

PAIRWISE NEUTROSOPHIC α -OPEN SET VIA NEUTROSOPHIC BITOPOLOGICAL SPACES

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Abstract. In this paper, an attempt is made to define a new class of set namely pairwise neutrosophic α -open set via neutrosophic bitopological space. Besides, we study some properties of them. Further, we formulate several results based on them.

1. Introduction

Smarandache [15] introduced the concept of neutrosophic set as a generalization of fuzzy set [19] and intuitionistic fuzzy set [3]. From then it becomes very useful in the area of decision making, artificial intelligence, etc. In the year 2005, Smarandache [16] further studied the concept of neutrosophy. Afterwards, the notion of n -valued refined neutrosophic logic was studied by Smarandache [17] in the year 2013. The notion of neutrosophic topological space was presented by Salama and Alblowi [13] in the year 2012. Salama and Alblowi [14] also studied the generalized neutrosophic set and generalized neutrosophic topological spaces. The concept of neutrosophic α -open set was first grounded by Arokiarani et al. [2]. They also defined neutrosophic semi-open functions and established some relation between them. Rao and Srinivasa [12] presented the neutrosophic pre-open sets and pre-closed sets in neutrosophic topological spaces. Iswaraya and Bageerathi [9] grounded the notion of neutrosophic semi-closed set and neutrosophic semi-open set. The concept of b -open set in general topological spaces was grounded by Andrijevic [1]. Later on, Ebenanjar et al. [8] introduced the neutrosophic b -open sets in neutrosophic topological spaces. The notion of neutrosophic simply b -open set via neutrosophic topological spaces was studied by Das and Tripathy [5]. The notion of bitopological space was introduced by Kelly [10] in the year 1963. Thereafter, Ozturk and Ozkan [11] introduced the idea of neutrosophic bitopological space (NBTS) in the year 2019. Recently, Das and Tripathy [6] presented the idea of pairwise neutrosophic

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b -open set and studied their different properties. Recently, Tripathy and Das [18] also introduced the concept of pairwise neutrosophic b -continuous function in NBTSSs.

The main aim of this article is to procure the notion of pairwise neutrosophic α -open set via neutrosophic bitopological spaces, and formulate some results on it.

2. Preliminaries and Definitions

In this section, we give some relevant definitions those are important for developing the main results of this paper.

Definition 2.1. [15] An neutrosophic set (NS) W over a fixed set X is defined by: $W = \{(m, T_W(m), I_W(m), F_W(m)) : m \in X\}$, where $T_W(m)$, $I_W(m)$ and $F_W(m)$ ($\in [0, 1]$) denotes the truth, indeterminacy and false membership values of each $m \in X$. So, $0 \leq T_W(m) + I_W(m) + F_W(m) \leq 3$, for each $m \in X$.

Definition 2.2. [13] Let τ be a family of NSs over X . Then, τ is called an neutrosophic topology (NT) on X if the following holds:

- (i) $0_N, 1_N \in \tau$;
- (ii) $W_1 \cap W_2 \in \tau$ whenever $W_1, W_2 \in \tau$;
- (iii) $\cup W_i \in \tau$ whenever $\{W_i : i \in \Delta\} \subseteq \tau$.

Then, (X, τ) is called an neutrosophic topological space (NTS). If $W \in \tau$, then W is called an neutrosophic open set (NOS) and its complement i.e., W^c is called an neutrosophic closed set (NCS) in (X, τ) .

Definition 2.3. [13] Let (X, τ) be an NTS, and U be a NS over X . Then, the neutrosophic interior (N_{int}) and neutrosophic closure (N_{cl}) of U are defined as follows:

- (i) $N_{int}(U) = \cup \{E : E \text{ is a NOS in } X \text{ and } E \subseteq U\}$;
- (ii) $N_{cl}(U) = \cap \{F : F \text{ is a NCS in } X \text{ and } U \subseteq F\}$.

Definition 2.4. [13] Suppose that (X, τ) be an NTS, and A be an neutrosophic subset of X . Then, A is called an neutrosophic α -open set in (X, τ) if and only if $A \subseteq N_{int}(N_{cl}(N_{int}(A)))$. The complement of an neutrosophic α -open set is called an neutrosophic α -closed set.

Definition 2.5. [13] A bijective mapping $\xi : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an neutrosophic α -continuous mapping if the inverse image of every NOS in Y is an neutrosophic α -open set in X .

Definition 2.6. [11] Suppose that τ_1 and τ_2 be two different NTs on X . Then, the triplet (X, τ_1, τ_2) is called an neutrosophic bitopological space (NBTS).

Remark 2.7. Suppose that (X, τ_1, τ_2) be an NBTS, and W be an neutrosophic subset of X . Then, $N_{int}^i(W)$ and $N_{cl}^i(W)$ indicates the neutrosophic interior and neutrosophic closure of W with respect to the NTs (X, τ_i) ($i=1, 2$).

Definition 2.8. [11] In an NBTS (X, τ_1, τ_2) , an NS Q is called a pairwise neutrosophic open set (in short pairwise NOS) if $Q = W \cup M$, where W is an NOS in (X, τ_1) and M is an NOS in (X, τ_2) .

