Poincare Journal of Analysis & Applications
Vol. 8, No. 1(II) (2021), 53-68

AN EFFICIENT SYNCHRONIZATIONAL APPROACH TO THE STABILITY ANALYSIS OF MICROSCOPIC CHAOS GENERATED IN CHEMICAL REACTOR SYSTEM VIA ACTIVE AND ADAPTIVE CONTROL METHODS

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Date of Receiving : 23. 03. 2021
Date of Revision : 20. 06. 2021
Date of Acceptance : 26. 06. 2021

Abstract. This study investigates the synchronization accomplishments of two broadly used chaos synchronization strategies: active control and adaptive control. It is demonstrated that the considered techniques have efficient performances in the synchronization of microscopic chaos generated in chemical reactor system (CRS) with the adaptive control method slightly outperforming the active control design in terms of transitory analysis. Nevertheless, the complexity of nonlinear adaptive control functions indicates that the active control would be more achievable in several engineering applications.

1. Introduction

Over the years, nonlinear chemical dynamics has progressed to be the most intriguing interdisciplinary field having enormous applications in all branches of chemistry, mathematics, biology, physics, engineering, and technology. During 1980 to 1995, macroscopic chaos is one of the illustrious phenomena found in several systems from laser optics [10] and fluid mechanics [6] to neurophysiology [9, 32] along with widely known Belousov-Zhabotinskii autocatalytic chemical reaction [38], the indium or gas-phase reactions [40] and the indium or thiocyanate electrochemical oscillator [22]. The aforesaid chaotic systems are indicated by high sensitivity to initial conditions which is prescribed as "Butterfly effect" in chaos literature. Fundamentally, this term is advocated by E.N. Lorenz [26] while studying a weather prediction model in 1963.

Microscopic chaos is specifically the irregularity or unpredictability of collisional motion of molecules and atoms presented in fluids. This existing chaos animated differential mesoscopic stochastic methods and reaction-diffusion development. These

2010 *Mathematics Subject Classification.* 34K23, 34K35, 37B25, 37N35.
Key words and phrases. Active control, Adaptive control, Chaotic system, Chemical reactor system, Complete synchronization, Lyapunov stability theory.
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Communicated by. Faisal Khan

procedures are of vital significance in chaos theory. The synchronization and control in chaotic systems are nowadays widely studied areas of nonlinear science that were firstly initiated in 1990 by Ott et al. [42] using a technique known as OGY method; and Pecora and Carroll [34] using a technique called APD scheme respectively. Numerous linear as well as nonlinear techniques [3, 4, 8, 14, 15, 17, 19, 20, 23, 28–30, 33, 37, 41, 43, 45, 47, 50] have appeared thereafter in exploration of extremely effective algorithms for controlling and synchronizing non-identical and identical chaotic systems. Typically, active control and adaptive control are two of the most classical methods. Chaos synchronization utilizing active control method was first introduced by Bai and Lonngren [1] in 1997 and has been hugely established as an effective approach for synchronizing the chaotic systems, because it may be used to control and synchronize different systems as well; a property which makes it more advantageous in comparison with other synchronization techniques. This technique has been applicable to several practical models, viz., spatio-temporal dynamical systems [5], nonlinear Bloch equations [49], the Rikitake system [21, 54], Van der Pol Dufffing oscillators [31], electric circuits [2], complex dynamos [27], modified Chua's circuit [57], El-Nino chaotic system [21], Josephson junctions [48], non-equilibrium physics-ratchets [53]. Furthermore, we have recently synchronized the identical hyperchaotic systems using active controlled hybrid projective synchronization technique [20]. On the other hand, adaptive control method was first described by Hubler [12] in 1989 and was recently used to synchronize the chaotic systems [23]. Adaptive control technique has been used recently to control 3-D chaotic generalized Lotka-Volterra biological systems [14, 16, 18, 19, 21, 51], chaotic satellite system [23], modified Chua's circuit [46, 57]; and to synchronize the four hyperchaotic systems [20] and illustrate the controlling in El-Nino chaotic systems [21]; among many applications. Specifically, the adaptive methodology is a comprehensive approach that prominently intertwines the preferred choice for a Lyapunov function along with the suitably designed controller. In fact, adaptive control approach may guarantee global stability, parameter identification and transient performances for a wide class of nonlinear chaotic systems [23].

Considering the above discussed facts and literature review, we in this work design systematically the active and adaptive controllers that will guarantee global synchronization in identical chaotic CRSs introduced by Singh and Roy [44] and compare numerical simulation results of these considered methods. In addition, adaptive control is very important approach for estimating the unknown parameters of chaotic systems. More importantly, many authors [7, 11, 13, 24, 25, 36, 39, 52, 55, 56] have studied and investigated the dynamics of numerous chaotic chemical reactors by using various synchronization and control techniques. Moreover, we use master-slave/ drive-response configuration in order to achieve global stability effectively.

The remainder of the present paper is arranged as: Sect. 2 indicates few preliminaries comprising of some basic notations and necessary terminology to be utilized in upcoming sections. A systematic methodology for adaptive control is explained in Sect. 3. Sect. 4 describes some fundamental features of discussed CRS. In Sect. 5, complete synchronization in identical CRS using active control design is presented; whereas in Sect. 6, adaptive control technique for synchronization among identical CRS is investigated. Sect. 7 deals with comparison analysis of simulation results. Lastly, the considered work is concluded in Sect. 8.

2. Preliminaries

In this section, we introduce notations and terminology and mention some basic results to be used in the subsequent sections of the paper. Consider the master system and the corresponding slave system as:

$$\dot{z}_m = F_1(z_m), \tag{2.1}$$

$$\dot{z}_s = F_2(z_s) + U, \tag{2.2}$$

where $z_m = (z_{m1}, z_{m2}, \dots, z_{mn})^T$, $z_s = (z_{s1}, z_{s2}, \dots, z_{sn})^T$ are the state variables of (2.1) and (2.2) respectively, $F_1, F_2 : R^n \to R^n$ are two nonlinear continuous vector functions and $U = (U_1, U_2, \dots, U_n) \in R^n$ is the suitable controller to be constructed.

