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MODELING UNCERTAINTY KNOWLEDGE OF THE TOPOLOGICAL METHODS

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Abstract. Most real- life situations have uncertain information; the world needs to be modeled every time to confront epidemics, such as the COVID-19. Measuring approximation accuracy is an important goal for re-searchers in many theoretical and applied areas. Thus, the aim of this study is to take advantage of the rough set; rough set theory can be considered as a topological method, which achieves a balance in digital infor-mation, so as to reduce superfluous information due to the denseness of digital information. This has been clarified by the application in this study. The results are obtained by using the MATLAB program.

1. Introduction

The topology was used in the fourth decade of the nineteenth century and the greatest evidence of the importance of topology [1]. The Nobel Prize in Physics in 2016 was awarded because of topological uses in the theory of the transformation of the material in addition to international schools in Germany and America are using topological applications in science and engineering. Information on the surrounding world is imprecise, incomplete or uncertain [[1], [2]]. Information granulation is very important for the solution of human, problems and therefore has a very significant impact on the design and implementation of intelligent systems. In information theory, one approach to supporting information reasoning and analysis based on varying levels of conceptualization is the emerging disciple of granular computing, a term coined together by [9]. To order to take a decision on incompleteness, or uncertainty, new mathematical methods were used for applications [[3], [5], [7]], Pawlak presented the notion of rough sets up to 1982 [[10], [11]] as a formal theory deriving from fundamental research into the logical properties of information systems. It is clear that there is another set, namely fuzzy theory [15], where this theory and the rough set theory complement classical sets generalizations. Where 21^{st} century's main significant factor. Throughout our lives,

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information technology is simply like all the expertise in this is available [4] as a method for conceptualizing, arranging, and evaluating various forms of data mining. Where the rough set approach has many uses for process control, economics, medical diagnosis, biology,... etc. Rough sets theory [11] can be considered as a topological method because it basically depends on the partition generated by the equivalence relationship and the topology generated by the partition [[6],[13]]. The concepts of core and reduced in the case of attributes and knowledge [[8],[14]] are two basic concepts of the rough set theory. The effectiveness of the rough collection of data analysis is based on topological approaches to solving uncertain problem solving. A rough set's performance in data analysis explicitly focuses on the focus on topological approaches to solving uncertainty issues. We're getting a proposed accuracy that depends on the neighborhood, and reduction method. Our research reports a new model that is more accurate, competing with reduction method. In addition, the lower (respectively upper) approximation of any subset is exactly the interior (respectively closure) of the subset. Therefore the starting point for applying topological definition in the approximation method is the use of closure and interior design [1].

2. Preliminaries

Now, we will provide the notes and definitions that will be used to present our work.

2.1. Rough sets and Approximations.

Definition 2.1. [10] Let U be a finite set, and let R be an equivalence relation on U. This relationship R generates a partition $U/R = \{Y_1, Y_2, ..., Y_m\}$ on U, where $Y_1, Y_2, ..., Y_m$are equivalence classes. These equivalence classes are also called the elementary sets of R. For any $X \subseteq U$, we can be describe X by the elementary sets of R and the following two sets

$$L(X) = \cup\{Y_i \in U/R : \{Y_i\} \subseteq X\},$$

$$U(X) = \cup\{Y_i \in U/R : \{Y_i\} \cap X \neq \phi\}.$$

Which are referred to as the lower and upper X approximation, respectively.

Definition 2.2. [11] If the lower and upper approximations are identical (i. e., $L(X) = U(X)), X$ is definable.

Otherwise, set X is undefinable in U, There are four kinds of undefinable sets in U:

(1) if $L(X) \neq \phi$, and $U(X) \neq U$, X is referred to as roughly undefinable in U;
(2) if $L(X) \neq \phi$, and $U(X) = U$, X is externally undefinable in U;
(3) if $L(X) = \phi$, and $U(X) \neq U$, in U , X is called internally undefinable;
(4) if $L(X) = \phi$, and $U(X) = U$, X is totally undefinable in U.

