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CONTRA ω_p-CONTINUOUS AND ALMOST CONTRA ω_p-CONTINUOUS FUNCTIONS

HALGWRD MOHAMMED DARWESH AND SHAGULL HOSSEIN MAHMOOD†

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Abstract. In this study, we apply the concept of ω_p-open sets to introduce two types of functions called contra ω_p-continuous and almost contra ω_p-continuous functions. We establish some fundamental properties of these functions and also explore their relationships with other types of functions.

1. Introduction

In 1996, K. Dontchev [2] defined contra-continuous functions and strongly s-closed topological spaces. K. Dontchev and T. Noiri [3] studied and introduced contra semicontinuous functions, which are a weaker form of contra-continuous functions.

H. M. Darwesh [1] used the concept of ω_p-open sets to introduce almost ω-continuous functions as a new type of continuity. M. Caldas and S. Jafari [4] examined the concept of contra-β-continuous functions. S. Jafari and T. Noiri [5, 6] defined and discussed contra super-continuous, contra pre-continuous, and contra α-continuous functions. The concept of function almost contra pre-continuous function was investigated by E. Ekici [7]. T. Noire and V. Popa [8] found out more about the almost-continuous functions. A. Nasef [9] presented the concept of contra γ-continuous functions.

However, the concept of $\omega-$open sets was used by A. Al-Omari and M. Noorani [10] to investigate and introduce new forms of functions named contra ω-continuous and almost contra ω-continuous functions. H. H. Aljarrah et al. [11] presented contra $\omega\beta$-continuous functions, which are a weak type of contra-continuity functions.

The purpose of this work is to define the new forms of functions named contra ω_p-continuous and almost contra ω_p-continuous functions by using ω_p-open sets. Likewise, we discuss some of their properties and relationships with the other types of functions.

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† *Corresponding author*

2. Preliminaries

Throughout this work, $(X, \Im)$ and (Y, τ) (or X and Y) are always topological spaces, $f : X \longrightarrow Y$ represent a function from a space X into a space Y. Let $\mathcal{A} \subseteq X$. Then, interior, closure, ω-closure and ω-interior of $\mathcal{A}$ are expressed as $Int(\mathcal{A})$ and $Cl(\mathcal{A})$, $\omega Cl(\mathcal{A})$ and $\omega Int(\mathcal{A})$, respectively. Also, ω_p-interior and ω_p-closure are defined in the same way as $Int(\mathcal{A})$ and $Cl(\mathcal{A})$, they are represented by $\omega_p Int(\mathcal{A})$ and $\omega_p Cl(\mathcal{A})$, respectively.

The family all preopen (resp. semi$-$open, $\alpha-$open, $\beta-$open, regular open, $\gamma-$open, $\omega-$open, $\omega\beta-$open, pre$-\omega-$open, ω_p-open) subsets of space $(X, \Im)$ represented by $PO(X)$ (resp. $SO(X)$, $\alpha O(X)$, $\beta O(X)$, $RO(X)$, $\gamma O(X)$, $\tau^\omega O(X)$, $\omega\beta O(X)$, $P\omega O(X)$, $\omega_p O(X)$). The complement of a preopen (resp. semi-open, $\alpha-$open, $\beta-$open, regular open, $\gamma-$open, $\omega-$open, $\omega\beta-$open, pre$-\omega-$open, ω_p-open) subset of space $(X, \Im)$ is called preclosed (resp. semi$-$closed, $\alpha-$closed, $\beta-$closed, regular closed, $\gamma-$closed, $\omega-$closed, $\omega\beta-$closed, pre$-\omega-$closed, ω_p-closed), and their family represented by $PC(X)$ (resp. $SC(X)$, $\alpha C(X)$, $\beta C(X)$, $RC(X)$, $\gamma C(X)$, $\tau^\omega C(X)$, $\omega\beta C(X)$, $P\omega C(X)$, $\omega_p C(X)$).

Definition 2.1. Let $V \subseteq X$. Then, a set V is called preopen [12] (resp. semi$-$open [13], $\alpha-$open [14], $\beta-$open [15], regular open [16], $\gamma-$open[17], pre$-\omega-$open [20], ω_p-open [1]) if $V \subseteq Int(Cl(V))$ (resp. $V \subseteq Cl(Int(V))$, $V \subseteq Int(Cl(Int(V)))$, $V \subseteq Cl(Int(Cl(V)))$, $V = Int(Cl(V))$, $V \subseteq Cl(Int(V)) \cup Int(Cl(V))$, $V \subseteq \omega Int(Cl(V))$, $V \subseteq Int(\omega Cl(V))$).

Definition 2.2. Let $V \subseteq X$. Then, V is said to be $\omega-$open [18] (resp. $\omega\beta-$open [19]) if for each x in V, there exits an open (resp. a $\beta-$open) set W in X containing x such that $W - V$ is countable.

Definition 2.3. Let $f : X \longrightarrow Y$ be a function. If for each open set G in Y, $f^{-1}(G)$ is closed (resp. preclosed, $\alpha-$closed, $\beta-$closed, *semi*$-$closed, $\omega-$closed, $\omega\beta-$closed, $\gamma-$closed, clopen) in X, then f is said to be contra$-$continuous [2] (resp. contra$-$precontinuous [5], contra$-\alpha-$continuous [6], contra$-\beta-$continuous [4], contra$-$semi continuous [3], contra $\omega-$continuous [10], contra $\omega\beta-$continuous [11], contra $\gamma-$continuous [9], perfectly continuous [22])

Definition 2.4. Let $f : X \longrightarrow Y$ be a function. If for each open subset G of Y, $f^{-1}(G)$ is open (resp. $\omega-$open, ω_p-open) in X, then f is called continuous [32] (resp. $\omega-$continuous [21], ω_p-continuous [1]).