Definition 2.1. The master system (2.1) and the slave system (2.2) are said to be in complete synchronization (CS) if

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|z_s(t) - z_m(t)\| = 0. \tag{2.3}$$

3. Synchronization Methodology

Suppose the chaotic (or hyperchaotic) master system and corresponding chaotic (hyperchaotic) slave system are considered as:

$$\dot{z}_m = F_1(z_m) + F_2(z_m)\xi, \tag{3.1}$$

$$\dot{z}_s = F_1(z_s) + F_2(z_s)\xi + U, \tag{3.2}$$

where $z_m = (z_{m1}, z_{m2}, \dots, z_{mn})^T$, $z_s = (z_{s1}, z_{s2}, \dots, z_{sn})^T$ are the state vectors, $U = (U_i, i = 1, 2, \dots, n) \in R^n$ is the controller vector, $F_1 : R^n \to R^n$, $F_2 : R^n \to R^{n\times p}$ are two nonlinear continuous vector functions, $\xi = (\xi_i; i = 1, 2, \dots, p)^T$ is the known parameter vector and $(.)^T$ denotes the transpose.

Error is defined by

$$e(t) = z_s(t) - z_m(t).$$

The systems (3.1) and (3.2) are in a synchronized state if

$$\lim_{t\to\infty} \|e(t)\| = 0,$$

where $e(t) = (e_i; i = 1, 2, \dots, n)^T$ is the error function and $\|.\|$ denotes vector norm.

Using (3.1) and (3.2), the error dynamics becomes:

$$\dot{e}(t) = F_1(z_s) + F_2(z_s)\xi + U - F_1(z_m) - F_2(z_m)\xi.$$

Now, we design the appropriate control function U and parameter update laws to ensure that master and slave systems get synchronized with unknown parameters.

Let us define the controller

$$U = -F_1(z_s) - F_2(z_s)\hat{\xi} + F_1(z_m) + F_2(z_m)\hat{\xi} - Ke,$$

where $\hat{\xi}(t) = (\hat{\xi}_i; i = 1, 2, \dots, p)$ is the uncertain time varying parameter and K is an arbitrary chosen positive number known as gain constant.

Define parameter update laws as

$$\dot{\hat{\xi}} = -[F_2(z_m)]^T + K_\xi \tilde{\xi},$$

where $K = diag(k_i; i = 1, 2, .., n)$, $K_\xi = diag(K_i; i = 1, 2, .., p)$ and $\tilde{\xi} = \xi - \hat{\xi}$.

Choosing the Lyapunov function as

$$V(t) = \frac{1}{2}(e^T e + \tilde{\xi}^T \tilde{\xi}),$$

which implies that V is positive definite.

Derivative of Lyapunov function takes the form:

$$\begin{aligned}
\dot{V}(t) &= e^T \dot{e} + \tilde{\xi}^T(-\dot{\hat{\xi}}) \\
&= e^T[-F_2(z_m)\tilde{\xi} - Ke] - \tilde{\xi}^T[-[F_2(z_m)]^T e + K_\xi \tilde{\xi}] \\
&= -e^T F_2(z_m)\tilde{\xi} - e^T K e + \tilde{\xi}^T [F_2(z_m)]^T e - \tilde{\xi}^T K_\xi \tilde{\xi} \\
&= -[\tilde{\xi}^T [F_2(z_m)]^T e]^T + \tilde{\xi}^T [F_2(z_m)]^T e - e^T K e - \tilde{\xi}^T K_\xi \tilde{\xi} \\
&= -e^T K e - \tilde{\xi}^T K_\xi \tilde{\xi} \\
&< 0,
\end{aligned}$$

which establishes that $\dot{V}$ is negative definite.

In view of Lyapunov stability theory [35], the error dynamics is globally asymptotically stable in the neighbourhood of the equilibrium points.

4. Model Analysis

In this section, we are describing in brief the chaotic system which is selected for CS technique using both the active and adaptive control techniques.

Introduced by Singh and Roy [44], the considered chaotic system is represented by

$$\begin{cases}
\dot{z}_{m1} = c_2 z_{m1} z_{m2} - 2c_6 z_{m1}^2 + (c_9 - c_{10}) z_{m1} \\
\dot{z}_{m2} = -c_1 z_{m1} z_{m2} z_{m3} + c_3 z_{m1} z_{m2} + c_5 z_{m2} z_{m3} - 2c_7 z_{m2}^2 - (c_{11} - c_{12}) z_{m2} \\
\dot{z}_{m3} = c_1 z_{m1} z_{m2} z_{m3} - c_4 z_{m1} z_{m3} - c_5 z_{m2} z_{m3} - 2c_8 z_{m3}^2 + c_{13} z_{m3},
\end{cases} \tag{4.1}$$

where $(z_{m1}, z_{m2}, z_{m3})^T \in R^4$ is the state vector and c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12} and c_{13} are positive parameters. When $c_1 = 0.88$, $c_2 = 10$, $c_3 = 29$, $c_4 = c_5 = 100$, $c_6 = 5$, $c_7 = 0.5$, $c_8 = 1.333$, $c_9 = c_{10} = 1000$, $c_{11} = 2900$, $c_{12} = 100$ and $c_{13} = 10002.667$, the system (4.1) exhibits chaos. Further, Fig. 1(a-d) depicts the phase portraits of (4.1). In addition, Lyapunov exponents of system (4.1) are determined as $L_1 = 92.298$, $L_2 = -5.665$ and $L_3 = -156.091$ which show the chaotic behaviour of (4.1). However, the detailed theoretical study along with numerical results can be found in [44] for system (4.1).

The coming up section presents the complete synchronization scheme to control chaos of (4.1) using active control method.