Lemma 2.9. [11] Let (X, τ_1, τ_2) be an NBTS. Then, every NOS in an NTS (X, τ_i) is a pairwise NOS in (X, τ_1, τ_2) .

Definition 2.10. [6] Let (X, τ_1, τ_2) be an NBTS. An NS P over X is called a

- (i) τ_{ij} neutrosophic semi-open set if and only if $P \subseteq N_{cl}^i(N_{int}^j(P))$;
- (ii) τ_{ij} neutrosophic pre-open set if and only if $P \subseteq N_{int}^j(N_{cl}^i(P))$.

Definition 2.11. [6] An NS L is said to be a pairwise neutrosophic pre-open set (pairwise neutrosophic semi open set) in an NBTS (X, τ_1, τ_2) if $L = K \cup M$, where K is a τ_{ij} neutrosophic pre-open set (τ_{ij} neutrosophic semi open set) and M is a τ_{ji} neutrosophic pre-open set (τ_{ji} neutrosophic semi-open set) in (X, τ_1, τ_2) .

Definition 2.12. [6] In an NBTS (X, τ_1, τ_2) , an NS W is called a τ_{ij} neutrosophic b -open set if and only if $W \subseteq N_{cl}^i N_{int}^j(W) \cup N_{int}^j N_{cl}^i(W)$. An neutrosophic set M over X is called a τ_{ij} neutrosophic b -closed set if and only if M^c is a τ_{ij} neutrosophic b -open set in (X, τ_1, τ_2) .

Definition 2.13. [6] An neutrosophic set L is called a pairwise neutrosophic b -open set in an NBTS (X, τ_1, τ_2) if $L = K \cup M$, where K is a τ_{ij} neutrosophic b -open set and M is a τ_{ji} neutrosophic b -open set in (X, τ_1, τ_2) . An neutrosophic set L is called a pairwise neutrosophic b -closed in (X, τ_1, τ_2) if and only if L^c is a pairwise τ_{ij} neutrosophic b -open set in (X, τ_1, τ_2) .

3. Pairwise Neutrosophic α -Open Set

In this section, we introduce the concept of pairwise neutrosophic α -open set, pairwise neutrosophic α -closed set, pairwise neutrosophic α -continuous function and pairwise neutrosophic α -open function via NBTSs, and formulate several results based on them.

Definition 3.1. Let (X, τ_1, τ_2) be an NBTS, and P be an neutrosophic subset of X . Then P , an NS is called a τ_{ij} neutrosophic α -open set iff $P \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(P)))$. If P is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) , then the complement of P i.e., P^c is called a τ_{ij} neutrosophic α -closed set in (X, τ_1, τ_2) .

Theorem 3.2. Let (X, τ_1, τ_2) be an NBTS. Then, every NOS in NTS (X, τ_j) is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) .

Proof. Suppose that (X, τ_1, τ_2) be an NBTS, and Q be an NOS in (X, τ_j) . Therefore, $N_{int}^j(Q) = Q$. It is known that, $N_{int}^j(Q) \subseteq N_{cl}^i(N_{int}^j(Q))$.

$$\begin{aligned}
 &\text{Now, } N_{int}^j(Q) \subseteq N_{cl}^i(N_{int}^j(Q)) \\
 &\implies Q = N_{int}^j(Q) \subseteq N_{cl}^i(N_{int}^j(Q)) \\
 &\implies Q \subseteq N_{cl}^i(N_{int}^j(Q)) \\
 &\implies N_{int}^j(Q) \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(Q))) \\
 &\implies Q = N_{int}^j(Q) \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(Q))) \\
 &\implies Q \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(Q)))
 \end{aligned}$$

Therefore, Q is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) .

Lemma 3.3. Let (X, τ_1, τ_2) be an NBTS. Then, every NCS in NTS (X, τ_j) is a τ_{ij} neutrosophic α -closed set in (X, τ_1, τ_2) .

Theorem 3.4. In an NBTS (X, τ_1, τ_2) ,

- (i) every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic semi-open set;
- (ii) every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic pre-open set.

Proof.(i) Suppose that (X, τ_1, τ_2) be an NBTS. Let Q be a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) . So, $Q \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(Q)))$. It is known that, $N_{int}^j(N_{cl}^i(N_{int}^j(Q))) \subseteq N_{cl}^i(N_{int}^j(Q))$. Therefore, $Q \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(Q))) \subseteq N_{cl}^i(N_{int}^j(Q))$. This implies, $Q \subseteq N_{cl}^i(N_{int}^j(Q))$. Hence, Q is a τ_{ij} neutrosophic semi-open set in (X, τ_1, τ_2) . Therefore, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic semi-open set.

(ii) Suppose that (X, τ_1, τ_2) be an NBTS. Let Q be a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) . So, $Q \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(Q)))$. It is known that, $N_{int}^j(Q) \subseteq Q$.

