(G) \in \overset{M}{S}\alpha O(X)$ for each $G \in \sigma$.

(2) Simply α-irresolute (for short, $\overset{M}{S}\alpha$- irresolute) if $f^{-1}(G) \in \overset{M}{S}PO(X)$ for every $G \in \sigma$.

(3) Simply pre-continuous (for short, $\overset{M}{S}P$- irresolute) if $f^{-1}(G) \in \overset{M}{S}\alpha O(X)$ for each $G \in \sigma$.

(4) Simply pre-irresolute (for short, $\overset{M}{S}P$- irresolute)if $f^{-1}(G) \in \overset{M}{S}PO(X)$ for every $G \in \overset{M}{S}PO(Y)$.

(5) α-simply continuous (for short, $\alpha\overset{M}{S}$- continuous) if $f^{-1}(G) \in \overset{M}{S}\alpha O(X)$ for every $G \in \alpha O(Y)$.

(6) pre-simply continuous (for short, $P\overset{M}{S}$- continuous)if $f^{-1}(G) \in \overset{M}{S}PO(X)$ for each $G \in PO(Y)$.

The following figure indicates the relation among the new types of continuity and the other types such that the implications are not reversible.

Example 3.7. (implication 2): Let $U = \{a_1, b_2, c_3\}$ and $Y = \{u_1, v_2, w_3\}$, such that $\tau = \{X, \phi, \{a_1\}, \{a_1, b_2\}\}$, and $\sigma = \{Y, \phi, \{u_1\}\}$, let $f : (X, \tau) \to (Y, \sigma)$

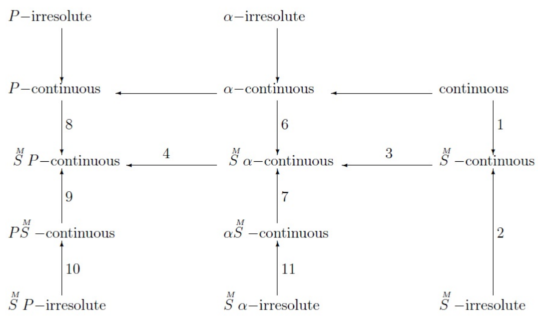

FIGURE 2. Shows the relations among the new types of continuity and other types

is defined as $f(a_1) = u_1, f(b_2) = v_2$,and $f(c_3) = w_3$. We have f is $\overset{M}{S}$-continuous but it is not $\overset{M}{S}$-irresolute, since there exist a set $\{w_3\} \in \overset{M}{S}O(Y)$ but $f^{-1}(\{w_3\}) = \{c_3\} \notin \overset{M}{S}O(X)$.

Example 3.8. (implication 6): Let $X = \{a_1, b_2, c_3, d_4\}$ and $Y = \{u_1, v_2, w_3, z_4\}$, such that $\tau = \{X, \phi, \{a_1\}, \{b_2, c_3\}, \{a_1, b_2, c_3\}\}$, and $\sigma = \{Y, \phi, \{u_1\}, \{v_2\}, \{u_1, v_2\}, \{v_2, w_3, z_4\}\}$, let $f : (X, \tau) \to (Y, \sigma)$ is defined as follows $f(a_1) = u_1, f(b_2) = w_3, f(c_3) = z_4$,and $f(d_4) = v_2$. Then we have f is $\overset{M}{S}\alpha$-continuous but it is not α-continuous, since there are a set $\{u_1, w_3, z_4\} \in \sigma$, but $f^{-1}(\{u_1, w_3, z_4\}) = \{b_2, c_3, d_4\} \notin \alpha O(X)$.

Example 3.9. (implication 7): Let $X = \{a, b, c\}$ and $Y = \{u, v, w\}$, such that $\tau = \{X, \phi, \{a, , c\}\}$, and $\sigma = \{Y, \phi, \{w\}\}$, let $f : (X, \tau) \to (Y, \sigma)$ is defined as $f(a) = u, f(b) = w$,and $f(c) = v$. We have f is $\overset{M}{S}\alpha$-continuous but it is not $\alpha\overset{M}{S}$-continuous, since there exist a set $\{v, w\} \in \alpha O(Y)$ but $f^{-1}(\{v, w\}) = \{b, c\} \notin \overset{M}{S}\alpha O(X)$.