5. Synchronization Phenomena using Active Control Method

The system (4.1) is conveniently considered as master system and the corresponding slave system may be defined as:

$$\begin{cases}
\dot{z}_{s1} = c_2 z_{s1} z_{s2} - 2c_6 z_{s1}^2 + (c_9 - c_{10}) z_{s1} + U_1 \\
\dot{z}_{s2} = -c_1 z_{s1} z_{s2} z_{s3} + c_3 z_{s1} z_{s2} + c_5 y_{s2} z_{s3} - 2c_7 z_{s2}^2 - (c_{11} - c_{12}) z_{s2} + U_2 \\
\dot{z}_{s3} = c_1 z_{s1} z_{s2} z_{s3} - c_4 z_{s1} z_{s3} - c_5 z_{s2} z_{s3} - 2c_8 z_{s3}^2 + c_{13} z_{s3} + U_3,
\end{cases} \tag{5.1}$$

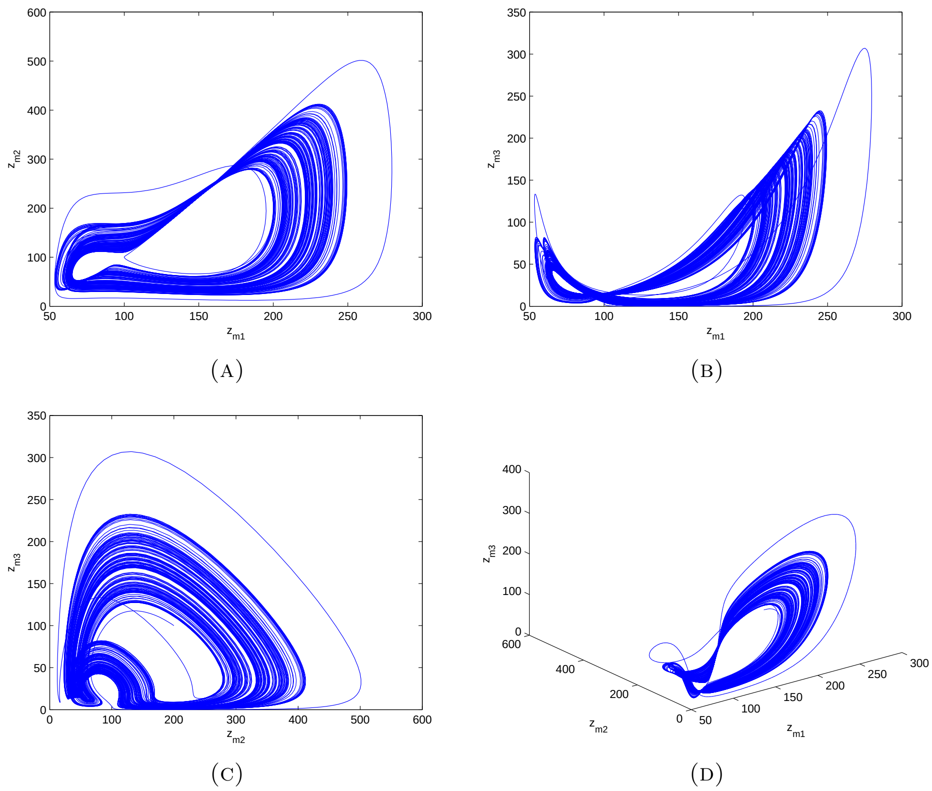


FIGURE 1. Phase portraits of chaotic chemical reactor system in (a) $z_{m1} - z_{m2}$ plane, (b) $z_{m1} - z_{m3}$ plane, (c) $z_{m2} - z_{m3}$ plane, (d) $z_{m1} - z_{m2} - z_{m3}$ space

where U_1, U_2 and U_3 are active controllers to be designed in such a manner that CS between two identical chaotic systems will be achieved. Further, Fig. 2(a-d) depict the phase portraits of (5.1) without controllers. Moreover, time series of master (4.1) and slave (5.1) systems are shown in Fig. 3(a-c) and Fig. 3(d-f) respectively.

Defining the state errors as

$$\begin{cases} e_1 = z_{s1} - z_{m1} \\ e_2 = z_{s2} - z_{m2} \\ e_3 = z_{s3} - z_{m3}. \end{cases} \tag{5.2}$$

The main target here is to introduce controllers U_i, $(i = 1, 2, 3)$ so that the errors defined in (5.2) satisfy

$$\lim_{t \to \infty} e_i(t) = 0, \quad \text{for } (i = 1, 2, 3).$$

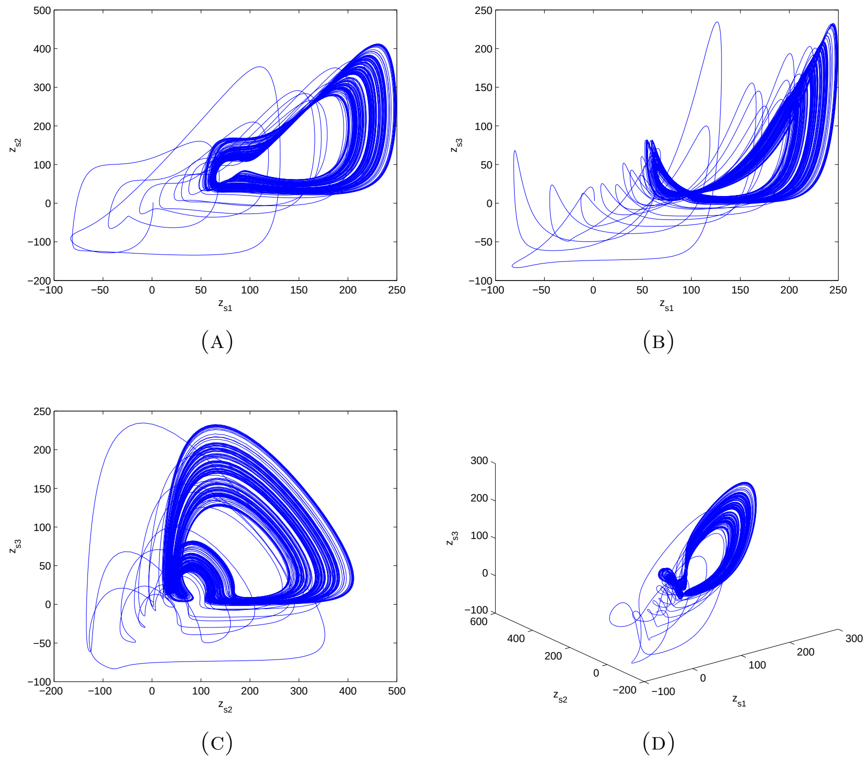


FIGURE 2. Phase portraits of chaotic chemical reactor system in (a) $z_{s1} - z_{s2}$ plane, (b) $z_{s1} - z_{s3}$ plane, (c) $z_{s2} - z_{s3}$ plane, (d) $z_{s1} - z_{s2} - z_{s3}$ space

