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# ON QUASI S-TOPOLOGICAL IP-LOOPS

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***Abstract***. In this paper, we have investigated that in what manner separately semi-continuous multiplication and semi continuous inverse mappings of topological spaces are defined over loops, in particular, over IP-loops. We have also constructed an example of quasi s-topological IP-loops by using zero dimensional additive metrizable perfect topological IP-loop $L^*$ with relative topology $\tau_{L'}$.

## 1. Introduction

With the introduction of semi open set by N. Levine, many of the mathematicians examined and explored several concepts by using semi continuity and semi open sets [8, 17, 19]. A number of new results are obtained when open set is replaced by semi open set and continuity is replaced by semi continuity [6, 10, 12, 22].

N. Levine defined semi open set as: A subset $M$ of $X$ is semi open, if there is an open set $O$ in $X$ such that

$$O \subseteq M \subseteq Cl(O)$$

or

$$M \subseteq Cl(Int(M)).$$

The collection of all semi open sets in $X$ is denoted by $SO(X)$ and $SO(X, t)$ denotes the collection of all semi open sets containing $t$.

A point $t \in X$ is a semi interior point of $M$, if there exists a semi open set $M'$ such that

$$t \in M' \subseteq M.$$

$sInt(M)$ is the set of all semi interior points of $M$. For any semi open set $M_t$, $t \in sCl(M')$ if and only if $M_t \cap M' \neq \phi$ [1].

A mapping $f : X \to Y$ is said to be

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- **Semi continuous**, if for each open set $M_2 \subseteq Y$ the set $f^{-1}(M_2)$ is semi open in $X$. Equivalently, $f$ is semi continuous if and only if for all $t \in X$, and for every open neighborhood $M_2$ of $f(t)$, there is a semi open neighborhood $M_1$ of $t$ such that $f(M_1) \subseteq M_2$ or $t \in M_1 \subseteq f^{-1}(M_2)$ [11];
- **Semi open**, if for each open set $M$ in $X$, $f(M)$ is semi open in $Y$ [7];
- **s-homeomorphism**, if $f$ is bijective, pre semi open, and semi continuous [3];
- **Quasi s-homeomorphism**, if $f$ is bijective, semi open, and semi continuous [3].

A subset $M$ of a topological space $X$ is said to be semi compact (semi-Lindelof), if there exists a finite(countable) subcover for any semi open cover of $M$ in $X$[4, 21]. A topological space $(L, \tau)$ is said to be s-regular, if for every $t \in L$ and every closed set $Q \subseteq L$, there exists semi open neighbourhoods $M$ containing $t$ and $N$ containing $Q$ with $t \notin Q$ such that $M \cap N = \phi$ [13]. A collection $\tau_w$ of subsets of a topological space $(L, \tau)$ is semi discrete, if every $t \in L$ has semi open neighbourhood that intersects at most one member of $\tau_w$ [9]. A topological space $(L, \tau)$ is called quasi s-homogeneous, if there is a quasi s-homeomorphism $f$ of $L$ onto itself such that for every $r, s \in L$ implies $f(r) = s$ [14]. Let $M \in SO(L, e)$ in a quasi s-topological loop $(L, *, \tau)$. A subset $S$ of $L$ is said to be M-semi disjoint, if for every disjoint $r, s \in S$ implies $r \notin s * M$ [2]. If a topological space $X$ cannot be expressed as a union of two nonempty disjoint semi open sets then it is said to be semi connected. Semi component are the maximal semi connected subsets of a topological space. $S.C.(x)$ denotes the semi component containing $x(\in X)$ [5].

A groupoid $(L, *)$ is a loop if the conditions given below are satisfied:

- $L$ contain an identity element.
- for every $t_1 \in L$, the mappings $l_{t_1} : L \to L$ and $r_{t_1} : L \to L$ are bijective, where $l_{t_1}(t_2) = t_1 * t_2$ and $r_{t_1}(t_2) = t_2 * t_1$ for all $t_2 \in L$ [18].

An inverse property loop (IP-loop) $L$ is a loop having two sided inverse $t^{-1}$ such that $(r * t) * t^{-1} = r = t^{-1} * (t * r)$ for all $r, t \in L$.