Example 3.10. (implication 8): Let $f : (X, \tau) \to (Y, \sigma)$ as in Example 3.2 $f(a) = h, f(b) = g, f(c) = i, f(d) = k$ and $f(e) = j$. We have f is α-continuous but it is not continuous,since there exist a set $\{g\} \in \sigma$ but $f^{-1}(\{g\}) = \{b\} \notin PO(X)$.

Example 3.11. (implication 1): Let $U = \{a_1, b_2, c_3, d_4\}$ and $V = \{u_1, v_2, w_3, z_4\}$, such that $\tau = \{U, \phi, \{d_4\}, \{a_1, d_4\}\}$, and $\sigma = \{V, \phi, \{u_1, v_2\}, \{w_3, z_4\}\}$, let $f : (U, \tau) \to (V, \sigma)$ is defined as $f(a_1) = u_1, f(b_2) = v_2, f(c_3) = w_3$,and $f(d_4) = z_4$. We get f is $\overset{M}{S}$-continuous but it is not continuous, since there are a set $\{u_1, v_2\} \in \sigma$, but $f^{-1}(\{u_1, v_2\}) = \{b_2, c_3\} \notin \tau$.

Example 3.12. (implication 4): Let $X = \{a, b, c, d, e\}$ and $Y = \{g, h, i, j, k\}$, such that $\tau = \{X, \phi, \{a\}, \{a, , b\}, \{a, c\ , d\}, \{a, b, c, d\}\}$, and $\sigma = \{Y, \phi, \{g\}, \{g, h\}, \{g, h, i, j\}, \{g, h, k\}\}$, let $f : (X, \tau) \to (Y, \sigma)$ is defined as $f(a) = g, f(b) = h$,and $f(c) = j, f(d) = k$ and $f(e) = i$. We have f is $\alpha-$continuous but it is not continuous,since there exist a set $\{g,h,i,j\} \in \sigma$ but $f^{-1}(\{g,h,i,j\}) = \{a,b,c,e\} \notin \tau$.

Remark 3.13.

(1) A continuous function and $\overset{M}{S}$-irresolute function are not comparable
(2) α-continuous function and $\overset{M}{S}$ α-irresolute function are not comparable
(3) α-irresolute function and pre irresolute function are not comparable
(4) $\overset{M}{S}$-irresolute function and $\overset{M}{S}\alpha$-irresolute function are not comparable
(5) Pre continuous function and $\overset{M}{S}P$-irresolute function are not comparable
(6) $\overset{M}{S}P$-irresolute function and $\overset{M}{S}\alpha$-irresolute function are not comparable.

Investigation the previous remark As one would assume using multiple examples: Here are two
of them.

Example 3.14. (Part 1):

(i) Assuming that $X = \{a_1, b_2, c_3, d_4\}$ and $Y = \{u_1, v_2, w_3, z_4\}$, such that $\tau = \{X, \phi, \{d_4\}, \{a_1, d_4\}\}$, and $\sigma = \{Y, \phi, \{u_1, v_2\}, \{w_3, z_4\}\}$, let $f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a_1) = u_1, f(b_2) = v_2,\ f(c_3) = w_3$,and $f(d_4) = z_3$. We have f is simply irresolute but it is not continuous, since there are a set $\{u_1, v_2\} \in \sigma$, but $f^{-1}(\{w_3, z_4\}) = \{c_3, d_4\} \notin \tau$.
(ii) Assuming that$X = \{a,b,c\}$ and $Y = \{u,v,w\}$, such that $\tau = \{X, \phi, \{a\}, \{b,c\}\}$, and $\sigma = \{Y, \phi, \{u\}\}$, let $f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a) = u, f(b) = v, f(c) = w$,and $f(d) = z$. We have f is continuous but it is not $\overset{M}{S}$-irresolute, since there is a set $\{u,v\} \in \overset{M}{S}O(Y)$, but $f^{-1}(\{u,v\} = \{a,b\} \notin \overset{M}{S}O(X)$.