Definition 2.3. [13] If U is a universe and R is a binary relationship on U, we define:

(1) "After set" as follows: ${}_xR = \{y : {}_xR_y\}$.
(2) "For set" as follows: $R_y = \{x : {}_xR_y\}$.

Definition 2.4. [11] Let (U, R, τ_R) be a topological space. If $B \subseteq U$, then the lower (resp. upper) approximation of B is defined by $\underline{R}B = B^o\,(resp.\ \overline{R}B = \overline{B})$. For a topological space$(U,\ R,\ \tau_R)$, the following description introduces new definability concepts for a subset $B \subseteq U$.

Remark 2.5. In general $\overline{R}(\underline{R}X) \neq \underline{R}X$ and $\underline{R}(\overline{R}X) \neq \overline{R}X$. The following is an example that illustrates this idea and also, it is computing the degree of certainty of uncertain concepts using topological approximations

Example 2.6. Let $U = \{x_1,\ x_2,\ x_3,\ x_4,\ x_5\}$ and R be a relation defined as follows:

$$
\begin{aligned}
R &= \{(x_1,\ x_2),(x_1,\ x_4),(x_2,x_5),(x_2,x_1),(x_3,x_3),(x_3,x_2),(x_4,x_5),(x_5x_5),(x_5,x_1)\},\\
S &= \{\{x_2,x_4\},\{x_1,x_5\},\{x_2,x_3\},\{x_5\}\}, B = \{\phi,\{x_2\},\{x_5\},\{x_1,x_5\},\{x_2,x_3\},\{x_2,x_4\}\},\\
\tau &= \{U,\phi,\{x_2\},\{x_5\},\{x_1,x_5\},\{x_2,x_3\},\{x_2,x_4\},\{x_2,x_5\},\{x_2,x_3,x_4\},\{x_2,x_4,x_5\},\\
&\qquad \{x_2,x_1,x_5\},\{x_2,x_3,x_5\},\{x_1,x_2,x_4,x_5\},\{x_1,x_2,x_3,x_5\},\{x_4,x_2,x_3,x_5\}\}.\\
\tau^c &= \{U,\phi,\{x_1\},\{x_3\},\{x_4\},\{x_1,x_4\},\{x_4,x_3\},\{x_1,x_3\},\{x_1,x_5\},\{x_1,x_3,x_4\},\{x_1,x_4,x_5\},\\
&\qquad \{x_2,x_3,x_4\},\{x_1,x_3,x_5\},\{x_1,x_2,x_4,x_3\},\{x_1,x_4,x_3,x_5\}\}.
\end{aligned}
$$

Let $X = \{x_4,\ x_5\}$, then $\underline{R}X = \{x_5\}, \overline{R}\,(\underline{R}X) = \{x_1,\ x_5\} \neq \underline{R}X$. $\overline{R}X = \{x_1,\ x_2,\ x_4,\ x_5\}, \underline{R}(\ \overline{R}X\ ) = \{x_1,\ x_2,\ x_3,\ x_4,\ x_5\} \neq \overline{R}X.$

3. Decision rough set

The decision-theoretic rough set provides new insights into the probabilistic rough set of approaches. One of the important applications of the basic set theory is the induction of decision [[1], [3]] or classification rules. Decisions will become very important in many applications in the years to come, such as water and breathing. These rough set models focus on extending the original model to Pawlak [11] and try to address its limitations, such as the handling of statistical distributions or noisy data.

Definition 3.1. [1] If (U, E, V, f) is an information system which defines function

${}_xR_{e_{iy}}$ iff ${}_xR_{e_{iy}} = \{y\ :\ |\,e_i\,(x) - e_i\,(y)\,| < \epsilon\}$, where ϵ is determined by the field expert. For example, if the information is in the medical field, a person interested in the medicine and dealing with the problem will be the expert. So, for each of them $e_i \in E$ we can get a classification O/R_{e_i} which is $\{{}_xR_{\,e_i}\ :\ x \in O\ \}$, where O is a finite set.