Now, $N_{int}^j(Q) \subseteq Q$

$$\Rightarrow N_{cl}^i(N_{int}^j(Q)) \subseteq N_{cl}^i(Q)$$

$$\Rightarrow N_{int}^j(N_{cl}^i(N_{int}^j(Q))) \subseteq N_{int}^j(N_{cl}^i(Q))$$

$$\Rightarrow Q \subseteq N_{int}^j(N_{cl}^i(Q)) \text{ [Since, } Q \subseteq N_{int}^j N_{cl}^i N_{int}^j(Q)\text{]}$$

Hence, Q is a τ_{ij} neutrosophic pre-open set in (X, τ_1, τ_2) . Therefore, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic pre-open set.

Theorem 3.5. *In an NBTS (X, τ_1, τ_2) , every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic b -open set.*

Proof. Suppose that (X, τ_1, τ_2) be an NBTS. Let Q be a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) . Since, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic semi-open set, and every τ_{ij} neutrosophic semi-open set is a τ_{ij} neutrosophic b -open set in (X, τ_1, τ_2) , so Q is a τ_{ij} neutrosophic b -open set in (X, τ_1, τ_2) . Hence, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic b -open set.

Theorem 3.6. *Let (X, τ_1, τ_2) be an NBTS. If G is both an NOS in (X, τ_j) and τ_{ij} neutrosophic semi-open set in (X, τ_1, τ_2) , then G is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) .*

Proof. Suppose that (X, τ_1, τ_2) be an NBTS. Let G be both NOS in (X, τ_j) and τ_{ij} neutrosophic semi-open set in (X, τ_1, τ_2) . Since, G is an NOS in (X, τ_j) , so $N_{int}^j(G) = G$. Further, since G is a τ_{ij} neutrosophic semi-open set in (X, τ_1, τ_2) , so $G \subseteq N_{cl}^i(N_{int}^j(G))$.

$$\text{Now, } G \subseteq N_{cl}^i(N_{int}^j(G))$$

$$\Rightarrow N_{int}^j(G) \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(G)))$$

$$\Rightarrow G = N_{int}^j(G) \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(G)))$$

$$\Rightarrow G \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(G)))$$

Therefore, G is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) .

Theorem 3.7. *Let (X, τ_1, τ_2) be an NBTS. If G is an NOS in (X, τ_j) and τ_{ij} neutrosophic pre-open set in (X, τ_1, τ_2) , then G is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) .*

Proof. Suppose that (X, τ_1, τ_2) be an NBTS. Let G be both NOS in (X, τ_j) and τ_{ij} neutrosophic pre-open set in (X, τ_1, τ_2) . Since, G is an NOS in (X, τ_j) , so $N_{int}^j(G) = G$. Further, since G is a τ_{ij} neutrosophic pre-open set in (X, τ_1, τ_2) , so $G \subseteq N_{int}^j(N_{cl}^i(G))$.

$$\text{Now, } G \subseteq N_{int}^j(N_{cl}^i(G))$$

$$\Rightarrow G \subseteq N_{int}^j(N_{cl}^i(N_{int}^j(G))) \text{ [By putting } G = N_{int}^j(G)\text{]}$$

Therefore, G is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) .

Definition 3.8. Let (X, τ_1, τ_2) be an NBTS. Then A , an NS over X is called a pairwise neutrosophic α -open set in (X, τ_1, τ_2) if there exists a τ_{ij} neutrosophic α -open set G and a τ_{ji} neutrosophic α -open set H in (X, τ_1, τ_2) such that $A = G \cup H$.

Theorem 3.9. Let (X, τ_1, τ_2) be an NBTS. Then, every pairwise neutrosophic open set in (X, τ_1, τ_2) is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) .

Proof. Suppose that (X, τ_1, τ_2) be an NBTS. Let G be a pairwise neutrosophic open set in (X, τ_1, τ_2) . Therefore, there exists an NOS Y in (X, τ_i) and an NOS R in (X, τ_j) such that $G = Y \cup R$. Since, every NOS in (X, τ_i) is a τ_{ji} neutrosophic α -open set in (X, τ_1, τ_2) , so Y is a τ_{ji} neutrosophic α -open set in (X, τ_1, τ_2) . Further, since every NOS in (X, τ_j) is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) , so R is a τ_{ij} neutrosophic α -open set in (X, τ_1, τ_2) . Therefore, G can be expressed as the union of a τ_{ij} neutrosophic α -open set R and a τ_{ji} neutrosophic α -open set Y . Hence, G is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) .

Theorem 3.10. Let (X, τ_1, τ_2) be an NBTS. Then,

- (i) every pairwise neutrosophic α -open set is a pairwise neutrosophic semi-open set in (X, τ_1, τ_2) ;
- (ii) every pairwise pairwise neutrosophic α -open set is a pairwise neutrosophic pre-open set in (X, τ_1, τ_2) ;
- (iii) every pairwise pairwise neutrosophic α -open set is a pairwise neutrosophic b-open set in (X, τ_1, τ_2) .