Example 3.15. (Part 2):

(i) let $f : (X,\tau) \to (Y,\sigma)$ as in Example 3.5 defined as follows $f(a) = u, f(b) = v,\ f(c) = w$,and $f(d) = z$. We have f is αcontinuous but it is not $\overset{M}{S}$-irresolute, since there are a set$\{v,w\} \in \overset{M}{S}\alpha O(Y)$, but $f^{-1}(\{v,w\}) = \{b,c\} \notin \overset{M}{S}\alpha O(X)$.
(ii) Let $X = \{a_1, b_2, c_3, d_4\}$ and $Y = \{u_1, v_2, w_3, z_4\}$, such that $\tau = \{X, \phi, \{a\}, \{c_3\}, \{d_4\}, \{a_1, c_3\}, \{c_3, d_4\}\ , \{a_1, d_4\}, \{a_1, c_3, d_4\}\}$, and $\sigma = \{Y, \phi, \{v_2\}, \{v_2, w_3\}\}$, Assuming that$f : (X,\tau) \to (Y,\sigma)$ is defined as $f(a_1) = u_1, f(b_2) = v_2, f(c_3) = w_3$, and $f(d_4) = z_4$. Then f is $\overset{M}{S}\alpha$-irresolute but it is not α-continuous, as there is a set $\{v\} \in \sigma$,but $f^{-1}(\{v\} = \{b\} \notin \alpha O(X)$.

4. Some types of near normality

In this section we present a new types of normality in a topological space (X,τ) based on $\overset{M}{S}O(X)$, $\overset{M}{S}PO(X)$ and $\overset{M}{S}\alpha O(Y)$.Also, we study their properties.

Definition 4.1. A topological space (X,τ) is called.

1. $P\overset{M}{S}P$-normal if for every $F_1, F_2 \in PC(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \overset{M}{S}PO(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.
2. α-$\overset{M}{S}\alpha$-normal if for every $F_1, F_2 \in \alpha C(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \overset{M}{S}\alpha O(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.
3. $\alpha\overset{M}{S}P$-normal if for every $F_1, F_2 \in \alpha C(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \overset{M}{S}PO(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

(1) $P\overset{M}{S}\alpha$-normal if for every $F_1, F_2 \in PC(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \overset{M}{S}\alpha O(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

(2) $\overset{M}{S}\alpha^*$-normal if for every $F_1, F_2 \in C(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \overset{M}{S}\alpha O(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

(3) $\overset{M}{S}P^*$-normal if for every $F_1, F_2 \in C(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \overset{M}{S}PO(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

(4) α^*-normal if for every $F_1, F_2 \in C(X), F_1 \cap F_2 = \phi$ then there exist $U, V \in \alpha O(X), U \cap V = \phi$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

(5) P^*-normal if for every $F_1, F_2 \in C(X), F_1 \cap F_2 = \phi$ then there are $U, V \in PO(X), U \cap V = \phi$ so that $F_1 \subseteq U$ and $F_2 \subseteq V$.

9. $\left(\overset{M}{S}\right)^*$-normal if for each $F_1, F_2 \in C(X), F_1 \cap F_2 = \phi$ then there are $U, V \in \overset{M}{S}O(X), U \cap V = \phi$ so that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Remark 4.2. The following figure indicates the relation among the new types of normality and the other types such that the implications are not reversible.

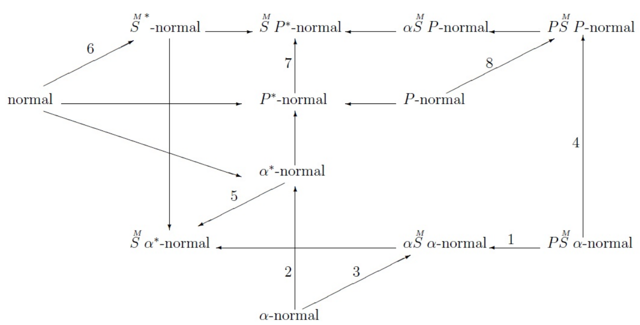

FIGURE 3. Shows the relation among various types of normality

The following examples show that the implications in Figure 3 need not be reversible.

Example 4.3. (implications 1, 4, 8) Let $X = \{a, b, c\}$,$\tau = \{X, \phi, \{a, b\}\}$. We have $(X, \tau)$is $P\overset{M}{S}P$-normal and $\alpha\overset{M}{S}\alpha$ -normal but not $P\overset{M}{S}\alpha$-normal, since there exist $\{a\}, \{b, c\} \in PC(X)$ but there is no disjoint simply α-open sets containing them,and (X, τ) is $P\overset{M}{S}P$-normal but it is not $P\overset{M}{S}\alpha$ -normal, since there exist $\{b\}, \{a, c\} \in PC(X)$ but there is no disjoint

simply α-open sets containing them. Also (X, τ) is $P\overset{M}{S}P$-normal but not P-normal, since there exist $\{a\}, \{c\} \in PC(X)$ but there is no disjoint pre open sets containing them.