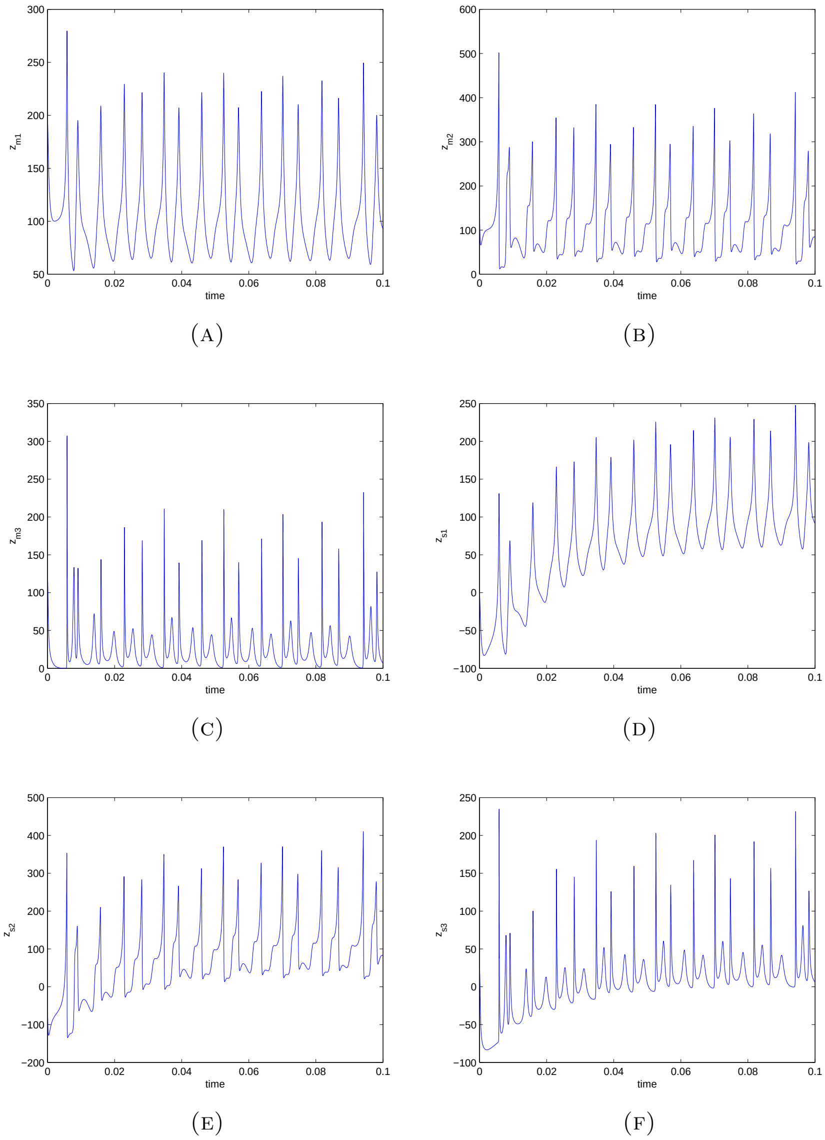
The subsequent error dynamics becomes

$$\begin{cases} \dot{e}_1 = c_2(z_{s1}z_{s2} - z_{m1}z_{m2}) - 2c_6(z_{s1}^2 - z_{m1}^2) + (c_9 - c_{10})e_1 + U_1 \\ \dot{e}_2 = -c_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) + c_3(z_{s1}z_{s2} - z_{m1}z_{m2}) \\ \qquad\quad +c_5(z_{s2}z_{s3} - z_{m2}z_{m3}) - (c_{11} - c_{12})e_2 - 2c_7(z_{s2}^2 - z_{m2}^2) + U_2 \\ \dot{e}_3 = +c_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) - c_4(z_{s1}z_{s3} - z_{m1}z_{m3}) \\ \qquad\quad -c_5(z_{s2}z_{s3} - z_{m2}z_{m3}) + c_{13}e_3 - 2c_8(z_{s3}^2 - z_{m3}^2) + U_3. \end{cases} \tag{5.3}$$

Now, we define the nonlinear active controllers by the rule:

$$\begin{cases} U_1 = -c_2(z_{s1}z_{s2} - z_{m1}z_{m2}) + 2c_6(z_{s1}^2 - z_{m1}^2) - (c_9 - c_{10})e_1 - K_1e_1 \\ U_2 = c_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) - c_3(z_{s1}z_{s2} - z_{m1}z_{m2}) - c_5(z_{s2}z_{s3} - z_{m2}z_{m3}) \\ \qquad +(c_{11} - c_{12})e_2 + 2c_7(z_{s2}^2 - z_{m2}^2) - K_2e_2 \\ U_3 = -c_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) + c_4(z_{s1}z_{s3} - z_{m1}z_{m3}) \\ \qquad +c_5(z_{s2}z_{s3} - z_{m2}z_{m3}) - z_{13}e_3 + 2c_8(z_{s3}^2 - z_{m3}^2) - K_3e_3, \end{cases} \tag{5.4}$$

where K_1, K_2, K_3 are positive gain constants.



(A) (B) (C) (D) (E) (F)

FIGURE 3. Time series of master (4.1) and slave (5.1) systems

By substituting the controllers (5.4) in error dynamics (5.3), we obtain

$$\begin{cases} \dot{e}_1 = -K_1 e_1 \\ \dot{e}_2 = -K_2 e_2 \\ \dot{e}_3 = -K_3 e_3. \end{cases} \tag{5.5}$$

Lyapunov function is defined as:

$$V = \frac{1}{2}[e_1^2 + e_2^2 + e_3^2], \tag{5.6}$$

which implies clearly that V is positive definite.