Left(right) translations and left(right) inverse mappings [15] are defined on a loop $(L, *)$ as follows:

- **Left translation** $l_{t_1} : L \to L$ is given by $l_{t_1}(t_2) = t_1 * t_2$;
- **Right translation** $r_{t_1} : L \to L$ is given as $r_{t_1}(t_2) = t_2 * t_1$;
- **Left inverse mapping** $i_L : L \to L$ is defined by $i_L(t) = (t)_L^{-1}$;
- **Right inverse mapping** $i_R : L \to L$ is defined as $i_R(t) = (t)_R^{-1}$;

where $t, t_1, t_2 \in L$.
We used the standard notions and terminologies as in [23].

## 2. Quasi s-topological IP-Loops

**Definition 2.1.** A triplet $(L, *, \tau)$ is said to be a quasi s-topological IP-loop, if the conditions given below are satisfied:

- $(L, *)$ is an IP-loop.
- $(L, \tau)$ is a topological space.
- Multiplication mapping is separately semi continuous in $(L, *, \tau)$.
- Inverse mapping is semi continuous in $(L, *, \tau)$.

**Example 2.2.** Consider a zero dimensional additive metrizable perfect topological IP-loop $L^*$ having commutative property with an infinite null sequence $\{N_0\}$. Here, we fix a sequence $S_n$ of clopen subsets in $L^*$ such that: for $n^* \neq n$, separate union with their additive inverses respectively, have empty intersection, and any open set $M$ containing identity contain all the terms of $S_n$ after some fixed term. Assume a base $M_n$ of clopen sets containing identity in $L^*$ with $S_n \subseteq M_n \subseteq M_n + M_n \subseteq M_{n-1} = -M_{n-1}$. Consider $\mu^*$ containing clopen sets $M$ in $(L^* \setminus \{0\}) \times L^*$ with $M = -M$ and $\cup\{((M_{m+1} \setminus \{0\}) \times \{0\}) \cup ((M_{2n+1} + M_{2n+1}) \times M_{n+1})\} \subseteq M$ for all $n > m-1$. Now, set $\mu$ consisting of $\{t\} \cup (M+t)$, where $t \in L^* \times L^*$ and $M \in \mu^*$ form a base for a topology $\tau$ on $L^* \times L^*$. Fix two dense-in-itself sub-loops $L_1, L_2$ of $L^*$ such that $\{N_0\} \subseteq Cl_{L^*} L_1$. For $L' = L_1 \times L_2$ as subspace of $\tau$ form quasi s-topological IP-loop with relative topology $\tau_{L'}$

**Lemma 2.3.** *Let $(L, *, \tau)$ be a quasi s-topological IP-loop and $M \subseteq L$. Then $(sCl(M))^{-1} \subseteq Cl(M^{-1})$.*

*Proof.* Let $t \in (sCl(M))^{-1}$ and $N$ is an open neighbourhood of $t$. So, $N^{-1}$ is semi open neighbourhood of $t^{-1}$. As $t^{-1} \in sCl(M)$, which implies $N^{-1} \cap M \neq \phi$. Therefore, $N \cap M^{-1} \neq \phi$. Hence, $t \in Cl(M^{-1})$. □

**Theorem 2.4.** *Inverse of a semi open set in a quasi s-topological IP-loop is semi open in conjugate topology.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological IP-loop and $M \in SO(L, \tau)$, then there exists an open set $N$ such that $N \subseteq M \subseteq Cl(N)$ equivalently, $N^{-1} \subseteq M^{-1} \subseteq (Cl(N))^{-1}$. Hence, $N^{-1} \subseteq M^{-1} \subseteq (Cl(N)^{-1})$. Now, $N^{-1} \in \tau^{-1}$ implies $M^{-1} \in SO(L, \tau^{-1})$. □

**Theorem 2.5.** *The property of being quasi s-topological IP-loop is preserved under topological conjugation.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological IP-loop and $N$ be an open set in $(L, *, \tau^{-1})$, then $M = N^{-1}$ is open in $(L, *, \tau)$. Therefore, $r_t^{-1}(M) \in SO(L, \tau)$ implies $(M * t^{-1})^{-1} \in SO(L, \tau^{-1})$, $t * M^{-1} \in SO(L, \tau^{-1})$, $t * N \in SO(L, \tau^{-1})$, $l_{t^{-1}}^{-1}(N) \in SO(L, \tau^{-1})$. Hence, $l_t$ is semi continuous in $(L, *, \tau^{-1})$. Similarly, $r_t$ is semi continuous in $(L, *, \tau^{-1})$. Let $M'$ is open in $(L, *, \tau^{-1})$ then $N' = M'^{-1}$ is open in $(L, *, \tau)$. So, $i^{-1}(N') = M' \in SO(L, \tau)$. Therefore, $M'^{-1} = i^{-1}(M') \in SO(L, \tau^{-1})$. Hence, inverse mapping is semi continuous in $(L, *, \tau^{-1})$. □