Definition 2.5. Let $f : X \longrightarrow Y$ be a function. If $f^{-1}(G)$ is pre$-$closed (resp. $\omega-$open, $\omega-$closed, regular open, open) in X, for every regular open set G in Y, then f is called almost contra$-$precontinuous [7] (resp. almost $\omega-$continuous [25], almost contra $\omega-$continuous [10], $R-$map [24], almost continuous [23]).

Definition 2.6. Let $f : (X, \Im) \longrightarrow (Y, \rho)$ be a function. If $f^{-1}(G)$ is clopen in X, for every regular open (or regular closed) subset G of Y, then f is called almost perfectly continuous ($\equiv$ regular set$-$connected [26]).

Definition 2.7. Let $(X, \Im)$ be a topological space. Then, X is called:

(1) strongly irresolvable [28] if no non$-$empty open set in X is resolvable, ($\equiv$ if every open set is $\alpha-$open).

(2) locally countable [30] if each point $x \in X$ has a countable open neighborhood.
(3) door space [27] if every set in X is either open or closed.
(4) anti$-$locally countable [30] if every non-empty open set in X is uncountable.
(5) submaximal [29] if every preopen set in X is open, ($\equiv$ if every dense subset of X is open in X).

Theorem 2.8. [31] *Let (X, τ) be a locally countable space. Then, $\tau^{\omega}O(X) = \tau_{dis}$.*

Lemma 2.9. [30] *Let (X, τ) be a locally countable space and $G \subseteq X$. Then,*
(1) $\omega Cl(G) = Cl(G)$, for every $G \in \tau^{\omega}O(X)$.
(2) $\omega Int(G) = Int(G)$, for every $G \in \tau^{\omega}C(X)$.

Lemma 2.10. [1] *Let $(X, \Im)$ be a topological space and $G \subseteq X$. Then,*
(1) If $G \in \Im$, then $G \in \omega_p O(X)$.
(2) If $G \in \omega_p O(X)$, then G is preopen, pre$-\omega-$open.

Proposition 2.11. [1] Let $(X, \Im)$ be a topological space and $\mathcal{D} \subseteq X$. Then, $\mathcal{D} \in \omega_p C(X)$ if and only if $Cl(\omega Int(\mathcal{D})) \subseteq \mathcal{D}$.

Proposition 2.12. [1] The family union of ω_p-open sets is ω_p-open.

Corollary 2.13. [1] Consider a topological space $(X, \Im)$. If $\mathcal{D} \in \Im$ and $\mathcal{E} \in \omega_p O(X)$, then $\mathcal{D} \cap \mathcal{E} \in \omega_p O(X)$.

Theorem 2.14. [1] *If $\mathcal{A} \in \omega_p O(X)$ and $\mathcal{V} \in \omega_p O(\mathcal{A})$, then $\mathcal{V} \in \omega_p O(X)$.*

Theorem 2.15. [1] *Let $\mathcal{G} \in \omega_p O(X)$ and $\mathcal{G} \subseteq \mathcal{V} \subseteq X$. Then, $\mathcal{G} \in \omega_p O(\mathcal{V})$.*

Proposition 2.16. [20] Let $(X, \Im)$ be a door space and $\mathcal{U} \subseteq X$. If $\mathcal{U} \in P\omega O(X)$, then $\mathcal{U} \in \Im^{\omega}O(X)$.

Theorem 2.17. [1] *Let $(X, \Im)$ be a topological space and $\mathcal{D} \subseteq X$. Then,*
(1) $\mathcal{D}$ is clopen;
(2) $\mathcal{D}$ is $\omega-$open and ω_p-open;
(3) $\mathcal{D}$ is closed and ω_p-open.

3. More properties of ω_p-open sets

Theorem 3.1. *Let $(X, \Im)$ be a locally countable space and $\mathcal{D} \subseteq X$.Then, $\mathcal{D} \in \omega_p O(X)$ if and only if $\mathcal{D} \in \Im$.*

Proof. If $\mathcal{D} \in \omega_p O(X)$ and $(X, \Im)$ is locally countable, so $\omega Cl(\mathcal{D}) = \mathcal{D}$. By Theorem 2.8, $\mathcal{D} \subseteq Int(\omega Cl(\mathcal{D})) = Int(\mathcal{D})$. Thus $\mathcal{D} \in \Im$. Conversely, let $\mathcal{D} \in \Im$. So, $\mathcal{D} = Int(\mathcal{D})$, and $\mathcal{D} \subseteq Int(\omega Cl(\mathcal{D}))$. Hence, $\mathcal{D} \in \omega_p O(X)$. $\square$

Corollary 3.2. Let $(X, \Im)$ be a door space and $\mathcal{D} \subseteq X$. If $\mathcal{D} \in \omega_p O(X)$, then $\mathcal{D} \in \Im^{\omega}O(X)$.

Proof. Let $\mathcal{D} \in \omega_p O(X)$. Then, by part (2) of Lemma 2.10, $\mathcal{D}$ is pre$-\omega-$open, and by Proposition 2.16, $\mathcal{D}$ is ω-open. $\square$

Proposition 3.3. Let $(X, \Im)$ be an anti-locally countable space and $V \in \Im^{\omega}O(X)$. Then, $V \in \omega_p O(X)$.

Proof. Let $V \in \Im^{\omega}O(X)$. Then, $V = \omega Int(V)$. By Lemma 2.9, $V = \omega Int(V) \subseteq \omega Int(\omega Cl(V)) = Int(\omega Cl(V))$. Hence, $V \subseteq Int(\omega Cl(V))$, and $V \in \omega_p O(X)$. □

Corollary 3.4. Let $(X, \Im)$ be a strongly irresolvable space. Then, $\omega_p O(X) \subseteq \Im^{\alpha} O(X)$.

Proof. It follows from definition of strongly irresolvable and part (2) of Lemma 2.10. □

Proposition 3.5. Let $(X, \Im)$ be a submaximal space and $V \subseteq X$. Then, $V \in \omega_p O(X)$ if and only if $V \in PO(X)$.