Example 4.4. (implications 5, 6, 7) Let $X = \{a, b, c, d, e\}, = \{X, , \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, e\},$

$\{a, b, c, d\}\}$. Then we have (X, τ) is $\left(\overset{M}{S}\right)^*$ -normal and $\overset{M}{S}P^*$-normal spaces but not normal nor P^*-normal, since there exist $\{b, e\}, \{c, d\} \in C(X)$ but there is no disjoint open sets containing them and also, there is no disjoint pre open sets containing them. Also, $(X, \tau)$is $\overset{M}{S}\alpha^*$-normal, since there exist $\{b, e\}, \{c, d\} \in C(X)$ but there is no disjoint -open sets

containing them.

Example 4.5. (implications 2, 3) Let $X = \{a, b, c\}, = \{X, , \{c\}, \{b, c\}\}$. We have (X, τ) is α^*-normal but not α-normal, since there exist $\{a\}, \{b\} \in C(X)$ but there is no disjoint α-open sets containing them. Also, (X, τ) is $\alpha\overset{M}{S}\alpha$-normal but not α-normal, since there exist $\{a\}, \{b\} \in \alpha C(X)$ but there is no disjoint α-open sets containing them.

In the following we give some properties of the new concepts.

Theorem 4.6. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed continuous, $\overset{M}{S}P$-open surjection and (X, τ) is $\overset{M}{S}P^*$-normal. Then (Y, σ) is $\overset{M}{S}P^*$-normal.*

Proof. Let $F_1, F_2 \in C(Y)$ and $F_1 \cap F_2 = \phi$. Since f is continuous subsequently, $f^{-1}(F_1)$, $f^{-1}(F_2) \in C(X)$ and $f^{-1}(F_1)$, $f^{-1}(F_2) = \phi$ as X is $\overset{M}{S}P^*$ -normal, then there are $W, N \in \overset{M}{S}PO(X)$ and $W \cap N = \phi$ so that $f^{-1}(F_1) \subseteq W$ and $f^{-1}(F_2) \subseteq N$. As f is surjective and $\overset{M}{S}P$- open, then we have $f(W), f(N) \in \overset{M}{S}PO(Y)$ and $f(W) \cap f(N) = \phi$. Consequently,(Y, σ) is $\overset{M}{S}P^*$-normal. □

Theorem 4.7. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed,P-irresolute, $\overset{M}{S}P$-open surjection and (X, τ) is $P\overset{M}{S}P$-normal. Then (Y, σ) is $P\overset{M}{S}P$-normal.*

Proof. Let $F_1, F_2 \in PC(Y)$ and $F_1 \cap F_2 = \phi$. Since f is irresolute, then $f^{-1}(F_1)$, $f^{-1}(F_2) \in PC(X)$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi$. Since X is $P\overset{M}{S}P$-normal, Consequently, there are $W, N \in \overset{M}{S}PO(X)$ and $W \cap N = \phi$. So that $f^{-1}(F_1) \subseteq W$ and $f^{-1}(F_2) \subseteq N$. As f is surjective □

Theorem 4.8. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed,α-irresolute, $\overset{M}{S}\alpha$-open surjection and (X, τ) is $\alpha\overset{M}{S}\alpha$-normal. Then (Y, σ) is $\alpha\overset{M}{S}\alpha$-normal.*

Proof. Let $F_1, F_2 \in \alpha C(Y)$ and $F_1 \cap F_2 = \phi$. Since f is irresolute, then $f^{-1}(F_1)$, $f^{-1}(F_2) \in \alpha C(X)$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi$. As X $\alpha\overset{M}{S}\alpha$-normal, then there are $W, N \in \overset{M}{S}\alpha O(X)$ and $W \cap N = \phi$ such that $f^{-1}(F_1) \subseteq W$ and $f^{-1}(F_2) \subseteq N$. Since f is surjective and $\overset{M}{S}\alpha$- open, then we have $f(W), f(N) \in \overset{M}{S}\alpha O(Y)$ and $f(W) \cap f(N) = \phi$. Consequently,(Y, σ) is $\alpha\overset{M}{S}\alpha$-normal. □