Definition 3.2. [11] Let $Z = \{X_1, X_2, ..., X_n\}$, be a set family like $X_i \subseteq U, i = 1, 2, 3..., n$. They say that X_i can be dispensed in Z, if $\cup(Z - X_i) = \cup Z$; otherwise the set X_i in Z is indispensable. The Z family is independent if all its components are indispensable to Z; otherwise Z is dependent. The family $H \subseteq Z$ is a reduct of Z, if H is independent and $\cup H = \cup Z$. The family of all indispensable ones set in Z will be called the core of Z.

Theorem 3.3. [10] *CORE* $(Z) = \cap RED(Z)$*, in which* $RED(Z)$ *is a family of all* Z *reduct.*

4. Information Data

4.1. Data. We're introducing a new reduction in this work. Information technology and how topology application areas are addressed by Pawlak in the current rough set theory. We start the application [12], beginning with an information system appropriation by translating the attribute values $\{e_1, e_{2,}..., e_4\}$ and the value of the decision attribute $\{d\}$ into qualitative terms. The data concerned the modeling of the energy of protein unfolding, where there are 9 coded amino acids (AAs). The AAs are described in terms of seven attributes: e_1= PIE, e_2= PIF, e_3= DGR of transfer from the inner protein to water, $e_{4=}$ SAC = surface area. The application starts with the translation into qualitative terms of the values of the attributes $\{e_1,\ e_{2,}...\ e_4\}$ and the value of the decision attribute $\{d\}$ into Table 1.

Table 1. Data sets on the decisions of the knowledge

Objects	**Attributes**				**Decision**	
	e_1	e_2	e_3	e_4	d	
w_1	.23	.31	$-.55$	2.126	8.5	8.5
w_2	$-.48$	$-.60$	0.51	2.994	8.2	8.2
w_3	$-.61$	$-.77$	1.20	2.994	8.5	8.5
w_4	.45	1.45	-1.4	2.933	11.0	11
w_5	$-.11$	$-.22$	0.29	3.458	6.3	6.3
w_6	$-.51$	$-.64$	0.76	3.243	8.8	8.8
w_7	0.00	0.00	0.00	1.662	7.1	7.1
w_8	.15	0.13	$-.25$	3.856	10.1	10.1
w_9	.38	.490	-1.5	2.876	8.2	8.2

State attributes are coded in four qualitative terms, for example: very low, low, high and very high, whereas decision attributes are coded in three qualitative terms such as: low, medium, and high. Alternatively, abstract representations of all properties are represented using natural numbers. Table 2, displays the set of code attributes that we selected for consistency.

Table 2. Description of the ranges of attribute coding

Attributes	**codes**			
	1	2	3	4
e_1	< -0.115	$[-0.115, 0.54)$	$[0.54, 1.195)$	> 1.195
e_2	< -0.18	$[-0.18, 0.63)$	$[0.63, 1.44)$	> 1.44
e_3	< -1.725	$[-1.725, -0.75)$	$[-0.75, 0.225)$	> 0.225
e_4	< 2.685	$[2.685, 3.708)$	$[3.708, 4.732)$	> 4.732
d	< 9.8	$[9.8, 13.3)$	$[13.3, 16.8)$	

In the Algorithm 1, a codding attributes is proposed based on rough set, and each process of the algorithm is described in detail as follows. The coded system for information is given in the table 3:

Algorithm 1 codding attributes based on the rough set

```
function [M] = coding(xapp,code);

[nl,nc] = size(xapp);
M = zeros(nl,nc);

for i = 1:nc
M(:,i) = (xapp(:,i) - min(xapp(:,i)))/(max(xapp(:,i)) - min(xapp(:,i)));
end
[I,J] = find(M >= ((code-1)/code) & M <= 1);
for i = 1:length(I)
M(I(i),J(i)) = code;
end
for i = 1:(code-1)
[I,J] = find(M >= ((i-1)/code) & M < ((i)/code));
for t = 1:length(I)
M(I(t),J(t)) = i;
end
end
```

Table 3. Data decisions about the details after converting by coding

Objects	**Attributes**				**Decision**
	e_1	e_2	e_3	e_4	d
w_1	2	2	3	1	1
w_2	1	1	4	2	1
w_3	1	1	4	2	1
w_4	2	4	2	2	2
w_5	2	1	4	2	1
w_6	1	1	4	2	1
w_7	2	2	3	1	1
w_8	2	2	3	3	2
w_9	2	2	2	2	1

Now, in table 4, we form the classification (elementary set) induced by $IND(E)$ relation to indiscernibility.