Proof. If $V \in \omega_p O(X)$, by Lemma 2.10, $V \in PO(X)$. Conversely, let $V \in PO(X)$. Since X is submaximal space, thus, $V \in \Im$. And, by part (1) of Lemma 2.10, $V \in \omega_p O(X)$. □

4. Contra ω_p-Continuous Functions

Definition 4.1. A function f from a topological space (X, τ) to a topological space $(Y, \Im)$ is called contra ω_p-continuous (briefly, $c.\omega_p.c$), if $f^{-1}(\mathcal{D}) \in \omega_p C(X)$, for each $\mathcal{D} \in \Im$.

Theorem 4.2. *The following statements for function $f : (X, \tau) \to (Y, \Im)$ are equivalent:*

(1) f is $c.\omega_p.c$;
(2) $f^{-1}(\mathcal{D}) \in \omega_p O(X)$, for every closed set $\mathcal{D}$ in Y;
(3) for each $x \in X$ and each closed subset $\mathcal{W}$ of Y containing $f(x)$ there exists $\mathcal{D} \in \omega_p O(X)$ contains x such that $f(\mathcal{D}) \subseteq \mathcal{W}$.

Proof. (1) ⇒ (2) If $\mathcal{D}$ is a closed set in Y, implies that $Y - \mathcal{D}$ is open in Y. Since f is $c.\omega_p c.$, so $f^{-1}(Y - \mathcal{D})$ is $\omega-$closed in X. But $X - f^{-1}(\mathcal{D}) = f^{-1}(Y - \mathcal{D})$, hence $f^{-1}(\mathcal{D}) \in \omega_p O(X)$.

(2) ⇒ (3) Let $x \in X$ and $\mathcal{D}$ be a closed set in Y which contains $f(x)$. By given, $f^{-1}(\mathcal{D}) \in \omega_p O(X)$, put $\mathcal{W} = f^{-1}(\mathcal{D}) \in \omega_p O(X)$ and $x \in \mathcal{W}$. Therefore, $f(\mathcal{W}) = f(f^{-1}(\mathcal{D})) \subseteq \mathcal{D}$.

(3) ⇒ (1) Suppose $\mathcal{D} \in \Im$. Then, $Y - \mathcal{D}$ is closed in Y. If $f^{-1}(Y - \mathcal{D}) = \emptyset$, then $f^{-1}(\mathcal{D}) = X$ which is ω_p-closed in X. Otherwise, for $x \in X$ such that $x \notin f^{-1}(\mathcal{D})$, then $x \in f^{-1}(\mathcal{D} - Y)$. By (c), there exists an ω_p-open set $\mathcal{V}$ in X such that $x \in \mathcal{V}$ and $f(\mathcal{V}) \subseteq f^{-1}(Y - \mathcal{D})$ then $\mathcal{V} \subseteq X - f^{-1}(\mathcal{D})$. Therefore, $\mathcal{V} \cap f^{-1}(\mathcal{D}) = \emptyset$ so $x \notin \omega_p Cl(f^{-1}(\mathcal{D}))$. Thus $\omega_p Cl(f^{-1}(\mathcal{D})) = f^{-1}(\mathcal{D})$, this is by part (1) of [1, Proposition 3.24], $f^{-1}(\mathcal{D}) \in \omega_p C(X)$. Hence, f is $c.\omega_p.c$. □

Theorem 4.3. *A function f from a locally countable space $(X, \Im)$ to any space (Y, τ) is $c.\omega_p.c$ if and only if its contra$-$continuous.*

Proof. Suppose f is $c.\omega_p.c$ and $\mathcal{D} \in \tau$. Then, $f^{-1}(\mathcal{D}) \in \omega_p C(X)$. Since $(X, \Im)$ is locally countable, so by Theorem 3.1, $f^{-1}(\mathcal{D})$ is closed in X. Hence, f is contra-continuous. Conversely, it follows from the fact that every closed set is ω_p-closed. □

Theorem 4.4. *A function f from a submaximal space $(X, \Im)$ to any space (Y, ρ) is contra pre$-$continuous if and only if its $c.\omega_p.c$.*

Proof. Let f be a contra pre−continuous and $\mathcal{D}$ be a closed set in Y. Then, $f^{-1}(\mathcal{D}) \in PO(X)$. Since $(X, \Im)$ is submaximal, so by Proposition 3.5, $f^{-1}(\mathcal{D}) \in \omega_pO(X)$. Hence, f is $c.\omega_p.c$. Conversely, it follows from part (2) of Lemma 2.10, every ω_p-open set is preopen. □

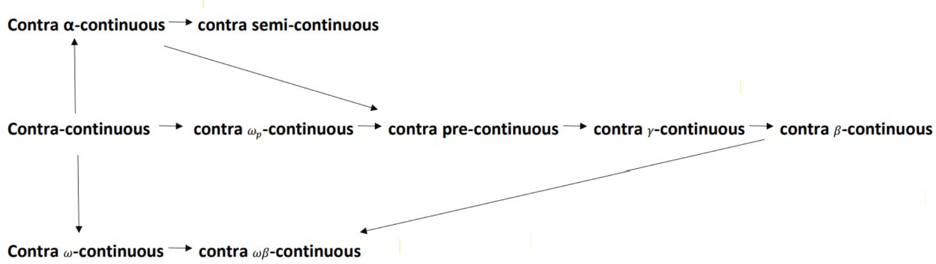

Figure 1. The relationships among contra ω_p-continuous and some other form of contra-continuous

The examples below explain the converse of these implications in Figure 1 is not true in general.

Example 4.5. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\Im = \{\emptyset, X, \{c\}, \{b, c\}\}$. The identity function $f : (X, \tau) \longrightarrow (X, \Im)$ is contra−precontinuous but not $c.\omega_p.c$. By Theorem 3.1, $\omega_pC(X, \tau) = \{\emptyset, X, \{b, c\}\}$ and $PC(X, \tau) = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$. Since $\{c\} \in \Im$, but $f^{-1}(\{c\}) = \{c\} \notin \omega_pC(X)$. Hence, f is not $c.\omega_p.c$.