Derivative of V is given by

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3. \tag{5.7}$$

Theorem 5.1. *The chaotic systems (4.1)-(5.1) are globally and asymptotically complete synchronized for all initial states* $(z_{m1}(0), z_{m2}(0), z_{m3}(0)) \in R^3$ *by the nonlinear active controller (5.4).*

Proof. The Lyapunov function V as defined in (5.6) is positive definite function. By solving equations (5.5) and (5.7), one may finds that

$$\begin{aligned}\dot{V} &= -K_1e_1^2 - K_2e_2^2 - K_3e_3^2 \\ &< 0,\end{aligned}$$

ensuring that $\dot{V}$ is negative definite.

Therefore, in view of Lyapunov stability theory [35], we conclude that the CS error $e(t) \to 0$ exponentially as $t \to \infty$ for each initial condition $e(0) \in R^3$. This signals the end of proof. □

5.1. Numerical Simulation and Results. Numerical simulations are conducted using MATLAB to illustrate the effectiveness of the proposed CS scheme via active control method. The initial states of the master and slave systems are $(200, 200, 100)$ and $(1, 2, 3)$ respectively. Also, K_i for $i = 1, 2, 3$ are taken to be 5. The state trajectories of master (4.1) and slave systems (5.1) are displayed to get complete synchronized in Fig. 4(a-c). Further, the synchronization error $(e_1, e_2, e_3) = (-199, -198, -97)$ plot converges to zero with time and has been shown in Fig. 4(d). Hence, the proposed CS scheme among master and slave systems is attained computationally.

6. Synchronization Phenomena using Adaptive Control Method

Here, we study CS technique to derive the laws in order to estimate system parameters along with adaptive controllers in such a prescribed manner that each state variable z_{m1}, z_{m2} and z_{m3} approach to equilibrium points as t tends to infinity.

The system (4.1) is conveniently considered as master system and the corresponding slave system can be expressed as:

$$\begin{cases}\dot{z}_{s1} = c_2z_{s1}z_{s2} - 2c_6z_{s1}^2 + (c_9 - c_{10})z_{s1} + V_1 \\ \dot{z}_{s2} = -c_1z_{s1}z_{s2}z_{s3} + c_3z_{s1}z_{s2} + c_5z_{s2}z_{s3} - 2c_7z_{s2}^2 - (c_{11} - c_{12})z_{s2} + V_2 \\ \dot{z}_{s3} = c_1z_{s1}z_{s2}z_{s3} - c_4z_{s1}z_{s3} - c_5z_{s2}z_{s3} - 2c_8z_{s3}^2 + c_{13}z_{s3} + V_3,\end{cases} \tag{6.1}$$

where V_1, V_2 and V_3 are adaptive controllers to be designed in such a manner that CS between two identical chaotic systems will be achieved.

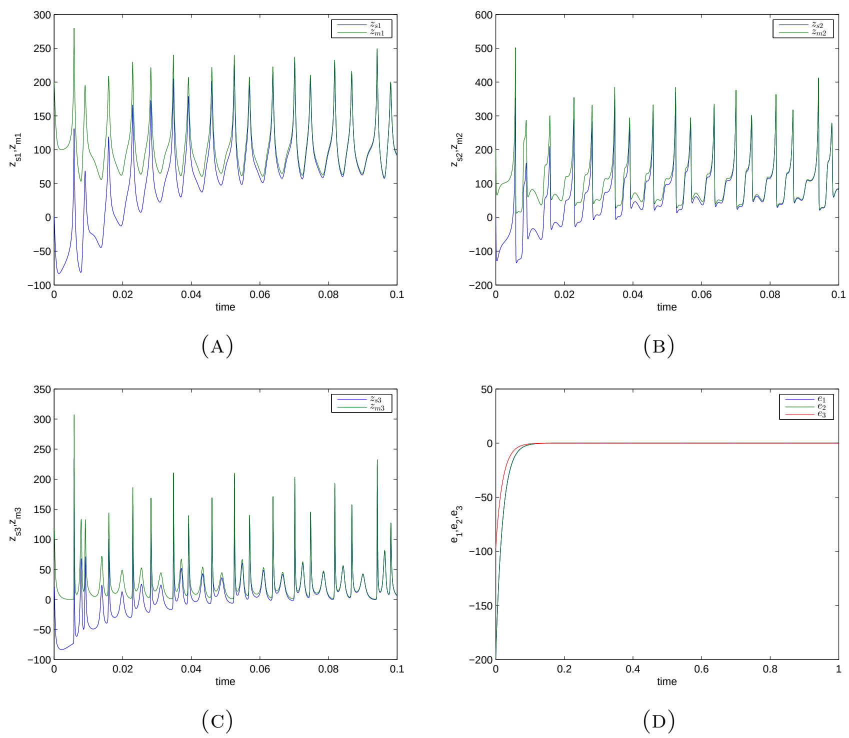


FIGURE 4. Complete synchronization of 3-D chaotic CRS using active control (a) between $z_{m1}(t) - z_{s1}(t)$, (b) between $z_{m2}(t) - z_{s2}(t)$, (c) between $z_{m3}(t) - z_{s3}(t)$, (d) synchronization error

Defining the state errors as:

$$\begin{cases} e_1 = z_{s1} - z_{m1} \\ e_2 = z_{s2} - z_{m2} \\ e_3 = z_{s3} - z_{m3}. \end{cases} \tag{6.2}$$