Table 4. elementary set induced by *IND* (E)

U/E	**E**			
	e_1	e_2	e_3	e_4
$W_1 = \{\ w_1, w_7\}$	2	2	3	1
$W_2 = \{\ w_2, w_3, w_6\}$	1	1	4	2
$W_3 = \{\ w_4\}$	2	4	2	2
$W_4 = \{\ w_5\}$	2	1	4	2
$W_5 = \{\ w_8\}$	2	2	3	3
$W_6 = \{\ w_9\}$	2	2	2	2

Next, by leaving out the e_1 attribute in table 5 as follows

Table 5. Removes from Table 3 attribute e_1

$U/E - \{e_1\}$	$E - \{e_1\}$		
	e_2	e_3	e_4
$W_1 = \{\ w_1, w_7\}$	2	3	1
$W_2 = \{\ w_2, w_3, w_5, w_6\}$	1	4	2
$W_3 = \{\ w_4\}$	4	2	2
$W_4 = \{\ w_8\}$	2	3	3
$W_5 = \{\ w_9\}$	2	2	2

Also next, by leaving out the e_2 attribute in table 6 as follows.

Table 6. Removes from Table 3 attribute e_2

$U/E - \{e_2\}$	$E - \{e_2\}$		
	e_1	e_3	e_4
$W_1 = \{\ w_1, w_7\}$	2	3	1
$W_2 = \{\ w_2, w_3, w_6\}$	1	4	2
$W_3 = \{\ w_4, w_9\}$	2	2	2
$W_4 = \{\ w_5\}$	2	4	2
$W_5 = \{\ w_8\}$	2	3	3

We get the $W_2$and W_4 objects to be equal when removing e_1, and we get the $W_3$and W_6 objects to be equal when removing e_2. We note that,
IND $(E) \neq$ *IND* $(E$ – $\{e_1\})$, ..., then e_1, e_2 and e_4 are indispensable. Anyway, we'll get Table 7 by leaving e_3. As shown below

Table 7. Removing Attribute $e_3$from Table 3

$U/E - \{e_3\}$	$E - \{e_3\}$		
	e_1	e_2	e_4
$W_1 = \{\ w_1, w_7\}$	2	2	1
$W_2 = \{\ w_2, w_3, w_6\}$	1	1	2
$W_3 = \{\ w_4\}$	2	4	2
$W_4 = \{\ w_5\}$	2	1	2
$W_5 = \{\ w_8\}$	2	2	3
$W_6 = \{\ w_9\}$	2	2	2

As similar, we get e_3 removed then we obtain $IND(E) \;\; = IND(E - \{e_3\})$, and there are superfluous e_3.

Algorithm 2 Core attributes one removal based on the rough set

```
function [core] = core_attributes_one_removal(M);
[pos] = object_reduction(M);
s = find(pos == 0);
pos(s) = [];
M = M(pos,:);
core = [];
M1 = M;
[nl,nc] = size(M1);
for i = 1:nc
M1(:,i) = [];
[pos] = object_reduction(M1);
if isempty(find(pos == 0)) == 0
core = [core;[i,length(find(pos == 0))]];
M1 = M;
end
```

If we remove attributes e_1, e_2 and $e_4$the number of elementary sets becomes smaller, but by removing attribute e_3,we do not change the elementary sets. Attribute e_3 is superfluous, whereas attributes e_1, e_2 and e_4 are indispensable.