Proof. (i) Suppose that (X, τ_1, τ_2) be an NBTS. Assume that G be a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic α -open set and Y is a τ_{ji} neutrosophic α -open set in (X, τ_1, τ_2) . Since, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic semi-open set in (X, τ_1, τ_2) , so R is a τ_{ij} neutrosophic semi-open set in (X, τ_1, τ_2) . Further, since every τ_{ji} neutrosophic α -open set is a τ_{ji} neutrosophic semi-open set in (X, τ_1, τ_2) , so Y is a τ_{ji} neutrosophic semi-open set in (X, τ_1, τ_2) . Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic semi-open set and Y is a τ_{ji} neutrosophic semi-open set in (X, τ_1, τ_2) . Hence, G is a pairwise neutrosophic semi-open set in (X, τ_1, τ_2) .

(ii) Assume that (X, τ_1, τ_2) be an NBTS. Let G be a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic α -open set and Y is a τ_{ji} neutrosophic α -open set in (X, τ_1, τ_2) . Since, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic pre-open set in (X, τ_1, τ_2) , so R is a τ_{ij} neutrosophic pre-open set in (X, τ_1, τ_2) . Further, since every τ_{ji} neutrosophic α -open set is a τ_{ji} neutrosophic pre-open set in (X, τ_1, τ_2) , so Y is a τ_{ji} neutrosophic pre-open set in (X, τ_1, τ_2) . Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic pre-open set and Y is a τ_{ji} neutrosophic pre-open set in (X, τ_1, τ_2) . Hence, G is a pairwise neutrosophic pre-open set in (X, τ_1, τ_2) .

(iii) Assume that (X, τ_1, τ_2) be an NBTS. Let G be a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic α -open set and Y is a τ_{ji} neutrosophic α -open set in (X, τ_1, τ_2) . Since, every τ_{ij} neutrosophic α -open set is a τ_{ij} neutrosophic b-open set in (X, τ_1, τ_2) , so R is

a τ_{ij} neutrosophic b -open set in (X, τ_1, τ_2) . Further, since every τ_{ji} neutrosophic α -open set is a τ_{ji} neutrosophic b -open set in (X, τ_1, τ_2) , so Y is a τ_{ji} neutrosophic b -open set in (X, τ_1, τ_2) . Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic b -open set and Y is a τ_{ji} neutrosophic b -open set in (X, τ_1, τ_2) . Hence, G is a pairwise neutrosophic b -open set in (X, τ_1, τ_2) .

Definition 3.11. A bijective mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called a pairwise neutrosophic α -continuous mapping iff $f^{-1}(D)$ is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) whenever D is a pairwise NOS in (Y, σ_1, σ_2) .

Theorem 3.12. (i) Every pairwise neutrosophic continuous mapping is also a pairwise neutrosophic α -continuous mapping;

(ii) Every pairwise neutrosophic α -continuous mapping is a pairwise neutrosophic semi-continuous mapping;

(iii) Every pairwise neutrosophic α -continuous mapping is a pairwise neutrosophic pre-continuous mapping;

(iv) Every pairwise neutrosophic α -continuous mapping is a pairwise neutrosophic b -continuous mapping.

Proof. (i) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic continuous mapping. Let D be a pairwise NOS in (Y, σ_1, σ_2) . By hypothesis, $f^{-1}(D)$ is a pairwise NOS in (X, τ_1, τ_2) . Since, every pairwise NOS in (X, τ_1, τ_2) is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) , so $f^{-1}(D)$ is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Therefore, $f^{-1}(D)$ is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) whenever D is a pairwise NOS in (Y, σ_1, σ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic α -continuous mapping.

(ii) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -continuous mapping. Let D be a pairwise NOS in (Y, σ_1, σ_2) . By hypothesis, $f^{-1}(D)$ is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Since, every pairwise pairwise neutrosophic α -open set in (X, τ_1, τ_2) is a pairwise neutrosophic semi-open set in (X, τ_1, τ_2) , so $f^{-1}(D)$ is a pairwise neutrosophic semi-open set in (X, τ_1, τ_2) . Therefore, $f^{-1}(D)$ is a pairwise neutrosophic semi-open set in (X, τ_1, τ_2) whenever D is a pairwise NOS in (Y, σ_1, σ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic semi-continuous mapping.

(iii) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -continuous mapping. Let D be a pairwise NOS in (Y, σ_1, σ_2) . By hypothesis, $f^{-1}(D)$ is a pairwise pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Since, every pairwise pairwise neutrosophic α -open set in (X, τ_1, τ_2) is a pairwise neutrosophic pre-open set in (X, τ_1, τ_2) , so $f^{-1}(D)$ is a pairwise neutrosophic pre-open set in (X, τ_1, τ_2) . Therefore, $f^{-1}(D)$ is a pairwise neutrosophic pre-open set in (X, τ_1, τ_2) whenever D is a pairwise NOS in (Y, σ_1, σ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic pre-continuous mapping.