Example 4.6. The function $f : (\mathbb{R}, \Im_u) \longrightarrow (Y, \tau)$ is defined as:

$$f(x) = \begin{cases} 1 & if \quad x \in \mathbb{Q} \\ 3 & if \quad x \in \mathbb{Q}^c \end{cases}$$

is $c.\omega_p.c.$, but not contra-continuous, where $Y = \{1, 2, 3\}$ and $\tau = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Since $\{3\}$ is closed subset of Y, $f^{-1}(\{3\}) = \mathbb{Q}^c$. Because $(\mathbb{R}, \Im_u)$ is anti locally countable and $\mathbb{Q}^c$ is $\omega-$open, so by Lemma 2.9, $\mathbb{Q}^c \subseteq Int(\omega Cl(\mathbb{Q}^c)) = Int(\mathbb{R}) = \mathbb{R}$, that is, $\mathbb{Q}^c$ is ω_p-open but not open in $\Im_u$.

The examples below show that $c.\omega_p.c$ is independent from contra−α−contionuous and contra−ω−continuous.

Example 4.7. Let $X = \{a, b, c\}$, $\Im = \{\emptyset, X, \{a\}\}$ and $\rho = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$. Then, the identity function $f : (X, \Im) \longrightarrow (X, \rho)$ is contra−α−continuous but not $c.\omega_p.c$. Since, $\omega_pO(X, \Im) = \Im$ and $\alpha O(X, \Im)) = \{\emptyset, X, \{a\}, \{a, c\}, \{a, b\}\}$, such that $\{a, b\}$ and $\{a, c\}$ are closed subsets of (X, ρ). So $f^{-1}(\{a, b\}) = \{a, b\}$ and $f^{-1}(\{a, c\}) = \{a, c\}$, thus $\{a, b\}, \{a, c\} \in \alpha O(X)$ but not ω_p-open.

Example 4.8. Let $Y = \{1, 2, 3\}$ and $\tau = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Then the function $f : (\mathbb{R}, \Im_u) \longrightarrow (Y, \tau)$ is defined in Example 4.6, is $c.\omega_p.c$ but not contra$-\alpha-$continuous. Since $\{3\}$ is closed subset of Y and $f^{-1}(\{3\}) = \mathbb{Q}^c$. So, $\mathbb{Q}^c \nsubseteq Int(Cl(Int(\mathbb{Q}^c))) = \emptyset$. That is $\mathbb{Q}^c$ is not $\alpha-$open in $(\mathbb{R}, \Im_u)$.

Example 4.9. Consider the function $f : (\mathbb{R}, \Im_{ind}) \longrightarrow (X, \tau)$, which is defined as;

$$f(x) = \begin{cases} a & if\ x \in \mathbb{R}\backslash(0, 1) \\ b & if\ x \in (0, 1) \end{cases}$$

Where $X = \{a, b\}$ and $\tau = \{\emptyset, X, \{a\}\}$. Since $f^{-1}(\{b\}) = (0, 1)$ is ω_p-open in $\mathbb{R}$ but not $\omega-$open. Thus f is *c.* $\omega_p.c$, but not contra$-\omega-$continuous.

Example 4.10. Let $X = \{a, b\}$, $\rho = \{\emptyset, X, \{a\}\}$, (X, τ) be an indiscrete topological space and (X, ρ) be a topological space. Then identity function $f : (X, \tau) \longrightarrow (X, \rho)$ is contra$-\omega-$continuous, but it is not *c.* $\omega_p.c$. By Theorem 3.1, $\omega_pO(X) = \tau$, and by Theorem 2.8, $\tau^\omega = \tau_{dis}$. so, $\{b\}$ is ρ-closed and $f^{-1}(\{b\}) = \{b\}$ is $\omega-$open in (X, τ) but not ω_p-open.

Corollary 4.11. For the function$f : X \rightarrow Y$, the followings are true:

(1) If f is *c.* $\omega_p.c.$ and X is door space, then f is contra-$\omega-$continuous.
(2) If f is contra$-\omega-$continuous and X is anti-locally countable, then f *c.* $\omega_p.c.$
(3) If f is *c.* $\omega_p.c.$ and X is strongly irresolvable, then f contra$-\alpha-$continuous.

Proof. It follows from Corollary 3.2, Proposition 3.3 and Corollary 3.4. □

Theorem 4.12. *If a function* $f : (X, \tau) \longrightarrow (Y, \Im)$ *is c.* $\omega_p.c.$ *and* $\mathcal{V} \in \tau$, *then* $f_\mathcal{V} : \mathcal{V} \longrightarrow Y$ *is c.* $\omega_p.c.$

Proof. Let $\mathcal{A}$ be any closed subset of Y. Since f is *c.* $\omega_p.c.$, $f^{-1}(\mathcal{A}) \in \omega_pO(X)$. So, by Corollary 2.13, $f_\mathcal{V}^{-1}(\mathcal{A}) = f^{-1}(\mathcal{A}) \cap \mathcal{V} \in \omega_pO(X)$. Thus, by Theorem 2.15, $f_\mathcal{V}^{-1}(\mathcal{A}) \in \omega_pO(\mathcal{V})$. Hence, $f_\mathcal{V}$ is *c.* $\omega_p.c.$. □

Theorem 4.13. *Let* $f : X \rightarrow Y$ *be a function and* $\{G_\kappa : \kappa \in K\}$ *be any* ω_p-*open cover for* X. *If* $f_{G_\kappa} : G_\kappa \longrightarrow Y$ *is c.* $\omega_p.c.$ *for each* $\kappa \in K$, *then* f *is c.* $\omega_p.c.$