Table 8. Eliminating Attributes

	Removal of attributes				
Number of elementary sets	None	e_1	e_2	e_3	e_4
	6	5	5	6	5

As represented in Table 9 In this way was introduced the set of all partitions induced by $IND(E^{'})$:

Table 9. The set of all partitions induced by $IND(E^{'})$

$U/E^{'}$	Attributes			Decision
	e_1	e_2	e_4	d
$T_{\mathbf{1}} = \{\, w_1, w_7\}$	2	2	1	1
$T_{\mathbf{2}} = \{\, w_2, w_3, w_6\}$	1	1	2	1
$T_3 = \{\, w_4\}$	2	4	2	2
$T_4 = \{\, w_5\}$	2	1	2	1
$T_5 = \{\, w_8\}$	2	2	3	2
$T_6 = \{\, w_9\}$	2	2	2	1

In this section we will generate the rules based on reduct and core of table
From Table 9 we calculate the strength of the attributes $e_{1,}e_2,\ e_4$ as follows:
We can find the strength of the rules for the attributes
$(e_1 = 2 \rightarrow D = 1)$ the strength of this particular rule comes out 60%.
$(e_1 = 2 \rightarrow D = 2)$ the strength of this particular rule comes out 40%.
$(e_1 = 1 \rightarrow D = 1)$ the strength of this particular rule comes out 100%.
Similarly we can find the strength of rules for strength of the particular e_2 *as follows:*
$(e_2 = 1 \rightarrow D = 1)$ the strength of this particular rule comes out 100%.
$(e_2 = 2 \rightarrow D = 1)$ the strength of this particular rule comes out 66%.
$(e_2 = 2 \rightarrow D = 2)$the strength of this particular rule comes out 33%.
$(e_2 = 4 \rightarrow D = 2)$ the strength of this particular rule comes out 100%.

Similarly we can find the strength of rules for strength of the particular e_4 as follows:
($e_4 = 1 \rightarrow D = 1$) the strength of this particular rule comes out 100%. ($e_4 = 2 \rightarrow D = 1$) the strength of this particular rule comes out 75%.
($e_4 = 2 \rightarrow D = 2$) the strength of this particular rule comes out 25%.
($e_4 = 3 \rightarrow D = 2$) the strength of this particular rule comes out 100%.
From these calculations we can find that the attributes $e_{1,}e_2$, and e_4 are indispensable and we easily find that attribute $e_4$is the power indispensable, but the next indispensable is e_2 and the third indispensable e_1.

4. Reduction Method

In the decision of the following Table 10, we illustrate reduction method, containing four attributes $\{e_1, e_2, e_3, e_4\}$ and one decision $\{d\}$ describes eight objects as belonging. We need for every superfluous condition values in any decision rule in the next. Even, by using definition 5 we measure core values of the condition attribute.

Table 10. Decision table of some objects

Objects	**Attributes**			
	e_1	e_2	e_3	e_4
w_1	2	2	3	1
w_2	1	1	4	2
w_3	1	1	4	2
w_4	2	4	2	2
w_5	2	1	4	2
w_6	1	1	4	2
w_7	2	2	3	1
w_8	2	2	3	3

Consider the sets families $F_1, ..., F_{8,}$ where
$F_1 = \{[1]_{e1}, [1]_{e2}, [1]_{e3}, [1]_{e4}\} = \{\{1, 4, 5, 7, 8\}, \{1, 7, 8\}, \{1, 7\}\}$.
$\cup F_1 = \{1, 4, 5, 7, 8\}$, since $e_1(1) = 2, e_2(1) = 2, e_3(1) = 3, e_4(1) = 1$.
To find it dispensable,
$\cup(F_1 - [1]e_1) = \{1, 7, 8\} \cup (F_1 - [1]e_1) = \{1, 7, 8\} \neq \cup F_1, \cup(F_1 - [1]e_2) = \{1, 4, 5, 7, 8\} = \cup F_1$,
$\cup(F_1 - [1]e_3) = \{1, 4, 5, 7, 8\} = \cup F_1, \cup(F_1 - [1]e_4) = \{1, 4, 5, 7, 8\} = \cup F_1$.
That means the core value is $e_1(\mathbf{1}) = 2$ and also find that
$F_2 = \{[2]_{e1}, [2]_{e2}, [2]_{e3}, [2]_{e4}\} = \{\{2, 3, 6\}, \{2, 3, 5, 6\}, \{2, 3, 4, 5, 6\}\}, \cup F_2 = \{2, 3, 4, 5, 6\}$,since $e_1(2) = 1, e_2(2) = 1, e_3(2) = 4, e_4(2) = 2$.
In order to find dispensable,
$\cup(F_2 - [2]e_1) = \{2, 3, 4, 5, 6\} = \cup F_2, \cup(F_2 - [2]e_2) = \{2, 3, 4, 5, 6\} = \cup F_2$,
$\cup(F_2 - [2]e_3) = \{2, 3, 4, 5, 6\} = \cup F_2, \cup(F_2 - [2]e_4) = \{2, 3, 5, 6\} \neq \cup F_2$.
This implies $e_4(\mathbf{2}) = 2$ is the core and we consider identical $F_3, F_4, .., F_8$.