**Theorem 2.6.** *Quasi s-topological IP-loop is a quasi s-homogeneous space.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological loop. Suppose $n = t^{-1} * m$ for $m, n, t \in L$. So $r_n(t) = t * n = t * (t^{-1} * m) = m$. □

**Theorem 2.7.** *Free product of an open subset with any subset in a quasi s-topological IP-loop is semi open.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological topological IP-loop, $M \in O(L)$ and $N \subseteq L$. If $n \in N$ and $t \in M * n$ then $t = m * n$ for some $m \in M$. This implies that $m = t * n^{-1} = r_{n^{-1}}(t)$ for fixed $n^{-1} \in L$, implies that there exists a semi open set $S_t$ such that $r_{n^{-1}}(S_t) \subseteq M$, $S_t * n^{-1} \subseteq M$, $S_t \subseteq M * n$. This implies that $t$ is semi interior point of $M * n$. Hence, $M * n \in SO(L)$. Therefore, $M * N = \cup_{n \in N} M * n$ is semi open. □

**Corollary 2.8.** Left and right translations are quasi s-homeomorphism in a quasi s-topological IP-loop.

**Theorem 2.9.** *Multiplication mapping is jointly semi open in a quasi s-topological IP-loop.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological IP-loop and $f : L \times L \to L$ be a multiplication mapping defined by $f(M \times N) = MN$. If $M, N \in O(L)$, then $M \times N$ is open in $L \times L$. So $f(M \times N) = MN$ is semi open in $L$ by Theorem 2.7. Hence, $f$ is semi open. □

**Theorem 2.10.** *Every sub-loop of a quasi s-topological IP-loop containing a non-empty open subset is semi open.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological IP-loop and $N$ is its sub-loop. $M$ is a non-empty open set in $L$ with $M \subseteq N$. Then by Theorem 2.8, for every $n \in N$ the set $r_n(M) = M * n$ is semi open in $(L, *, \tau)$. Therefore, $N = \cup_{n \in N}(M * n)$ is semi open in $L$. □

**Corollary 2.11.** Every open sub-loop in a quasi s-topological IP-loop is semi closed.

*Proof.* Let $S$ be an open sub-loop of a quasi s-topological IP-loop $L$. Since, $l_t$ is semi open so $t * S$ is semi open $\forall t \in L - S$. Therefore, $F = \cup_{t \in L-S}(t * S)$ is semi open. Thus, $S = L - F$ is semi closed. □

**Theorem 2.12.** *If a homomorphism $f$ from $L$ to $L'$ (quasi s-topological IP-loops) is irresolute at $e_L$, then is semi continuous on $L$.*

*Proof.* Let for all $t_1 \in L$, suppose $M_2' \in O(L', t_2)$ where $t_2 = f(t_1)$. Then, there is $M_1' \in SO(L', e_{L'})$ such that $l'_{t_2}(M_1') = t_2 * M_1' \subseteq M_2'$. This implies $f(M_1) \subseteq M_1'$ for some $M_1 \in SO(L, e_L)$. So, $t_1 * M_1 \in SO(L, t_1)$ and $f(t_1 * M_1) = f(t_1) * f(M_1) = t_2 * f(M_1) \subseteq t_2 * M_1' \subseteq M_2'$. Hence, $f$ is semi continuous on $L$. □

**Remark 2.13.** Above result also holds for a homomorphism from a quasi irresolute topological IP-loop to quasi s-topological IP-loop.