Theorem 4.9. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed,continuous, $\overset{M}{S}\alpha$-open surjection and (X, τ) is $\overset{M}{S}\alpha^*$-normal.Then (Y, σ) is $\overset{M}{S}\alpha^*$-normal.*

Proof. Let $F_1, F_2 \in C(Y)$ and $F_1 \cap F_2 = \phi$. Since f continuous, then $f^{-1}(F_1)$, $f^{-1}(F_2) \in C(X)$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi$. Since X is $\overset{M}{S}\alpha^*$-normal, then there are $W, N \in \overset{M}{S}\alpha O(X)$ and $W \cap N = \phi$ So that $f^{-1}(F_1) \subseteq W$ and $f^{-1}(F_2) \subseteq N$. As f is surjective and $\overset{M}{S}\alpha$- open, consequently, $f(W), f(N) \in \overset{M}{S}\alpha O(Y)$ and $f(W) \cap f(N) = \phi$. Consequently,(Y, σ) is $\overset{M}{S}\alpha^*$-normal. □

Theorem 4.10. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed,α-irresolute, $\overset{M}{S}P$-open surjection and (X, τ) is $\alpha\overset{M}{S}P$-normal.Then (Y, σ) is $\alpha\overset{M}{S}P$-normal.*

Proof. Let $F_1, F_2 \in \alpha C(Y)$ and $F_1 \cap F_2 = \phi$. Since f is α-irresolute, then $f^{-1}(F_1)$, $f^{-1}(F_2) \in \alpha C(X)$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi$. Since X is $\alpha\overset{M}{S}P$-normal, then there exist $U, V \in \overset{M}{S}PO(X)$ and $U \cap V = \phi$ such that $f^{-1}(F_1) \subseteq U$ and $f^{-1}(F_2) \subseteq V$. Since f is surjective and $\overset{M}{S}P$- open, then we have $f(U), f(V) \in \overset{M}{S}PO(Y)$ and $f(U) \cap f(V) = \phi$. Consequently,(Y, σ) is $\alpha\overset{M}{S}P$-normal. □

Theorem 4.11. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed,continuous, pre-open surjection and (X, τ) is P^*-normal.Then (Y, σ) is P^*-normal.*

Proof. Let $F_1, F_2 \in C(Y)$ and $F_1 \cap F_2 = \phi$. Since f is continuous, then $f^{-1}(F_1)$, $f^{-1}(F_2) \in C(X)$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi$. Since X is P^*-normal, then there are $W, N \in PO(X)$ and $W \cap N = \phi$ such that $f^{-1}(F_1) \subseteq W$ and $f^{-1}(F_2) \subseteq N$. Since f is surjective and pre open, then we have $f(W), f(N) \in PO(Y)$ and $f(U) \cap f(V) = \phi$. Consequently,(Y, σ) is P^*-normal. □

Theorem 4.12. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed,continuous, $\overset{M}{S}$-open surjection and (X, τ) is $\left(\overset{M}{S}\right)^*$-normal.Then (Y, σ) is $\left(\overset{M}{S}\right)^*$-normal.*

Proof. Let $F_1, F_2 \in C(Y)$ and $F_1 \cap F_2 = \phi$. Since f is continuous, then $f^{-1}(F_1)$, $f^{-1}(F_2) \in C(X)$ and $f^{-1}(F_1) \cap f^{-1}(F_2) = \phi$, as X is $(\overset{M}{S})^*$-normal, then there exist $W, N \in \overset{M}{S}O(X)$ and $W \cap N = \phi$ so that $f^{-1}(F_1) \subseteq W$ and $f^{-1}(F_2) \subseteq N$. As f is surjective and $\overset{M}{S}$ open, consequently, $f(W), f(N) \in \overset{M}{S}O(Y)$ and $f(W) \cap f(N) = \phi$. Consequently,(Y, σ) is $\left(\overset{M}{S}\right)^*$-normal. □

5. Some types of near regularity

In this part we present some new types of regularity in a topological space (X, τ) based on $\overset{M}{S}O(X)$, $\overset{M}{S}PO(X)$ and $\overset{M}{S}\alpha O(Y)$.Also, we some study their properties.

Definition 5.1. A topological space (X, τ) is called.