(iv) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -continuous mapping. Let D be a pairwise NOS in (Y, σ_1, σ_2) . By hypothesis, $f^{-1}(D)$ is a pairwise

neutrosophic α -open set in (X, τ_1, τ_2) . Since, every pairwise pairwise neutrosophic α -open set in (X, τ_1, τ_2) is a pairwise neutrosophic b -open set in (X, τ_1, τ_2) , so $f^{-1}(D)$ is a pairwise neutrosophic b -open set in (X, τ_1, τ_2) . Therefore, $f^{-1}(D)$ is a pairwise neutrosophic b -open set in (X, τ_1, τ_2) whenever D is a pairwise NOS in (Y, σ_1, σ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic b -continuous mapping.

Theorem 3.13. *Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -continuous mapping, and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \theta_1, \theta_2)$ be a pairwise neutrosophic continuous mapping, then the composition mapping $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is a pairwise neutrosophic α -continuous mapping.*

Proof. Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -continuous mapping, and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \theta_1, \theta_2)$ be a pairwise neutrosophic continuous mapping. Let D be a pairwise NOS in (Z, θ_1, θ_2) . Since $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \theta_1, \theta_2)$ is a pairwise neutrosophic continuous mapping, so $g^{-1}(D)$ is a pairwise NOS in (Y, σ_1, σ_2) . Further, since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -continuous mapping, so $f^{-1}(g^{-1}(D)) = (g \circ f)^{-1}(D)$ is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Therefore, $(g \circ f)^{-1}(D)$ is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) whenever D is a pairwise NOS in (Z, θ_1, θ_2) . Hence, the composition mapping $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is a pairwise neutrosophic α -continuous mapping.

Definition 3.14. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be a pairwise neutrosophic α -open mapping iff $f(D)$ is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) whenever D is a pairwise NOS in (X, τ_1, τ_2) .

Theorem 3.15. (i) *Every pairwise neutrosophic open mapping is also a pairwise neutrosophic α -open mapping;*
(ii) *Every pairwise neutrosophic α -open mapping is a pairwise neutrosophic semi-open mapping;*
(iii) *Every pairwise neutrosophic α -open mapping is a pairwise neutrosophic pre-open mapping;*
(iv) *Every pairwise neutrosophic α -open mapping is a pairwise neutrosophic b -open mapping.*

Proof. (i) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic open mapping. Let D be a pairwise NOS in (X, τ_1, τ_2) . By hypothesis, $f(D)$ is a pairwise NOS in (Y, σ_1, σ_2) . Since, every pairwise NOS in (Y, σ_1, σ_2) is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) , so $f(D)$ is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) . Therefore, $f(D)$ is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) whenever D is a pairwise NOS in (X, τ_1, τ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic α -open mapping.

(ii) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -open mapping. Let D be a pairwise NOS in (X, τ_1, τ_2) . By hypothesis, $f(D)$ is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) . Since, every pairwise pairwise neutrosophic α -open set in (Y, σ_1, σ_2) is a pairwise neutrosophic semi-open set in (Y, σ_1, σ_2) , so $f(D)$ is a pairwise neutrosophic semi-open set in (Y, σ_1, σ_2) . Therefore, $f(D)$ is a pairwise neutrosophic semi-open set in (Y, σ_1, σ_2) whenever D is a pairwise NOS in (X, τ_1, τ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic semi-open mapping.

(iii) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -open mapping. Let D be a pairwise NOS in (X, τ_1, τ_2) . By hypothesis, $f(D)$ is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) . Since, every pairwise neutrosophic α -open set in (Y, σ_1, σ_2) is a pairwise neutrosophic pre-open set in (Y, σ_1, σ_2) , so $f(D)$ is a pairwise neutrosophic pre-open set in (Y, σ_1, σ_2) . Therefore, $f(D)$ is a pairwise neutrosophic pre-open set in (Y, σ_1, σ_2) whenever D is a pairwise NOS in (X, τ_1, τ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic pre-open mapping.

(iv) Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic α -open mapping. Let D be a pairwise NOS in (X, τ_1, τ_2) . By hypothesis, $f(D)$ is a pairwise neutrosophic α -open set in (Y, σ_1, σ_2) . Since, every pairwise neutrosophic α -open set in (Y, σ_1, σ_2) is a pairwise neutrosophic b -open set in (Y, σ_1, σ_2) , so $f(D)$ is a pairwise neutrosophic b -open set in (Y, σ_1, σ_2) . Therefore, $f(D)$ is a pairwise neutrosophic b -open set in (Y, σ_1, σ_2) whenever D is a pairwise NOS in (X, τ_1, τ_2) . Hence, the mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic b -open mapping.

Theorem 3.16. *Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic open mapping, and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \theta_1, \theta_2)$ be a pairwise neutrosophic α -open mapping, then the composition mapping $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is a pairwise neutrosophic α -open mapping.*

Proof. Suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise neutrosophic open mapping, and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \theta_1, \theta_2)$ be a pairwise neutrosophic α -open mapping. Let D be a pairwise NOS in (X, τ_1, τ_2) . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise neutrosophic open mapping, so $f(D)$ is a pairwise NOS in (Y, σ_1, σ_2) . Further, since $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \theta_1, \theta_2)$ is a pairwise neutrosophic α -open mapping, so $g(f(D)) = (g \circ f)(D)$ is a pairwise neutrosophic α -open set in (Z, θ_1, θ_2) . Therefore, $(g \circ f)(D)$ is a pairwise neutrosophic α -open set in (Z, θ_1, θ_2) whenever D is a pairwise NOS in (X, τ_1, τ_2) . Hence, the composition mapping $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is a pairwise neutrosophic α -open mapping.