Proof. Let V be closed subset of Y. Since f_{G_κ} is *c.* $\omega_p.c.$ for each $\kappa \in K$, then $f_{G_\kappa}^{-1}(V)$ is ω_p-open subset of G_κ, for each $\kappa \in K$. So, by using Theorem 2.14, $f_{G_\kappa}^{-1}(V) \in \omega_pO(X)$. By Proposition 2.12, $f^{-1}(V) = \bigcup_{\kappa \in K} f_{G_\kappa}^{-1}(V)$. Hence, $f^{-1}(V) \in \omega_pO(X)$ subset of X and f is *c.* $\omega_p.c.$ □

Corollary 4.14. Let $\zeta : X \longrightarrow Y$ be a function and let $G,\ V \in \omega_pO(X)$ such that $X = G \cup V$. If $\zeta_G : G \longrightarrow Y$ and $\zeta_V : V \longrightarrow Y$ are $c.\omega_p.c.$, then ζ is $c.\omega_p.c.$

Proof. It follows from Theorem 4.13. □

Proposition 4.15. Let $f : (X, \tau) \longrightarrow (Y, \rho)$ be a *c.* $\omega_p.c.$ function. Then, $f^{-1}(Int(\mathcal{U})) \subseteq \omega_pCl(f^{-1}(\mathcal{U}))$ for all $\mathcal{U} \subseteq Y$.

Proof. Let $\mathcal{U} \subseteq Y$ and $Int(\mathcal{U}) \in \rho$. By hypothesis, $f^{-1}(Int(\mathcal{U})) \in \omega_pC(X)$. Thus,

$$\omega_pCl(f^{-1}(Int(\mathcal{U}))) = f^{-1}(Int(\mathcal{U})) \quad (4.1)$$

since $f^{-1}(Int(\mathcal{U})) \subseteq f^{-1}(\mathcal{U})$, then $\omega_p Cl(f^{-1}(Int(\mathcal{U}))) \subseteq \omega_p Cl(f^{-1}(\mathcal{U}))$, therefore, by (4.1), $f^{-1}(Int(\mathcal{U})) \subseteq \omega_p Cl(f^{-1}(\mathcal{U}))$. □

The converse of Proposition 4.15 is not true generally. In Example 4.7, the function f holds $f^{-1}(Int(\mathcal{U})) \subseteq \omega_p Cl(f^{-1}(\mathcal{U}))$, for each $\mathcal{U} \subseteq Y$, but f is not *c.* ω_p*.c*.

Proposition 4.16. Let $\zeta : X \longrightarrow Y$ be a *c.*ω_p*.c* function. Then, $\zeta(\omega_p Int(\mathcal{D})) \subseteq Cl(\zeta(\mathcal{D}))$, for each $\mathcal{D} \subseteq X$.

Proof. Let $\mathcal{D} \subseteq X$. Then $Cl(\zeta(\mathcal{D}))$ is a closed subset of Y, by given, $\zeta^{-1}(Cl(\zeta(\mathcal{D}))) \in \omega_p O(X)$. Thus,

$$\omega_p Int\zeta^{-1}(Cl(\zeta(\mathcal{D}))) = \zeta^{-1}(Cl(\zeta(\mathcal{D}))) \tag{4.2}$$

since $\mathcal{D} \subseteq \zeta^{-1}Cl(\zeta(\mathcal{D}))$, then $\omega_p Int(\mathcal{D}) \subseteq \omega_p Int(\zeta^{-1}Cl(\zeta(\mathcal{D})))$. Hence, by (4.2), $\zeta(\omega_p Int(\mathcal{D})) \subseteq Cl(\zeta(\mathcal{D}))$. □

In general, the converse of Proposition 4.16 is not true. The function f in Example 4.10, holds $f(\omega_p Int(\mathcal{D})) \subseteq Cl(f(\mathcal{D}))$, for all $\mathcal{D} \subseteq X$, but is not *c.* ω_p*.c*.

Theorem 4.17. *Let $\zeta : (X, \tau) \longrightarrow (Y, \rho)$ be a c. ω_p.c function. If ζ holds one of the following, then its ω_p-continuous.*

(1) $\zeta^{-1}(\mathcal{D}) \subseteq \omega_p Cl(\zeta^{-1}(Int(\mathcal{D}))$, for each closed set $\mathcal{D}$ in (Y, ρ).
(2) $\omega_p Int(\zeta^{-1}(Cl(\mathcal{D}))) \subseteq \zeta^{-1}(\mathcal{D})$, for each $\mathcal{D} \in \rho$.

Proof. (1) If $\mathcal{D}$ is a closed set in Y, then $Int(\mathcal{D}) \in \rho$. Since ζ is *c.* ω_p*.c*, $\zeta^{-1}(Int(\mathcal{D})) \in \omega_p C(X)$. Thus,

$$\omega_p Cl(\zeta^{-1}(Int(\mathcal{D}))) = \zeta^{-1}(Int(\mathcal{D})) \tag{4.3}$$

by given and (4.3), $\zeta^{-1}(\mathcal{D}) \subseteq \omega_p Cl(\zeta^{-1}Int(\mathcal{D})) = \zeta^{-1}(Int(\mathcal{D}))) \subseteq \zeta^{-1}(\mathcal{D})$, that is, $\zeta^{-1}(\mathcal{D}) = \omega_p(\zeta^{-1}(Int(\mathcal{D})))$ and $\zeta^{-1}(\mathcal{D}) \in \omega_p C(X)$. Hence, ζ is ω_p-continuous.

(2) Obvious.