1. $P\overset{M}{S}P$-regular if for every $A \in PC(X), x \notin A$ then there are $U, V \in \overset{M}{S}PO(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(1) α-$\overset{M}{S}\alpha$-Regular if for every $A \in \alpha C(X), x \notin A$ then there exist $U, V \in \overset{M}{S}\alpha O(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(2) $\alpha\overset{M}{S}P$-Regular if for every $A \in \alpha C(X)$,then there exist $U, V \in \overset{M}{S}PO(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(3) $P\overset{M}{S}\alpha$-Regular if for every $A \in PC(X)$,then there exist $U, V \in \overset{M}{S}\alpha O(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(4) $\overset{M}{S}\alpha^*$-Regular if for every $A \in C(X), x \notin A$ then there are $U, V \in \overset{M}{S}\alpha O(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(5) $\overset{M}{S}P^*$-Regular if for every $A \in C(X), x \notin A$ then there are $U, V \in \overset{M}{S}PO(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(6) α^*-Regular if for every $A \in C(X)$, $x \notin A$ then there are $U, V \in \alpha O(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(7) P^*-Regular if for every $A \in C(X)$, $x \notin A$ then there exist $U, V \in PO(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

(8) $\left(\overset{M}{S}\right)^*$-Regular if for every $A \in C(X)$,then there are $U, V \in \overset{M}{S}O(X), U \cap V = \phi$ such that $x \in U$ and $A \subseteq V$.

Remark 5.2. The following figure indicates the relation among the new types of regularity and the other types such that the implications are not reversible.

The following examples show that the implications in Figure 4 need not be reversible.

Example 5.3. (implication 7, 3): Let $X = \{a, b, c\}, = \{X, , \{b, c\}\}$. We have (X, τ) is $\alpha\overset{M}{S}\alpha$-regular but not $P\overset{M}{S}\alpha$ -regular, since there exist $\{a, b\} \in PC(X)$ and $c \notin \{a, b\}$, the only simply α-open set containing $\{a, b\}$is X and $c \in \{b, c\}$ such that $\{b, c\} \cap X \neq \phi$.

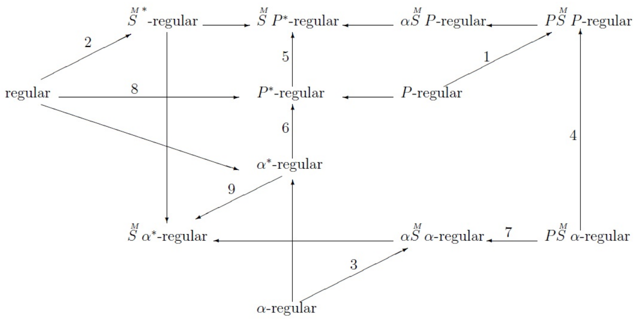

FIGURE 4. Shows the relation among various types of regularity

Also, $\alpha\overset{M}{S}\alpha$-regular but not α-regular, since there exist $\{a\} \in \alpha C(X)$ and $b \notin \{a\}$, the only -open set containing $\{a\}$ is X and $b \in \{b, c\}$ such that $\{b, c\} \cap X \neq \phi$.

(implication 6, 9) We have (X, τ) is $\overset{M}{S}\alpha^*$-regular and P -regular but not α^*-regular,since there exist $\{a\} \in C(X)$ and $b \notin \{a\}$, the only α-open set containing $\{a\}$ is X and $b \in \{b, c\}$ such that $\{b, c\} \cap X \neq \phi$.

(implications 2) We have (X, τ) is $\left(\overset{M}{S}\right)^*$-regular but not regular, since there exist $\{a\} \in C(X)$ and $b \notin \{a\}$, the only open set containing $\{a\}$ is X and $b \in \{b, c\}$ such that $\{b, c\} \cap X \neq \phi$.

(implication 5) Let $X = \{a, b, c, d\}$,$\tau = \{X, \varphi, \{d\}, \{a, d\}\}$. Then we have (X, τ) is $\overset{M}{S}P^*$-regular but not P-regular, since there is $\{b, c\} \in C(X)$ and $a \notin \{b, c\}$,the only pre-open set containing $\{b, c\}$ is $\{b, c, d\}$ and $a \in \{a, d\}$ but $\{a, d\} \cap \{b, c, d\} \neq \varphi$.

(implication 1) Let $X = \{a, b, c\}, = \{X, , \{b\}\}$. Then we have (X, τ) is $p\overset{M}{S}P$-regular but not P-regular, since there exist $\{c\} \in PC(X)$ and $b \notin \{c\}$, the only preopen set containing $\{c\}$ is $\{b, c\}$ and $b \in \{b\}$ but $\{b\} \cap \{b, c\} \neq \varphi$.