The ultimate objective here is to introduce controllers $V_i, (i = 1, 2, 3)$ so that state errors defined in (6.2) satisfy

$$\lim_{t\to\infty} e_i(t) = 0, \quad \text{for } (i = 1, 2, 3).$$

The subsequent error dynamics becomes

$$\begin{cases} \dot{e}_1 = c_2(z_{s1}z_{s2} - z_{m1}z_{m2}) - 2c_6(z_{s1}^2 - z_{m1}^2) + (c_9 - c_{10})e_1 + V_1 \\ \dot{e}_2 = \ -c_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) + c_3(z_{s1}z_{s2} - z_{m1}z_{m2}) \\ \qquad +c_5(z_{s2}z_{s3} - z_{m2}z_{m3}) - (c_{11} - c_{12})e_2 - 2c_7(z_{s2}^2 - z_{m2}^2) + V_2 \\ \dot{e}_3 = +c_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) - c_4(z_{s1}z_{s3} - z_{m1}z_{m3}) \\ \qquad -c_5(z_{s2}z_{s3} - z_{m2}z_{m3}) + c_{13}e_3 - 2c_8(z_{s3}^2 - z_{m3}^2) + V_3. \end{cases} \tag{6.3}$$

Now, we define the adaptive controllers as:

$$\begin{cases} V_1 = -\hat{c}_2(z_{s1}z_{s2} - z_{m1}z_{m2}) + 2\hat{c}_6(z_{s1}^2 - z_{m1}^2) - (\hat{c}_9 - \hat{c}_{10})e_1 - K_1 e_1 \\ V_2 = \hat{c}_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) - \hat{c}_3(z_{s1}z_{s2} - z_{m1}z_{m2}) - \hat{c}_5(z_{s2}z_{s3} - z_{m2}z_{m3}) \\ \qquad + (\hat{c}_{11} - \hat{c}_{12})e_2 + 2\hat{c}_7(z_{s2}^2 - z_{m2}^2) - K_2 e_2 \\ V_3 = -\hat{c}_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) + \hat{c}_4(z_{s1}z_{s3} - z_{m1}z_{m3}) \\ \qquad + \hat{c}_5(z_{s2}z_{s3} - z_{m2}z_{m3}) - \hat{c}_{13}e_3 + 2\hat{c}_8(z_{s3}^2 - z_{m3}^2) - K_3 e_3, \end{cases} \tag{6.4}$$

where $\hat{c}_1$, $\hat{c}_2$, $\hat{c}_3$, $\hat{c}_4$, $\hat{c}_5$, $\hat{c}_6$, $\hat{c}_7$, $\hat{c}_8$, $\hat{c}_9$, $\hat{c}_{10}$, $\hat{c}_{11}$, $\hat{c}_{12}$ and $\hat{c}_{13}$ are estimated values of unknown parameter c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12} and c_{13} respectively and K_1, K_2, K_3 are positive gain constants.

By substituting the controllers (6.4) in error dynamics (6.3), we get

$$\begin{cases} \dot{e}_1 = (c_2 - \hat{c}_2)(z_{s1}z_{s2} - z_{m1}z_{m2}) - 2(c_6 - \hat{c}_6)(z_{s1}^2 - z_{m1}^2) \\ \qquad + ((c_9 - \hat{c}_9) - (c_{10} - \hat{c}_{10}))e_1 - K_1 e_1 \\ \dot{e}_2 = -(c_1 - \hat{c}_1)(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) + (c_3 - \hat{c}_3)(z_{s1}z_{s2} - z_{m1}z_{m2}) \\ \qquad + (c_5 - \hat{c}_5)(z_{s2}z_{s3} - z_{m2}z_{m3}) - ((c_{11} - \hat{c}_{11}) - (c_{12} - \hat{c}_{12}))e_2 \\ \qquad - 2(c_7 - \hat{c}_7)(z_{s2}^2 - z_{m2}^2) - K_2 e_2 \\ \dot{e}_3 = (c_1 - \hat{c}_1)(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) - (c_4 - \hat{c}_4)(z_{s1}z_{s3} - z_{m1}z_{m3}) \\ \qquad - (c_5 - \hat{c}_5)(z_{s2}z_{s3} - z_{m2}z_{m3}) + (c_{13} - \hat{c}_{13})e_3 \\ \qquad - 2(c_8 - \hat{c}_8)(z_{s3}^2 - z_{m3}^2) - K_3 e_3. \end{cases} \tag{6.5}$$

We define parameter estimation error as follows:

$$\begin{cases} \tilde{c_1} = c_1 - \hat{c1}, \tilde{c_2} = c_2 - \hat{c_2}, \tilde{c_3} = c_3 - \hat{c_3}, \tilde{c_4} = c_4 - \hat{c_4}, \tilde{c_5} = c_5 - \hat{c_5}, \\ \tilde{c_6} = c_6 - \hat{c_6}, \tilde{c_7} = c_7 - \hat{c_7}, \tilde{c_8} = c_8 - \hat{c_8}, \tilde{c_9} = c_9 - \hat{c_9}, \\ \tilde{c_{10}} = c_{10} - \hat{c_{10}}, \tilde{c_{11}} = c_{11} - \hat{c_{11}}, \tilde{c_{12}} = c_{12} - \hat{c_{12}}, \tilde{c_{13}} = c_{13} - \hat{c_{13}}. \end{cases} \tag{6.6}$$

Using (6.6), the error dynamics (6.5) can be written as:

$$\begin{cases} \dot{e}_1 = \tilde{c}_2(z_{s1}z_{s2} - z_{m1}z_{m2}) - 2\tilde{c}_6(z_{s1}^2 - z_{m1}^2) + (\tilde{c}_9 - \tilde{c}_{10})e_1 - K_1 e_1 \\ \dot{e}_2 = -\tilde{c}_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) + \tilde{c}_3(z_{s1}z_{s2} - z_{m1}z_{m2}) \\ \qquad + \tilde{c}_5(z_{s2}z_{s3} - z_{m2}z_{m3}) - (\tilde{c}_{11} - \tilde{c}_{12})e_2 - 2\tilde{c}_7(z_{s2}^2 - z_{m2}^2) - K_2 e_2 \\ \dot{e}_3 = -\tilde{c}_1(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3}) - \tilde{c}_4(z_{s1}z_{s3} - z_{m1}z_{m3}) \\ \qquad - \tilde{c}_5(z_{s2}z_{s3} - z_{m2}z_{m3}) + \tilde{c}_{13}e_3 - 2\tilde{c}_8(z_{s3}^2 - z_{m3}^2) - K_3 e_3. \end{cases} \tag{6.7}$$