Definition 3.17. A family $\{A_i : i \in \Delta\}$, where Δ is an index set and A_i is a pairwise neutrosophic open set in an NBTS (X, τ_1, τ_2) , for each $i \in \Delta$, is called a pairwise neutrosophic open cover of an NS A if $A \subseteq \cup_{i \in \Delta} A_i$.

Definition 3.18. An NBTS (X, τ_1, τ_2) is called a pairwise neutrosophic compact space if each pairwise neutrosophic open cover of 1_N has a finite sub-cover.

Definition 3.19. A neutrosophic sub-set B of an NBTS (X, τ_1, τ_2) is called a pairwise neutrosophic compact set relative to X if every pairwise neutrosophic open cover of B has a finite pairwise neutrosophic open sub-cover.

Theorem 3.20. *Every pairwise neutrosophic closed sub-set of a pairwise neutrosophic compact space (X, τ_1, τ_2) is a pairwise neutrosophic compact relative to X .*

Proof. Assume that (X, τ_1, τ_2) be a pairwise neutrosophic compact space. Suppose that K be a pairwise neutrosophic closed sub-set of (X, τ_1, τ_2) . Therefore, K^c is a pairwise neutrosophic open set in (X, τ_1, τ_2) . Suppose that $U = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic open set in } X\}$ be a pairwise neutrosophic open cover of K . Then,

$H = \{K^c\} \cup U$ is a pairwise neutrosophic open cover of 1_N . Since (X, τ_1, τ_2) is a pairwise neutrosophic compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \dots, H_n, K_c\}$. This implies, $\{H_1, H_2, H_3, \dots, H_n\}$ is a finite pairwise neutrosophic open cover of K . Hence, K is a pairwise neutrosophic compact set relative to X .

Theorem 3.21. *If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a pairwise neutrosophic continuous mapping, then for each pairwise neutrosophic compact set Q relative to X , $\xi(Q)$ is a pairwise neutrosophic compact set relative to Y .*

Proof. Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a pairwise neutrosophic continuous mapping. Let Q be a pairwise neutrosophic compact set relative to X . Suppose that $H = \{H_i : i \in \Delta, \text{ and } H_i \text{ is a pairwise neutrosophic open set in } Y\}$ be a pairwise neutrosophic open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(H) = \{\xi^{-1}(H_i) : i \in \Delta \text{ and } \xi^{-1}(H_i) \text{ is a pairwise neutrosophic open set in } X\}$ is a pairwise neutrosophic open cover of $\xi^{-1}(\xi(Q)) = Q$. Since Q is a pairwise neutrosophic compact set relative to X , so there exists a finite pairwise neutrosophic open sub-cover of Q say $\{\xi^{-1}(H_1), \xi^{-1}(H_2), \xi^{-1}(H_3), \dots, \xi^{-1}(H_n)\}$ such that $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$.
Now $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$
 $\implies \xi(Q) \subseteq \cup_i \{\xi(\xi^{-1}(H_i)) : i = 1, 2, \dots, n\}$
 $\implies \xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$
Therefore, there exist a finite pairwise neutrosophic open sub-cover $\{H_1, H_2, H_3, \dots, H_n\}$ of $\xi(Q)$ such that $\xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$. Hence, $\xi(Q)$ is a pairwise neutrosophic compact set relative to Y .

Theorem 3.22. *If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a pairwise neutrosophic open function, and B is a pairwise neutrosophic compact set relative to (Y, δ_1, δ_2) , then $\xi^{-1}(B)$ is also a pairwise neutrosophic compact set relative to (X, τ_1, τ_2) .*

Proof. Assume that $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a pairwise neutrosophic open function. Let B is a pairwise neutrosophic compact set relative to (Y, δ_1, δ_2) . Suppose that $H = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic open set in } X\}$ be a pairwise neutrosophic open cover of $\xi^{-1}(B)$. By hypothesis, $\xi(H) = \{\xi(H_i) : i \in \Delta \text{ and } \xi(H_i) \text{ is a pairwise neutrosophic open set in } Y\}$ is a pairwise neutrosophic open cover of $\xi(\xi^{-1}(B)) = B$. Since, B is a pairwise neutrosophic compact set relative to (Y, δ_1, δ_2) , so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \dots, \xi(H_n)\}$ such that $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$.
Now, $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$
 $\implies \xi^{-1}(B) \subseteq \cup \{\xi^{-1}(\xi(H_i)) : i = 1, 2, \dots, n\}$
 $\implies \xi^{-1}(B) \subseteq \cup \{H_i : i = 1, 2, \dots, n\}$
This implies, $\{H_1, H_2, \dots, H_n\}$ is a finite sub-cover for $\xi^{-1}(B)$. Hence, $\xi^{-1}(B)$ is a pairwise neutrosophic compact set relative to (X, τ_1, τ_2) .