**Theorem 2.14.** *Let $(L, *, \tau)$ be a quasi s-topological IP-loop. Then for each open neighbourhod $M$ of $e$ and for each $S \subseteq L$, $sCl(S) \subseteq S * M$.*

*Proof.* Consider $t \in sCl(S)$ and for an open neighbourhood $N$ of $e$, $t * N$ is semi open neighbourhood of $t$. So, there exists $s \in S \cap t * N$, that is $s \in t * N$. It gives $s = t * n$ for some $n \in N$. Therefore, $t = s * n^{-1} \in s * N^{-1} \subseteq S * M$ for $M \in O(L, e)$. □

**Theorem 2.15.** *Let $\mu_e$ is collection of open sets containing $e$ in quasi s-topological IP-loop $(L, *, \tau)$. Then for each $S \subseteq L$, $sCl(S) = \cap\{S * M : M \in \mu_e\}$.*

*Proof.* For $t \notin sCl(S)$ implies $t \notin S$, and there exists $N \in \mu_e$ such that $t * N \cap S = \phi$. As $M \in \mu_e$ satisfies $M^{-1} \subseteq N$. Therefore, $t * M^{-1} \cap S = \phi$, $\{t\} \cap S * M = \phi$. So $t \notin S * M$. Moreover, by Theorem 2.14 $sCl(S) \subseteq S * M$. Thereupon, $sCl(S) = \cap\{S * M : M \in \mu_e\}$. □

**Theorem 2.16.** *Semi closure of inverse of every open neighbourhood of identity element in a quasi s-topological IP-loop contain in free product of its inverse with itself.*

*Proof.* Let $M^{-1}$ is an open neighbourhood of $e$ and $t \in sCl(M)$. As $t * M^{-1}$ is semi open neighbourhood of $t$ so it meets $M$. Furthermore, there exists $r \in M$ such that $r = t * s^{-1}$ for $s \in M$. Then, $t = r * s \in M * M$. □

**Theorem 2.17.** *For $N \in O(L, e)$ and $M \subseteq L$ in quasi s-topological IP-loop $(L, *, \tau)$ such that $N^4 \subseteq M$, and $N$ is symmetric. If $S \subseteq L$ is M-semi disjoint, then $\{s * N : s \in S\}$ being the family of semi open sets is semi discrete in $L$.*

*Proof.* To prove semi discreteness of $\{s * N : s \in S\}$, it is sufficient to prove that for every $t \in L$, $t * N$ intersects at most one of $\{s * N : s \in S\}$. Assume contrarily, for some $t \in L$, there exists $r, s \in S$ such that $t * N \cap s * N \neq \phi$ and $t * N \cap r * N \neq \phi$, then $t^{-1} * s \in N^2$ and $t \in r * N^2$. Here, $s = t * (t^{-1} * s) \in r * N^4 \subseteq r * M$. Hence, $s \in r * M$. A contradiction to the fact that $S$ is M-semi disjoint. So, $\{s * N : s \in S\}$ being the family of semi open sets is semi discrete in $L$. □

## 3. Quasi s-topological Loops

**Definition 3.1.** A triplet $(L, *, \tau)$ is said to be a quasi s-topological loop, if the conditions given below are satisfied:

- $(L, *)$ is a loop.
- $(L, \tau)$ is a topological space.
- Multiplication mapping is separately semi continuous in $(L, *, \tau)$.
- Left and right inverse mappings $i_L$, $i_R$ defined on $(L, *, \tau)$ are semi continuous.

**Theorem 3.2.** *In a quasi s-topological loop, openness is sufficient condition for a sub-loop to be a quasi s-topological loop.*

*Proof.* Let $(L, *, \tau)$ be a quasi s-topological loop and $L'$ be an open sub-loop of $L$. So, for every open neighbourhood $M_1$ in $L$ with $t \in L'$ and $M_1 \subseteq L'$, $l_t^{-1}(M_1) \in SO(L)$ and $l_t^{-1}(M_1) \subseteq L'$ gives $l_t^{-1}(M_1) \in SO(L')$ [16]. Clearly, $l_t$ is semi continuous in $L'$. In the same way, $r_t$, $i_L$ and $i_R$ are semi continuous in $L'$. □

**Remark 3.3.** Every quasi irresolute topological loop is quasi s-topological loop.

**Lemma 3.4.** *Left and right inverse mappings are quasi s-homeomorphisms in a quasi s-topological loop.*

**Remark 3.5.** In a loop with topology, if left(right) inverse mapping is semi open, then right(left) inverse mapping is semi continuous.