By differentiating parameter estimation error (6.6), one finds that

$$\begin{cases} \dot{\tilde{c_1}} = -\dot{\hat{c_1}}, \dot{\tilde{c_2}} = -\dot{\hat{c_2}}, \dot{\tilde{c_3}} = -\dot{\hat{c_3}}, \dot{\tilde{c_4}} = -\dot{\hat{c_4}}, \dot{\tilde{c_5}} = -\dot{\hat{c_5}}, \\ \dot{\tilde{c_6}} = -\dot{\hat{c_6}}, \dot{\tilde{c_7}} = -\dot{\hat{c_7}}, \dot{\tilde{c_8}} = -\dot{\hat{c_8}}, \dot{\tilde{c_9}} = -\dot{\hat{c_9}}, \dot{\tilde{c_{10}}} = -\dot{\hat{c_{10}}}, \\ \dot{\tilde{c_{11}}} = -\dot{\hat{c_{11}}}, \dot{\tilde{c_{12}}} = -\dot{\hat{c_{12}}}, \dot{\tilde{c_{13}}} = -\dot{\hat{c_{13}}} \end{cases} \tag{6.8}$$

Lyapunov function is defined by

$$\begin{aligned} V = \frac{1}{2}[e_1^2 + e_2^2 + e_3^2 + \tilde{c_1}^2 + \tilde{c_2}^2 + \tilde{c_3}^2 + \tilde{c_4}^2 + \tilde{c_5}^2 + \tilde{c_6}^2 \\ + \tilde{c_7}^2 + \tilde{c_8}^2 + \tilde{c_9}^2 + \tilde{c_{10}}^2 + \tilde{c_{11}}^2 + \tilde{c_{12}}^2 + \tilde{c_{13}}^2], \end{aligned} \tag{6.9}$$

which implies clearly that V is positive definite.

Derivative of Lyapunov function V is expressed by

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 - \tilde{c_1}\dot{\hat{c_1}} - \tilde{c_2}\dot{\hat{c_2}} - \tilde{c_3}\dot{\hat{c_3}} - \tilde{c_4}\dot{\hat{c_4}} - \tilde{c_5}\dot{\hat{c_5}} - \tilde{c_6}\dot{\hat{c_6}} \\ - \tilde{c_7}\dot{\hat{c_7}} - \tilde{c_8}\dot{\hat{c_8}} - \tilde{c_9}\dot{\hat{c_9}} - \tilde{c_{10}}\dot{\hat{c_{10}}} - \tilde{c_{11}}\dot{\hat{c_{11}}} - \tilde{c_{12}}\dot{\hat{c_{12}}} - \tilde{c_{13}}\dot{\hat{c_{13}}}. \tag{6.10}$$

In view of (6.10), parameter estimates law is defined as :

$$\begin{cases} \dot{\hat{c_1}} = -(z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3})e_2 + (z_{s1}z_{s2}z_{s3} - z_{m1}z_{m2}z_{m3})e_3 + K_4\tilde{c}_1 \\ \dot{\hat{c_2}} = (z_{s1}z_{s2} - z_{m1}z_{m2})e_1 + K_5\tilde{c}_2 \\ \dot{\hat{c_3}} = (z_{s1}z_{s2} - z_{m1}z_{m2})e_2 + K_6\tilde{c}_3 \\ \dot{\hat{c_4}} = -(z_{s1}z_{s3} - z_{m1}z_{m3})e_3 + K_7\tilde{c}_4 \\ \dot{\hat{c_5}} = (z_{s2}z_{s3} - z_{m2}z_{m3})e_2 - (z_{s2}z_{s3} - z_{m2}z_{m3})e_3 + K_8\tilde{c}_5 \\ \dot{\hat{c_6}} = -2(z_{s1}^2 - z_{m1}^2)e_1 + K_9\tilde{c}_6 \\ \dot{\hat{c_7}} = -2(z_{s2}^2 - z_{m2}^2)e_2 + K_{10}\tilde{c}_7 \\ \dot{\hat{c_8}} = -2(z_{s3}^2 - z_{m3}^2)e_3 + K_{11}\tilde{c}_8 \\ \dot{\hat{c_9}} = e_1^2 + K_{12}\tilde{c}_9 \\ \dot{\hat{c_{10}}} = -e_1^2 + K_{13}\tilde{c}_{10} \\ \dot{\hat{c_{11}}} = -e_2^2 + K_{14}\tilde{c}_{11} \\ \dot{\hat{c_{12}}} = e_2^2 + K_{15}\tilde{c}_{12}, \quad \dot{\hat{c_{13}}} = e_3^2 + K_{16}\tilde{c}_{13}, \end{cases} \tag{6.11}$$

where $K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, K_{15}$ and K_{16} are positive gain constants.

Theorem 6.1. *The chaotic systems (4.1) and (6.1) are globally and asymptotically complete synchronized for all initial states $(z_{m1}(0), z_{m2}(0), z_{m3}(0)) \in R^3$ by the adaptive controller (6.4) and the parameter update law (6.11).*

Proof. The Lyapunov function V as described in (6.9) is positive definite. On simplification, the equations (6.7), (6.10) and (6.11) give rise to

$$\dot{V} = -K_1e_1^2 - K_2e_2^2 - K_3e_3^2 - K_4\tilde{c_1}^2 - K_5\tilde{c_2}^2 - K_6\tilde{c_3}^2 - K_7\tilde{c_4}^2 - K_8\tilde{c_5}^2 - K_9\tilde{c_6}^2 \\ - K_{10}\tilde{c_7}^2 - K_{11}\tilde{c_8}^2 - K_{12}\tilde{c_9}^2 - K_{13}\tilde{c_{10}}^2 - K_{14}\tilde{c_{11}}^2 - K_{15}\tilde{c_{12}}^2 - K_{16}\tilde{c_{13}}^2 < 0,$$

showing that $\dot{V}$ is negative definite.