□

Theorem 4.18. *Let $\zeta : (X, \tau) \longrightarrow (Y, \Im)$ be a c. ω_p.c function and Y be a regular space. Then, ζ is ω_p-continuous.*

Proof. Let x be any point of X and $\mathcal{M} \in \Im$ which contains $\zeta(x)$. By given, then there exists $\mathcal{N} \in \Im$ containing $\zeta(x)$ such that $Cl(\mathcal{N}) \subseteq \mathcal{M}$. Since ζ is *c.*ω_p*.c.*, and $Cl(\mathcal{N})$ is closed in Y containing $\zeta(x)$, thus, by (c) of Theorem 4.2, there exists an ω_p-open subset $\mathcal{U}$ in X contains x such that $\zeta(\mathcal{U}) \subseteq Cl(\mathcal{N}) \subset \mathcal{M}$, that is $\zeta(\mathcal{U}) \subseteq \mathcal{M}$. Hence, ζ is ω_p-continuous. □

Theorem 4.19. *The following is true, for the functions $\zeta : X \to Y$ and $\eta : Y \to Z$:*

(1) If η is continuous and ζ is c. ω_p.c, then $\eta \circ \zeta$ is c. ω_p.c.
(2) If η and ζ are c. ω_p.c. and Y is locally-countable space, then $\eta \circ \zeta$ is c. ω_p.c.

Proof.

(1) Let $\mathcal{D}$ be any closed subset of Z. By hypothesis, $\eta^{-1}(\mathcal{D})$ is closed in Y. For the reason that, ζ is *c.* ω_p*.c.*, $\zeta^{-1}(\eta^{-1}(\mathcal{D})) \in \omega_p O(X)$. But, $\zeta^{-1}(\eta^{-1}(\mathcal{D})) = (\eta \circ \zeta)^{-1}(\mathcal{D})$. Hence $\eta \circ \zeta$ is *c.* ω_p*.c*.

(2) Let $\mathcal{D}$ be closed in Z. Since η is $c.\omega_p.c.$, so $\eta^{-1}(\mathcal{D}) \in \omega_p O(Y)$. However, Y is locally-countable, and by Theorem 3.1, $\eta^{-1}(\mathcal{D})$ is open in Y. Thus, since ζ is $c.\omega_p.c.$, so $(\eta \circ \zeta)^{-1}(\mathcal{D}) = \zeta^{-1}(\eta^{-1})(\mathcal{D}) \in \omega_p C(X)$. That is, $\eta \circ \zeta$ is $c.\omega_p.c.$

□

Theorem 4.20. *Let $\eta : X \longrightarrow Y$ be a function. Then, the followings are equivalent:*

(1) η is perfectly continuous.
(2) η is $\omega-$continuous and contra ω_p-continuous.
(3) η is continuous and $c.\omega_p.c.$

Proof. It follows from Theorem 2.17. □

Corollary 4.21. If $\eta : (X, \Im) \longrightarrow (Y, \rho)$ is a $c.\omega_p.c.$ function and $(X, \Im)$ is a locally countable space, then

(1) η is contra semi$-$continuous.
(2) η is contra $\alpha-$continuous.
(3) η is contra b-continuous.
(4) η is contra $\omega-$continuous.

Proof. It follows from the fact that every ω_p-open set is open in locally countable space Theorem 3.1. □

Theorem 4.22. *Let $\eta : (X, \Im) \longrightarrow (Y, \rho)$ be a function and for each $\mathcal{D} \subseteq Y$. If $\omega_p Cl(\eta^{-1}(\mathcal{D})) \subseteq \eta^{-1}(Int(\mathcal{D}))$, then η is $c.\omega_p.c.$*

Proof. Let $\mathcal{D} \in \rho$ and $\omega_p Cl(\eta^{-1}(\mathcal{D})) \subseteq \eta^{-1}(Int(\mathcal{D}))$. Then, $\omega_p Cl(\eta^{-1}(\mathcal{D})) \subseteq \eta^{-1}(Int(\mathcal{D})) = \eta^{-1}(\mathcal{D})$. Thus, $\eta^{-1}(\mathcal{D}) = \omega_p Cl(\eta^{-1}(\mathcal{D}))$, therefore $\eta^{-1}(\mathcal{D})$ is ω_p-closed. Hence, η is $c.\omega_p.c.$ □

Generally, the converse of Theorem 4.22 need not to be true.

Example 4.23. A function $\eta : (X, \tau) \longrightarrow (Y, \rho)$, defined as:

$$\eta(x) = \begin{cases} 1 & if\ x = 2 \\ 2 & if\ x = 1 \end{cases}$$

Where, $X = \{1, 2\}$, $\tau = \{\emptyset, \{1\}, X\}$ and $Y = \{1, 2, 3\}$, $\rho = \{\emptyset, Y, \{1\}, \{1, 2\}\}$. Then, η is $c.\omega_p.c.$, since $\{\eta^{-1}(\emptyset) = \emptyset, \eta^{-1}(\{1\}) = \{2\}, \eta^{-1}(\{1, 2\}) = X,\ \eta^{-1}(Y) = X\} \subseteq \omega_p C(X)$. Although, if $\mathcal{D} = \{2\} \subseteq Y$, $\eta^{-1}(\mathcal{D}) = \{1\}$ and $\eta^{-1}(Int(\mathcal{D})) = \emptyset$. Thus, $\omega_p Cl(\eta^{-1}(\mathcal{D})) = Cl(\eta^{-1}(\mathcal{D})) = Cl(\{1\}) = X \nsubseteq \eta^{-1}(Int(\mathcal{D})) = \emptyset$.

5. Almost contra ω_p-continuous

Definition 5.1. Let $\zeta : X \longrightarrow Y$ be a function. If $\zeta^{-1}(\mathcal{D}) \in \omega_p C(X)$, for all $\mathcal{D} \in RO(Y)$, then ζ is called almost contra ω_p-continuous (briefly, $a.c.\omega_p.c.$).

Theorem 5.2. *Let $\zeta : (X, \tau) \longrightarrow (Y, \Im)$ be a function, then followings are true:*

(1) If ζ is a $c.\omega_p.c.$ function, then its $a.c.\omega_p.c.$
(2) If ζ is a regular set-connected function, then its $a.c.\omega_p.c.$
(3) If ζ is an $a.c.\omega_p.c.$ function, then its almost contra pre$-$continuous.

Proof.