Algorithm 3 union method based on an information system

```
function [MR] = union_method(M);
[nl,nc] = size(M);
for i = 1:nl
eval(['F' num2str(i) '= []']);
U = eval(['F' num2str(i)]);
for j = 1:nc
eval(['F' num2str(i) num2str(j) '= find(M(:,j) == M(i,j))']);
V = eval(['F' num2str(i) num2str(j)]);
eval(['F' num2str(i) '= [U,[V;NaN(nl-length(V),1)]]']);
U = eval(['F' num2str(i)]);
longueur(1,j) = length(V);
end
s = max(longueur);
U = eval(['F' num2str(i)]);
eval(['F' num2str(i) '= U(1:s,:)']);
end
for i = 1:nl
[I] = union_multiple(eval(['F' num2str(i)]));
eval(['Inter_F' num2str(i) '= I']);
end
for i = 1:nl
A = eval(['F' num2str(i)]);
B = A;
for j = 1:nc
A(:,j) = [];
[II] = union_multiple(A);
eval(['Inter_F' num2str(i) num2str(j) '= II']);
A = B;
end
MR = M;
for i = 1:nl
AA = eval(['Inter_F' num2str(i)]);
for j = 1:nc
BB = eval(['Inter_F' num2str(i) num2str(j)]);
if isequal(AA,BB) == 1
MR(i,j) = 0;
end
```

Table 11 makes the final decision, as follows

Table 11. Decision table by Reduction method

Objects	$\cup(F_i - e_k) = \cup F_i$			
	e_1	e_2	e_3	e_4
w_1	2	$-$	$-$	$-$
w_2	$-$	$-$	$-$	2
w_3	$-$	$-$	$-$	2
w_4	2	$-$	$-$	2
w_5	2	$-$	$-$	$-$
w_6	$-$	$-$	$-$	2
w_7	2	$-$	$-$	$-$
w_8	2	$-$	$-$	$-$

5. Neighborhood method

We notice that, the coded method of converting objectives are the same as goals w_2, w_3, w_6 and also objectives w_1, w_7 are the same as shown in Table 3. So, we thought to solve the problems for the other methods. By using neighborhood, we need to make a reduction in tables in the information system. The next is determining the proposal's method per description. As shown below, let's illustrate Table 1.

Table 12. Decision table of some objects original data

Objects	**Attributes**			
	e_1	e_2	e_3	e_4
w_1	.23	.31	$-.55$	2.126
w_2	$-.48$	$-.60$	0.51	2.994
w_3	$-.61$	$-.77$	1.20	2.994
w_4	.45	1.54	-1.40	2.933
w_5	$-.11$	$-.22$	0.29	3.458
w_6	$-.51$	$-.64$	0.76	3.243
w_7	0.00	0.00	0.00	1.662
w_8	.15	0.13	$-.25$	3.856