In the following theorems we illustrate some properties of the new types of regularity.

Theorem 5.4. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a closed, $\overset{M}{S}P$-irresolute injective map and (Y, σ) is $\overset{M}{S}P^*$-regular. Then (X, τ) is $\overset{M}{S}P^*$-regular.*

Proof. Let $A \in C(X)$ and $x \notin A$. Since f is closed injective, then $\forall x \in X$ $\exists y \in Y$ so that $y = f(x) \notin f(A)$ and $f(A) \in C(Y)$. As (Y, σ) is $\overset{M}{S}P^*$-regular, then there are $W, N \in \overset{M}{S}PO(Y)$ so that $W \cap N = \phi$ and $f(A) \subseteq W, y \in N$. Since f is a $\overset{M}{S}P$-irresolute injection, then $f^{-1}(W)$, $f^{-1}(N) \in \overset{M}{S}PO(X)$ such that $A \subseteq f^{-1}(W), x = f^{-1}(y) \in f^{-1}(N)$ with $f^{-1}(U) \cap f^{-1}(V) = .$ Hence (X, τ) is $\overset{M}{S}P^*$ -regular □

Theorem 5.5. *Let* $f : (X,\tau) \longrightarrow (Y,\sigma)$ *is a* α*-closed,* $\overset{M}{S}\alpha$*-irresolute injective map and* (Y,σ) *is* $\alpha\overset{M}{S}\alpha$*-regular. Then* (X,τ) *is* $\alpha\overset{M}{S}\alpha$*-regular.*

Proof. Let $A \in \alpha C(X)$ and $x \notin A$. Since f is α-closed injective, then $\forall x \in X\ \exists y \in Y$ such that $y = f(x) \notin f(A)$ and $f(A) \in \alpha C(Y)$. Since (Y,σ) is $\alpha\overset{M}{S}\alpha$-regular, then there exists $W, N \in \overset{M}{S}\alpha O(Y)$ so that $W \cap N = \phi$ and $f(A) \subseteq W, y \in N$ As f is a $\overset{M}{S}\alpha$-irresolute injection, then $f^{-1}(W)$, $f^{-1}(N) \in \overset{M}{S}\alpha O(X)$ so that $A \subseteq f^{-1}(W), x = f^{-1}(y) \in f^{-1}(N)$ with $f^{-1}(W) \cap f^{-1}(N) = \varphi$. This implies that (X,τ) is $\alpha\overset{M}{S}\alpha$ -regular □

Theorem 5.6. *Let* $f : (X,\tau) \longrightarrow (Y,\sigma)$ *is a preclosed,* $\overset{M}{S}$*-pre irresolute injective map and* (Y,σ) *is* $P\overset{M}{S}P$*-regular. Then* (X,τ) *is* $P\overset{M}{S}P$*-regular.*

Proof. Let $A \in PC(X)$ and $x \notin A$. Since f is pre closed injective, then $\forall x \in X$ $\exists y \in Y$ such that $y = f(x) \notin f(A)$ and $f(A) \in PC(Y)$. Since (Y,σ) is $P\overset{M}{S}P$-regular, then there exists W, N $\overset{M}{S}PO(Y)$ so that $W \cap N = \phi$ and $f(A) \subseteq W, y \in N$. As f is a $\overset{M}{S}$-pre irresolute injection, then $f^{-1}(W)$, $f^{-1}(N) \in \overset{M}{S}PO(X)$ such that $A \subseteq f^{-1}(W), x = f^{-1}(y) \in f^{-1}(N)$ and $f^{-1}(W) \cap f^{-1}(N) = \varphi$. Thus (X,τ) is $P\overset{M}{S}P$ -regular. □

Theorem 5.7. *Let* $f : (X,\tau) \longrightarrow (Y,\sigma)$ *is a closed,* $\overset{M}{S}\alpha$*-irresolute injective map and* (Y,σ) *is* $\overset{M}{S}\alpha^*$*-regular. Then* (X,τ) *is* $\overset{M}{S}\alpha^*$*-regular.*