Definition 3.23. A family $\{A_i : i \in \Delta\}$, where Δ is an index set and A_i is a pairwise neutrosophic α -open set in an NBTS (X, τ_1, τ_2) , for each $i \in \Delta$, is called a pairwise neutrosophic α -open cover of an NS A if $A \subseteq \cup_{i \in \Delta} A_i$.

Definition 3.24. An NBTS (X, τ_1, τ_2) is called a pairwise neutrosophic α -compact space if each pairwise neutrosophic α -open cover of 1_N has a finite sub-cover.

Definition 3.25. An neutrosophic sub-set B of an NBTS (X, τ_1, τ_2) is called a pairwise neutrosophic α -compact relative to X if every pairwise neutrosophic α -open cover of B has a finite pairwise neutrosophic α -open sub-cover.

Theorem 3.26. Every pairwise neutrosophic α -closed sub-set of a pairwise neutrosophic α -compact space (X, τ_1, τ_2) is a pairwise neutrosophic α -compact set relative to X .

Proof. Assume that (X, τ_1, τ_2) be a pairwise neutrosophic α -compact space. Suppose that K be a pairwise neutrosophic α -closed sub-set of (X, τ_1, τ_2) . Therefore, K^c is a pairwise neutrosophic α -open set in (X, τ_1, τ_2) . Suppose that $U = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic } \alpha\text{-open set in } X\}$ be a pairwise neutrosophic α -open cover of K . Then, $H = \{K^c\} \cup U$ is a pairwise neutrosophic α -open cover of 1_N . Since (X, τ_1, τ_2) is a pairwise neutrosophic α -compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \dots, H_n, K_c\}$. This implies, $\{H_1, H_2, H_3, \dots, H_n\}$ is a finite pairwise neutrosophic α -open cover of K . Hence, K is a pairwise neutrosophic α -compact set relative to X .

Theorem 3.27. If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a pairwise neutrosophic α -continuous mapping, then for each pairwise neutrosophic α -compact set Q relative to X , $\xi(Q)$ is a pairwise neutrosophic α -compact set relative to Y .

Proof. Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a pairwise neutrosophic α -continuous mapping. Let Q be a pairwise neutrosophic α -compact set relative to X . Suppose that $H = \{H_i : i \in \Delta, \text{ and } H_i \text{ is a pairwise neutrosophic } \alpha \text{ open set in } Y\}$ be a pairwise neutrosophic α -open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(H) = \{\xi^{-1}(H_i) : i \in \Delta \text{ and } \xi^{-1}(H_i) \text{ is a pairwise neutrosophic } \alpha\text{-open set in } X\}$ is a pairwise neutrosophic α -open cover of $\xi^{-1}(\xi(Q)) = Q$. Since Q is a pairwise neutrosophic α -compact set relative to X , so there exists a finite pairwise neutrosophic α -open sub-cover of Q say $\{\xi^{-1}(H_1), \xi^{-1}(H_2), \xi^{-1}(H_3), \dots, \xi^{-1}(H_n)\}$ such that $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$.
Now $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$
 $\implies \xi(Q) \subseteq \cup_i \{\xi(\xi^{-1}(H_i)) : i = 1, 2, \dots, n\}$
 $\implies \xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$

Therefore, there exist a finite pairwise neutrosophic α -open sub-cover $\{H_1, H_2, H_3, \dots, H_n\}$ of $\xi(Q)$ such that $\xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$. Hence, $\xi(Q)$ is a pairwise neutrosophic α -compact set relative to Y .

Theorem 3.28. If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a pairwise neutrosophic α -open function, and B is a pairwise neutrosophic α -compact set relative to (Y, δ_1, δ_2) , then $\xi^{-1}(B)$ is also a pairwise neutrosophic α -compact set relative to (X, τ_1, τ_2) .

Proof. Assume that $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a pairwise neutrosophic α -open function. Let B is a pairwise neutrosophic α -compact set relative to (Y, δ_1, δ_2) . Suppose that $H = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic } \alpha\text{-open set in } X\}$ be a pairwise neutrosophic α -open cover of $\xi^{-1}(B)$. By hypothesis, $\xi(H) = \{\xi(H_i) : i \in \Delta \text{ and } \xi(H_i) \text{ is a pairwise neutrosophic } \alpha\text{-open set in } Y\}$ is a pairwise neutrosophic α -open cover of $\xi(\xi^{-1}(B)) = B$. Since, B is a pairwise neutrosophic α -compact set relative to (Y, δ_1, δ_2) , so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \dots, \xi(H_n)\}$ such that $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$.

Now, $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$
 $\implies \xi^{-1}(B) \subseteq \cup \{\xi^{-1}(\xi(H_i)) : i = 1, 2, \dots, n\}$

$$\implies \xi^{-1}(B) \subseteq \cup\{H_i : i = 1, 2, \dots, n\}$$

This implies, $\{H_1, H_2, \dots, H_n\}$ is a finite sub-cover for $\xi^{-1}(B)$. Hence, $\xi^{-1}(B)$ is a pairwise neutrosophic α -compact set relative to (X, τ_1, τ_2) .