**Theorem 3.6.** *For a semi discrete sub-loop $L^{**}$, $sCl(L^{**}) = L^*$, where $L^*$ is a sub-loop and $(L, *, \tau)$ is a quasi s-topological loop having semi open left translation.*

*Proof.* Let $L'$ be a discrete sub-loop of a quasi s-topological loop $(L, *, \tau)$. For $t_1, t_2 \in sCl(L')$, if $M_1$ and $M_2$ are open neighborhoods of $t_1$ and $t_2$, then for $(t_1)_L^{-1}, (t_2)_L^{-1} \in L$, $l_{(t_1)_L^{-1}}(M_1) = (t_1)_L^{-1} * M_1$, and $l_{(t_2)_L^{-1}}(M_2) = (t_2)_L^{-1} * M_2$ are semi open neighborhoods of $e$. So, $((t_1)_L^{-1} * M_1) \cap L' \neq \phi$ and $((t_2)_L^{-1} * M_2) \cap L' \neq \phi$. Therefore, $((t_1 * (t_2)_L^{-1}) * ((t_1)_L^{-1} * M_1) \cap (t_1 * (t_2)_L^{-1}) * L') \cup ((t_1 * (t_2)_L^{-1}) * ((t_2)_L^{-1} * M_2) \cap (t_1 * (t_2)_L^{-1}) * L') \neq \phi$. By using distributive law, $((t_1 * (t_2)_L^{-1}) * ((t_1)_L^{-1} * M_1) \cup ((t_1 * (t_2)_L^{-1}) * ((t_2)_L^{-1} * M_2))) \cap ((t_1 * (t_2)_L^{-1}) * L') \neq \phi$. That is $H \cap ((t_1 * (t_2)_L^{-1}) * L') \neq \phi$, where

$H = (t_1 * (t_2)_L^{-1}) * ((t_1)_L^{-1} * M_1) \cup ((t_1 * (t_2)_L^{-1}) * ((t_2)_L^{-1} * M_2))$ is a semi open neighborhood of $t_1 * (t_2)_L^{-1}$, implies $t_1 * (t_2)_L^{-1} \in sCl(L')$. Accordingly, $t_1 * (t_2)_R^{-1} \in sCl(L')$. □

**Remark 3.7.** The sufficient condition for a sub-loop of a quasi s-topological loop with semi open left translation to be semi clopen is its openness.

**Theorem 3.8.** *In a quasi s-topological loop, inverses and free product with finite (countable) set of a semi compact (semi-Lindelof) set is compact (Lindelof).*

*Proof.* Let $M$ is semi compact (semi-Lindelof), this implies $i_L(M) = M_L^{-1}$, $i_R(M) = M_R^{-1}$ are compact (Lindelof) [4, 21]. Moreover, let $N$ is finite (countable) and for all $n \in N$, $N.M$ as a finite union of the sets $nM = l_n(M)$ is compact (Lindelof) [4, 21]. □

**Theorem 3.9.** *Let $(L, *, \tau)$ be a quasi s-topological loop and $S.C.(e)$ is open. Then $S.C.(e)$ is sub-loop, if for all $t \in S.C.(e)$, $r_{(t)_L^{-1}}$ and $r_{(t)_R^{-1}}$ are open.*

*Proof.* Let $t \in S.C.(e)$ then $(S.C.(e)).(t)_L^{-1} = r_{(t)_L^{-1}}(S.C.(e))$ is semi connected and contain $e$ [**?**]. Hence, $(S.C.(e)).(t)_L^{-1} \subseteq S.C.(e)$. As $(t)_L^{-1} \in (S.C.(e))_L^{-1}$, therefore, $(S.C.(e)).(S.C.(e))_L^{-1} \subseteq S.C.(e)$. Similarly,
$(S.C.(e)).(S.C.(e))_R^{-1} \subseteq S.C.(e)$. □

## 4. Conclusion

In this study, we have investigate and explore many significant results on quasi topological loops with respect to semi-continuity. We have discussed that the property of being quasi s-topological IP-loop is preserved under topological conjugation, and show that multiplication mapping is jointly semi open in a quasi s-topological IP-loop. We have also introduce the concept of semi discrete sub-loop and have proved that for a semi discrete sub-loop $L^{**}$, $sCl(L^{**}) = L^*$, where $L^*$ is a sub-loop and $(L, *, \tau)$ is a quasi s-topological loop having semi open left translation. Moreover, we have observed that every quasi irresolute topological loop is quasi s-topological loop, and have been discussed many properties of neighbourhood of identity in a quasi s-topological IP-loop as well. We illustrate quasi s-topological IP-loop with an interesting example by using a zero dimensional additive metrizable perfect topological IP-loop $L^*$ having commutative property with an infinite null sequence $\{N_0\}$ with a new base $\mu$. As a matter of fact, these results have a useful contribution in the field of topological algebra.

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