(1) Let $\mathcal{D} \in RO(Y)$. Then, $\mathcal{D} \in \Im$ and by hypothesis, $\zeta^{-1}(\mathcal{D}) \in \omega_p C(X)$. Thus, ζ is $a.c.\omega_p.c.$
(2) It follows the definitions of regular set$-$connected and almost contra ω_p-continuous function.
(3) It follows from part (1) of Lemma 2.10.

□

Remark 5.3. The identity function of Example 4.10 is $a.c.\omega_p.c.$ which is not $c.\omega_p.c$ and the function of Example 4.6 is $a.c.\omega_p.c.$, but not regular set$-$connected. This means that, the converse of parts (1) and (2) of Theorem 5.2 are not true. However, the following example shows that the converse of part (3) of Theorem 5.2, is not true in general.

Example 5.4. The usual topology $\Im_u$ on $\mathbb{R}$ and $X = \{a, b, c, d\}$ with the topology $\rho = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the function $\zeta : (\mathbb{R}, \Im_u) \longrightarrow (X, \rho)$ defined as

$$\zeta(x) = \begin{cases} c, & if \quad x \in \mathbb{Q} \\ a, & if \quad x \in \mathbb{Q}^c \end{cases}$$

is almost contra pre$-$continuous but not $a.c.\omega_p.c.$ Since $RO(X, \rho) = \{\emptyset, X, \{a\}, \{b\}\}$ and $\zeta^{-1}(\{a\}) = \mathbb{Q}^c$. But $Cl(\omega Int(\mathbb{Q}^c)) = \mathbb{R} \nsubseteq \mathbb{Q}^c$, thus $\mathbb{Q}^c$ is not ω_p-closed in $\Im_u$.

Theorem 5.5. *Let $\zeta : X \to Y$ be a function. Then, followings are equivalent:*

(1) ζ is $a.c.\omega_p.c.$;
(2) $\zeta^{-1}(\mathcal{D}) \in \omega_p O(X)$, for every $\mathcal{D} \in RC(Y)$;
(3) for each $a \in X$ and each regular$-$closed $\mathcal{D}$ in Y containing $\zeta(a)$, there exists an $\omega_p -$ openset $\mathcal{U}$ in X containing a, such that $\zeta(\mathcal{U}) \subseteq \mathcal{D}$.
(4) for each $x \in X$ and each regular open set $\mathcal{D}$ in Y non-containing $\zeta(x)$, there exists an ω_p-open set $\mathcal{U}$ in X non-containing x such that $\zeta^{-1}(\mathcal{D}) \subseteq \mathcal{U}$.

Proof. (1) $\Rightarrow$ (2) Let $\mathcal{D} \in RC(Y)$. Then, $Y - \mathcal{D} \in RO(Y)$ and by (1), $\zeta(Y - \mathcal{D}) \in \omega_p C(Y)$, since, $X - \zeta^{-1}(\mathcal{D}) = \zeta^{-1}(Y - \mathcal{D})$. Therefore, $\zeta^{-1}(\mathcal{D}) \in \omega_p O(X)$.

(2) $\Rightarrow$ (3) Suppose $a \in X$ and $\mathcal{D} \in RC(Y)$ containing $\zeta(a)$. By (2), $\zeta^{-1}(\mathcal{D}) \in \omega_p O(X)$ $a \in \zeta^{-1}(\mathcal{D})$. Let $\mathcal{U} = \zeta^{-1}(\mathcal{D})$. Then, $\zeta(\mathcal{U}) \subseteq \zeta(\zeta^{-1}(\mathcal{D})) \subseteq \mathcal{D}$. Thus $f(\mathcal{U}) \subseteq \mathcal{D}$.

(3) $\Rightarrow$ (4) Suppose $a \in X$ and $\mathcal{D} \in RO(Y)$ not containing $\zeta(a)$, so $Y - \mathcal{D}$ is regular$-$closed set in Y and $\zeta(a) \in Y - B$, by (3), there exists an ω_p-open set $\mathcal{U}$ in X contains a such that $\zeta(\mathcal{U}) \subseteq Y - \mathcal{D}$. Thus $\mathcal{U} \subseteq \zeta^{-1}(\zeta(\mathcal{U})) \subseteq \zeta^{-1}(Y - \mathcal{D}) = X - \zeta^{-1}(\mathcal{D})$, that is, $\zeta^{-1}(\mathcal{D}) \subseteq X - \mathcal{U}$, therefore, $X - \mathcal{U} \in \omega_p C(X)$ and $a \notin X - \mathcal{U}$.

(4) $\Rightarrow$ (1) Let $\mathcal{D} \in RO(Y)$. If $\mathcal{D} = \emptyset$, then $\zeta^{-1}(\mathcal{D}) = X \in \omega_p C(X)$. Otherwise, $\zeta^{-1}(\mathcal{D}) \in \omega_p C(X)$. For this, let $x \notin \zeta^{-1}(\mathcal{D})$ then $\zeta(x) \in \mathcal{D}$. By (4), there exits an ω_p-closed subset $\mathcal{U}$ of X such that $\zeta(\mathcal{D}) \subseteq \mathcal{U}$. That is, $X - \mathcal{U} \in \omega_p O(X)$ which contains x and $\zeta^{-1}(\mathcal{D}) \cap X - \mathcal{U} = \emptyset$. Then, $x \notin \omega_p Cl(\zeta^{-1}(\mathcal{D}))$. Thus, $\omega_p Cl(\zeta^{-1}(\mathcal{D})) \subseteq \zeta^{-1}(\mathcal{D})$, therefore $\zeta^{-1}(\mathcal{D}) \in \omega_p C(X)$. Hence, ζ is $a.c.\omega_p.c.$ □

Theorem 5.6. *Let $\zeta : X \longrightarrow Y$ be a function. Then, the followings are equivalent:*

(1) ζ is $a.c.\omega_p.c$;
(2) $\zeta^{-1}(Int(Cl(\mathcal{D}))) \in \omega_p C(X)$, for each $\mathcal{D} \subseteq Y$;
(3) $\zeta^{-1}(Cl(lnt(\mathcal{D}))) \in \omega_p O(X)$, for each $\mathcal{D} \subseteq Y$.