Theorem 3.29. *Let (X, τ_1, τ_2) be an NBTS. Then, every pairwise neutrosophic α -compact set relative to X is also a pairwise neutrosophic compact set relative to X .*

Proof. Let (X, τ_1, τ_2) be an NBTS. Suppose that B be a pairwise neutrosophic α -compact set relative to X . Let $U = \{E_i : i \in \Delta \text{ and } E_i \text{ is a pairwise neutrosophic open set in } X\}$ be a pairwise neutrosophic open cover of B . Since every pairwise neutrosophic open set in (X, τ_1, τ_2) is also a pairwise neutrosophic α -open set in (X, τ_1, τ_2) , so $U = \{E_i : i \in \Delta \text{ and } E_i \text{ is a pairwise neutrosophic } \alpha\text{-open set in } X\}$ is a pairwise neutrosophic α -open cover of B . Further, since B is a pairwise neutrosophic α -compact set relative to X , so there exist a finite pairwise neutrosophic α -open sub-cover say $\{E_1, E_2, E_3, \dots, E_n\}$ such that $B \subseteq \cup_{i=1}^n E_i$. Therefore, $\{E_1, E_2, E_3, \dots, E_n\}$ is a finite pairwise neutrosophic open sub-cover of B . This implies, B is a pairwise neutrosophic compact set relative to X . Hence, every pairwise neutrosophic α -compact set relative to (X, τ_1, τ_2) is also a pairwise neutrosophic compact set relative to (X, τ_1, τ_2) .

4. Conclusions

In this article, we have introduced a new class of set namely pairwise neutrosophic α -open set via neutrosophic bitopological spaces. Besides, we have studied some properties of them, and formulated several interesting results based on them. It is hoped that, these notions can also be applied in the field of neutrosophic infi topological space, quadripartitioned neutrosophic topological space, pentapartitioned neutrosophic topological space, neutrosophic tri-topological space, etc.

References

- [1] D. Andrijevic, On b -open sets, *Matemacki Vesnik*, 48 (1996), 59-64.
- [2] I. Arokianani, R. Dhavaseelan, S. Jafari and M. Parimala, On some new notations and functions in neutrosophic topological spaces, *Neutrosophic Sets and Systems*, 16 (2017), 16-19.
- [3] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986), 87-96.
- [4] S. Das, R. Das and C. Granados, Topology on quadripartitioned neutrosophic Sets, *Neutrosophic Sets and Systems*, 45, 54-61.
- [5] S. Das and B. C. Tripathy, Neutrosophic simply b -open set in neutrosophic topological spaces, *Iraqi Journal of Science*, 62 (2021) 12, 4830-4838. DOI: 10.24996/ij.s.2021.62.12.21
- [6] S. Das and B. C. Tripathy, Pairwise neutrosophic- b -open set in neutrosophic bitopological spaces, *Neutrosophic Sets and Systems*, 38 (2020), 135-144.
- [7] A. Dutta and B. C. Tripathy, On fuzzy $b - \theta$ open sets in fuzzy topological space, *Journal of Intelligent and Fuzzy Systems*, 32 (2017), no. 1, 137-139.
- [8] E. Ebenanjar, J. Immaculate and C.B. Wilfred, On neutrosophic b -open sets in neutrosophic topological space, *Journal of Physics Conference Series*, 1139 (2018), no. 1, 012062.
- [9] P. Iswarya and K. Bageerathi, On neutrosophic semi-open sets in Neutrosophic Topological Spaces, *International Journal of Mathematics Trends and Technology*, 37 (2016), no. 3, 214-223.

- [10] J. C. Kelly, Bitopological Spaces, Proceedings of the London Mathematical Society, 3 (1963), no. 1, 71-89.
- [11] T. Y. Ozturk and A. Ozkan, Neutrosophic bitopological Spaces, Neutrosophic Sets and Systems, 30 (2019), 88-97.
- [12] V. V. Rao and R. Srinivasa, Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology, International Journal of Chemical Technology Research, 10 (2017), no. 10, 449-458.
- [13] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological space, ISOR Journal of Mathematics, 3 (2012), no. 4, 31-35.
- [14] A. A. Salama and S. A. Alblowi, Generalized neutrosophic set and generalized neutrosophic topological space, Computer Science and Engineering, 2 (2012), 129-132.
- [15] F. Smarandache, *A unifying field in logics, neutrosophy: neutrosophic probability, set and logic*, Rehoboth: American Research Press, 1998.
- [16] F. Smarandache, Neutrosophic set: a generalization of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics, 24 (2005), 287-297.
- [17] F. Smarandache, n -valued refined neutrosophic logic and its applications in physics, Progress in Physics, 4(2013), 143-146.
- [18] B. C. Tripathy and S. Das, Pairwise neutrosophic b -continuous mapping in neutrosophic bitopological spaces, Neutrosophic Sets and Systems, 43 (2021), 82-92.
- [19] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338-353.

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