Look at the $e_1$attribute in the Table 12: we're taking the neighborhoods between them as follows: $|x_i - x_j| \leq 0.6$,
Consider the $F_1,...,F_8$ sets families, where
$F_1 = \{[1]_{e1}, [1]_{e2}, [1]_{e3}, [1]_{e4}\} = \{\{1,4,5,7,8\}, \{1,5,7,8\}, \{1,7,8\}, \{1,7\}\}$,
$\cup F_1 = \{1,4,5,7,8\}$, since $e_1(1) = 0.23, e_2(1) = 0.31, e_3(1) = -0.55, e_4(1) = 2.126$.
In order to find dispensable,
$\cup(F_1 - [1]e_1) = \{1,5,7,8\} \neq \cup F_1, \cup(F_1 - [1]e_2) = \{1,4,5,7,8\} = \cup F_1$,
$\cup(F_1 - [1]e_3) = \{1,4,5,7,8\} = \cup F_1, \cup(F_1 - [1]e_4) = \{1,4,5,7,8\} = \cup F_1$.
This implies $e_1(1) = 0.23$ is the core value, and we also find that
$F_2 = \{[2]_{e1}, [2]_{e2}, [2]_{e3}, [2]_{e4}\} = \{\{2,3,5,6,7\}, \{2,5,6,7\}, \{2,3,4,6\}\}$,
$\cup F_2 = \{2,3,4,5,6,7\}$,
since $e_1(2) = -0.48, e_2(2) = -0.6, e_3(2) = 0.51, e_4(2) = 2.994$.
In order to find dispensable,
$\cup(F_2 - [2]e_1) = \{2,3,4,5,6,7\} = \cup F_2, \cup(F_2 - [2]e_2) = \{2,3,4,5,6,7\} = \cup F_2$,
$\cup(F_2 - [2]e_3) = \{2,3,4,5,6,7\} = \cup F_2, \cup(F_2 - [2]e_4) = \{2,3,5,6,7\} \neq \cup F_2$.
This means that the core values is $e_4(\mathbf{2}) = 2.994$. By similar, we can find $F_3, F_4, ..., F_8$.
In table 13, the final decision is given by the neighborhood method as in the following

Table 13. Decision table by the method of neighborhood

Objects	$\cup(F_i - e_k) = \cup F_i$			
	e_1	e_2	e_3	e_4
w_1	0.23	–	–	–
w_2	–	–	–	2.994
w_3	–	–	–	2.994
w_4	0.45	–	–	2.933
w_5	–	–	–	–
w_6	−0.51	–	–	3.243
w_7	0	–	–	–
w_8	0.15	–	–	–

The final decision, as indicated in the following

Table 14. Decision final table by neighborhood method

Objects		$\cup(F_i - e_k) = \cup F_i$	
		e_1	e_4
$I.$	w_1, w_7, w_8	2	–
$II.$	w_2, w_3	–	2
$III.$	w_5	–	–
$IV.$	w_4	2	2
$V.$	w_6	1	2

Table 15. Decision final table by Reduction method

Objects		$\cup\ (F_i - [e_k]) = \cup F_i$	
		e_1	e_4
$I.$	w_1, w_7, w_8, w_5	2	–
$II.$	w_2, w_3, w_6	–	2
$III.$	w_4	2	2

Let us assume that the following two sets of objects are to classified using the Reduction method data from Table 15 and the neighborhood method data from Table 14, We split the decision accordingly $Y_1 = \{w_1, w_2,\ w_3,\ w_5,\ w_6,\ w_7\} = d_1 = \{1\}$ and $Y_2 = \{w_4,\ w_8\} = d_2 = \{2\}$.