Proof. Let $A \in C(X)$ and $x \notin A$. Since f is closed injective, then $\forall x \in X\ \exists y \in Y$ so that $y = f(x) \notin f(A)$ and $f(A) \in C(Y)$.As (Y,σ) is $\overset{M}{S}\alpha^*$-regular, then there are $W, N \in \overset{M}{S}\alpha O(Y)$ that $W \cap N = \phi$ and $f(A) \subseteq W, y \in N$. As f is a $\overset{M}{S}\alpha$-irresolute injection, then $f^{-1}(W)$, $f^{-1}(N) \in \overset{M}{S}\alpha O(X)$ so that $A \subseteq f^{-1}(W), x = f^{-1}(y) \in f^{-1}(N)$ with $f^{-1}(W) \cap f^{-1}(N) = \varphi$. Hence (X,τ) is $\overset{M}{S}\alpha^*$ -regular. □

Theorem 5.8. *Let* $f : (X,\tau) \longrightarrow (Y,\sigma)$ *is a* α*-closed,* $\overset{M}{S}$*-pre irresolute injective map and* (Y,σ) *is* $\alpha\overset{M}{S}P$*-regular. Then* (X,τ) *is* $\alpha\overset{M}{S}P$*-regular.*

Proof. Let $A \in \alpha C(X)$ and $x \notin A$. Since f is α-closed injective, then $\forall x \in X\ \exists y \in Y$ such that $y = f(x) \notin f(A)$ and $f(A) \in \alpha C(Y)$. As (Y,σ) is $\alpha\overset{M}{S}P$-regular, then there are $W, N \in \overset{M}{S}PO(Y)$ that $W \cap N = \phi$ and $f(A) \subseteq W, y \in N$. Since f is a $\overset{M}{S}$-pre irresolute injection, then $f^{-1}(W)$, $f^{-1}(N) \in \overset{M}{S}PO(X)$ so that $A \subseteq f^{-1}(W), x = f^{-1}(y) \in f^{-1}(N)$ and $f^{-1}(W) \cap f^{-1}(N) = \varphi$. Hence (X,τ) is $\alpha\overset{M}{S}P$ -regular. □

6. Conclusion

In this paper we intorduce new types of sets based on near open sets namely simply open, simply α-open and simply pre open sets where these concepts play a major role in topology such that we used these conceptes in introducing new types in natural and regular spaces, we studied the properties of these new concepts and we study the relationship between them and other spaces, as shown in Figure 3 and Figure 4. Also we introduce new types of continuous function and the function of irresolutness based on the simply open sets,simply α- open and simply pre open sets.Finally we use these in soft topology as in [1], [2] and [3].

References

[1] M. El Sayed. Soft simply* generalized continuous mappings in soft topological space.Journal of engineering and applied science 14(9)(2019)3104-3112,https://doi.org/10.36478/jeasci.2019.3104.3112.

[2] M. El Sayed and Abdul Gawad A.Q. Al Qubati. Some properties on soft simply star open sets in soft topological Space Nanoscience and Nanotechnology Letters Vol. 12,(2020)113-119 https://doi.org/10.1166/nnl.2020.3079.

[3] M. El Sayed and Mostafa K. El-Bably. Soft simply open sets in soft topological space. Journal of Computational and Theoretical Nanoscience Vol. 14,(2017) 4100–4103.

[4] N. Levine. Semi open sets and semi continuous mappings. Amr. Math.Monthly 70 (1963) 36-41

[5] A. S. Mashhour, I. A. Hassanein, and S. N. El-Deeb,. On α-continuous and α-open mappings, Acta Math. Hunger 41,(1983), NO 3-4, 213-218.

[6] A. S. Mashhour, M. E. Abd El-Monsef, and S. N. El Deeb. on precontinuous and weak pre continuous mappings. Pro. Math. and Phys. Sci. Egypt 53(1982) 47-53.

[7] S. N. Masheshhwari and S. S. Thakur. on α-irresolute mappings. Tankang J. Math.11, (1980), 209-214.

[8] A. Neubrunove on transfinite sequence of certain types of functions Acta. Fac. Nature Univ. Com. Math, 30(1975)121-126.

[9] G. B. Navalgi Completely pre-irresolute functions and completely gp-irresolute functions International Journal of Mathematics and Computing Applications. January-December, Vol. 3, Nos. 1-2, 2011, pp. 55-65

[10] O. Njastad. On some classes of nearly open sets. Pacific J. Math. 15 (1965) 961-970.

[11] M. H. Stone, Application of the theory of Boolean ranges to general topology. TAMS 41,(1937), 375-810.

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