Therefore, by Lyapunov stability theory [35], we conclude that the CS error $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$. This signals the ending of the proof. □

6.1. Numerical Simulation and Results. Numerical simulations are conducted using MATLAB to illustrate the effectiveness of the proposed CS scheme via adaptive control method. The initial states of the master and slave systems are $(200, 200, 100)$ and $(1, 2, 3)$ respectively. Also, K_i for $i = 1, 2, 3$ are taken to be 5. The state trajectories of master (4.1) and slave systems (6.1) are displayed to get complete synchronized in Fig. 5(a-c). Further, the synchronization error $(e_1, e_2, e_3) = (-199, -198, -97)$ plot converges to zero with time and has been shown in Fig. 5(d). Also, in Fig. 5(e-g), it is noted that the estimated values $\hat{c}_1$, $\hat{c}_2$, $\hat{c}_3$, $\hat{c}_4$, $\hat{c}_5$, $\hat{c}_6$, $\hat{c}_7$, $\hat{c}_8$, $\hat{c}_9$, $\hat{c}_{10}$, $\hat{c}_{11}$, $\hat{c}_{12}$, and $\hat{c}_{13}$ of unknown parameters converging to their original values asymptotically with time. Hence, the proposed CS scheme among master and slave systems is attained computationally.

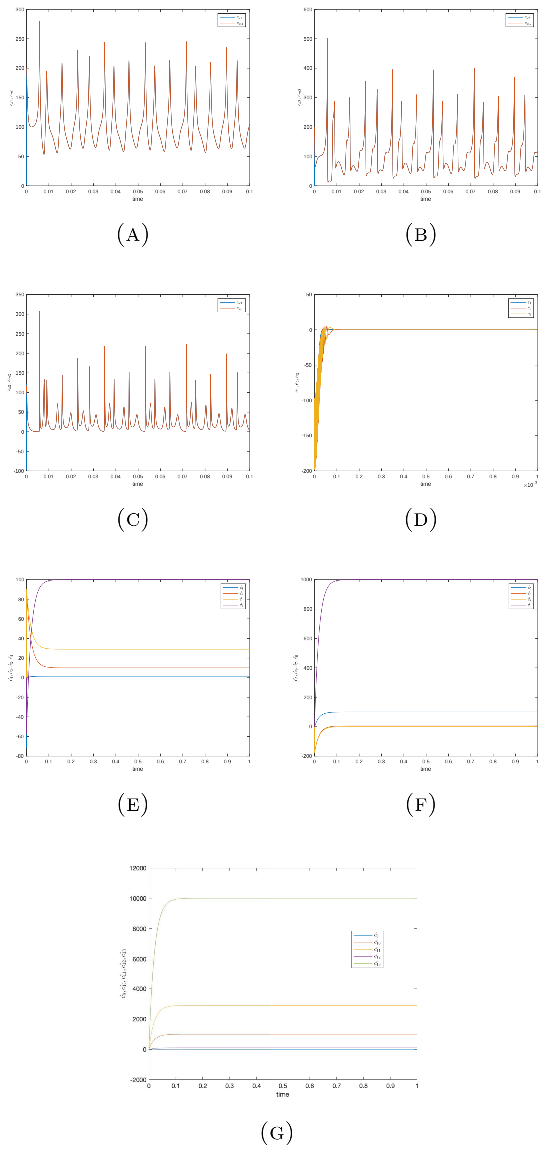
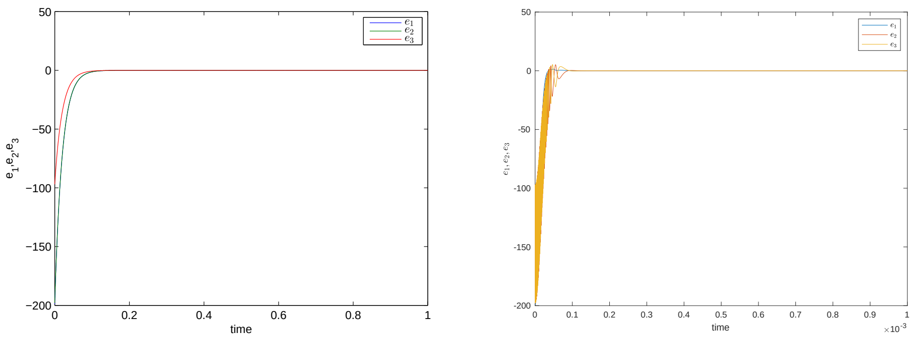


FIGURE 5. Complete synchronization of 3-D chaotic CRS using adaptive control (a) between $z_{m1}(t)-z_{s1}(t)$, (b) between $z_{m2}(t)-z_{s2}(t)$, (c) between $z_{m3}(t)-z_{s3}(t)$, (d) synchronization error, (e-g) parameter estimation

7. A Comparative Study

In [44], the chaos phenomenon in the considered CRS is established by utilizing phase portraits, Lyapunov exponents and bifurcation diagrams. Also, nonlinear active along with integral sliding mode control is investigated for system (4.1) and (5.1) considering Lyapunov stability theory. It is noted that the synchronization errors converge to zero at $t = 0.005$ (approx) which is displayed in [44], while in this study, the CS scheme is obtained using both active and adaptive control techniques when performed on the same chaotic CRS. It is worth noticing that synchronization errors are converging to zero at $t = 0.1(approx)$ and $t = 0.00001(approx)$ respectively as shown in Fig. 6(a) and (b). Additionally, our discussed approach describes the parameter updating law which identifies the given unknown parameter data of the system. This exhibits that our proposed CS approach via adaptive control is more preferred over existing publications.



(A) (B)

FIGURE 6. Time history of synchronization errors (a) using active control, (b) using adaptive control

8. Conclusion

In this paper, complete synchronization in identical chaotic chemical reactor systems have been investigated via active and adaptive control techniques. By designing proper control inputs based on Lyapunov stability theory, the considered complete synchronization technique among the given systems has been achieved. The feasibility and efficacy of theoretical results are verified in simulations by using MATLAB. Significantly, the theoretical analysis and the numerical results both are in excellent arrangement. Additionally, in this direction, we would extend these investigations on chaotic complex chemical reactor system interfered with external disturbances and model uncertainties.

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