Algorithm 4 Lower, upper and accuracy based on rough set

```
function [core,Acc,MR,Lower5,Upper5,Class100] =
Lower_Upper(xapp,yapp,code);
[M] = coding(xapp,code);
M = xapp;
[core] = core_attributes_one_removal(M);
[posss] = object_reduction(M);
tt = find(posss==0);
M(tt,:) = [];MR = M;
yapp(tt) = [];
[MR] = coding(xapp,code);
[posss] = object_reduction(MR);
tt = find(posss==0);
[nl,nc] = size(MR);
D = unique(yapp);
Acc = zeros(1,length(D));
for i = 1:length(D)
eval(['D' num2str(D(i)) '= find(yapp == D(i));']);
end
MR1 = MR;
obs = [1:nl]';
t = 0;
while isempty(obs) == 0
pos = [];
for i = 1:length(obs)
if isequal(MR1(1,:),MR1(i,:)) == 1
pos = [pos,i];
end
t = t + 1;
eval(['Class' num2str(t) '= obs(pos)']);
obs(pos) = [];
MR1(pos,:) = [];
end
for i = 1:length(D)
eval(['Lower' num2str(i) '= []']);
eval(['Upper' num2str(i) '= []']);
end
for i = 1:length(D)
for j = 1:t
j
A = eval(['D' num2str(D(i))]);
B = eval(['Class' num2str(j)]);
if isequal(intersect(A,B),B) == 1
L = eval(['Lower' num2str(i)]);
eval(['Lower' num2str(i) '= [L;B]']);
end
if isempty(intersect(A,B)) == 0
U = eval(['Upper' num2str(i)]);
eval(['Upper' num2str(i) '= [U;B]']);
end
for i = 1:length(D)
R = eval(['Lower' num2str(i)]);
S = eval(['Upper' num2str(i)]);
Acc(i) = length(R)/length(S);
end
```

The following table get the lower, upper approximations and the accuracy of their classification:

Table 16. Accuracy of Reduction and Neighborhood methods

Class number	**Objects**	**Pawlak method**			**Neighborhood method**		
		Lower	***Upper***	***Accuracy***	***Lower***	***Upper***	***Accuracy***
d_1	6	3	7	3/7	4	7	4/7
d_2	2	1	5	1/5	1	4	1/4

Comparing between Tables 14 and Tables 15, we find that the new work satisfy all aims from saving time and effort. The new method is best to deal with the fact information system. Table 16 clearly shows that the neighborhood approach is better than the reduction method.
Removed attribute e_4: from using the data of Reduction method from Table 15 and the data of neighborhood method from Table 14, we get the results shown in the following table

Table 17. Removed attribute e_4

Neighborhood method		**Reduction method**	
$U/C-\{e_4\}$	e_1	$U/C-\{e_4\}$	e_1
$X_1=\{w_1,w_4,w_7,w_8\}$	2	$X_1=\{w_1,w_4,w_5,w_7,w_8\}$	2
$X_2=\{w_2,w_3,w_5\}$	$-$	$X_2=\{w_2,w_3,w_6\}$	$-$
$X_3=\{w_6\}$	1	-	-

From Neighborhood method, we find that:
The lower approximation is $L(Y_1)=4\neq\phi$,the upper approximation is $U(Y_1)=8=U$, Then X is called externally undefinable in U. Accuracy of approximation $\mu(Y_1)={}^{L(Y_1)}\!/_{U(Y_1)}={}^1\!/_2$.
From Reduction method, we find that:
The lower approximation is $L(Y_1)=3\neq\phi$, the upper approximation is $U(Y_1)=8=U$, Then X is called externally undefinable in U.
Accuracy of approximation$\mu(Y_1)={}^{L(Y_1)}\!/_{U(Y_1)}={}^3\!/_8$.
From the illustration above, we show that the neighborhood reduction is better than of reduction method of finding reduction.

6. Conclusion

This may be a time-consuming task to use all knowledge arising from an experimental calculation. Hence the removal of superfluous attributes process is necessary to save time and energy. Topological methods (rough set) are very important and useful for solving unknown problems. This research offers new insight into the issue of rising attributes. This implies that more semantic properties conserved by that an attribute should be carefully investigated. Knowledge reduction involves the elimination of superfluous partitions (equivalence relationships). This procedure enables us to remove from the knowledge base all unnecessary knowledge, preserving only that part of the knowledge which is really useful.
Looking at this paper, we'll find it serves all branches of science. In energy networks, pharmaceutical plants, for instance, patients care, ... etc. We show that if you use medical research, we find that there is a difference with experience between medical practitioners' research and doctoral advisors, and this paper may help practitioner' skills to manage their patients, as well as to assess the right drug in the treatment of disease.

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