Proof. (1) $\Rightarrow$ (2) Let $\mathcal{D} \subseteq Y$. Then, $Int(Cl(\mathcal{D})) \in RO(Y)$. By (1), $\zeta^{-1}(Int(Cl(\mathcal{D}))) \in \omega_p C(X)$.

(2) $\Rightarrow$ (3) Let $\mathcal{D} \subseteq Y$. Then, $Y - \mathcal{D}$ is also a subset of Y. By (2), $\zeta^{-1}(Int(Cl(Y - \mathcal{D}))) \in \omega_p C(X)$. But, since $Int(Cl(Y - \mathcal{D})) = Y - Cl(Int(\mathcal{D}))$ and $\zeta^{-1}(Y - \mathcal{D}) = X - \zeta^{-1}(\mathcal{D})$, we obtaining that $X - \zeta^{-1}(Cl(Int(\mathcal{D}))) \in \omega_p C(X)$. That is, $\zeta^{-1}(Cl(Int(\mathcal{D}))) \in \omega_p O(X)$.

(3) $\Rightarrow$ (1) Let $\mathcal{D} \subseteq RC(Y)$. Then, $Cl(Int(\mathcal{D})) = \mathcal{D}$. By hypothesis, $\zeta^{-1}(\mathcal{D}) = \zeta^{-1}(Cl(Int(\mathcal{D}))) \in \omega_p O(X)$. So, by part (2) of Theorem 5.5, ζ is $a.c.\omega_p.c.$ $\square$

Theorem 5.7. *Let $\zeta : X \to Y$ be an $a.c.\omega_p.c$ and $R-$map function. Then, ζ is regular set$-$connected.*

Proof. Let $\mathcal{D} \in RO(Y)$. Then, by hypothesis, $\zeta^{-1}(\mathcal{D})$ is both ω_p-closed and open subset of X. Therefore, $\zeta^{-1}(\mathcal{D})$ is clopen. Hence, ζ is *regular set $-$ connected.* $\square$

Remark 5.8. In part (1) of Theorem 4.19, if ζ is a $a.c.\omega_p.c.$ and η is a continuous function, then $\eta \circ \zeta$ is contra ω_p-continuous. By part (1) of Theorem 5.2, $\eta \circ \zeta$ is $a.c.\omega_p.c.$

Theorem 5.9. *Let $\zeta : X \to Y$ be a contra continuous function and $\eta : Y \to Z$ be a $R-$map function, then $\eta \circ \zeta$ is $a.c.\omega_p.c.$*

Proof. Let $\mathcal{D} \in RO(Z)$. Then, by given, $\eta^{-1}(\mathcal{D})$ is regular open in Y. Since, ζ is contra continuous, $(\eta \circ \zeta)^{-1}(\mathcal{D}) = \zeta^{-1}(\eta^{-1}(\mathcal{D}))$ is closed in X. Thus, $(\eta \circ \zeta)^{-1}(\mathcal{D}) \in \omega_p C(X)$ Therefore, $\eta \circ \zeta$ is $a.c.\omega_p.c.$

$\square$

Proposition 5.10. Let $\zeta : X \to Y$ be a surjective open function and $\eta : Y \to Z$ be any function. If $\eta \circ \zeta : X \to Z$ is a contra-continuous, then η is $a.c.\omega_p.c.$

Proof. Suppose, $\mathcal{D} \in RC(Z)$, then $\mathcal{D}$ is closed in Z. By hypothesis, $(\eta \circ \zeta)^{-1}(\mathcal{D})$, is open in X. Since, ζ is surjective open, this implies that $\zeta((\eta \circ \zeta)^{-1}(\mathcal{D})) = \eta^{-1}(\mathcal{D})$ and $\eta^{-1}(\mathcal{D})$ is an open subset of X, then $\eta^{-1}(\mathcal{D}) \in \omega_p O(X)$. Therefore, by part (2) of Theorem 5.5, η is $a.c.\omega_p.c.$ $\square$

Theorem 5.11. *Let $\zeta : X \to Y$ be an $a.c.\omega_p.c.$ function and $\mathcal{K}$ be an open set in X. Then, $\zeta_{\mathcal{K}} : \mathcal{K} \longrightarrow Y$ is $a.c.\omega_p.c.$*

Proof. Let $\mathcal{D} \in RC(Y)$. Then, by hypothesis, $\zeta^{-1}(\mathcal{D}) \in \omega_p O(X)$. By Corollary 2.13, $\zeta_{\mathcal{K}}^{-1}(\mathcal{D}) = \zeta^{-1}(\mathcal{D}) \cap \mathcal{K} \in \omega_p O(X)$. By Theorem 2.15, $\zeta^{-1}(\mathcal{D}) \in \omega_p O(\mathcal{K})$. By part (2) of Theorem 5.5, $\zeta_{\mathcal{K}}$ is $a.c.\omega_p.c.$ $\square$

Theorem 5.12. *Let $\zeta : X \longrightarrow Y$ be a function and $\{\mathcal{K}_\kappa : \kappa \in \mathcal{K}\}$ be any ω_p-cover for X. If $\zeta_{\mathcal{K}_\kappa} : \mathcal{K}_\kappa \longrightarrow Y$ is $a.c.\omega_p.c.$ for each $\kappa \in \mathcal{K}$, then, $\zeta : X \longrightarrow Y$ is $a.c.\omega_p.c.$*

Proof. Let $\mathcal{D} \in RC(Y)$ and since $\zeta_{\mathcal{K}_\kappa}$ is $a.c.\omega_p.c.$, so $\zeta_{/\mathcal{K}_\kappa}(\mathcal{D})$ is $\omega-$open set in $\mathcal{K}_\kappa$ for each $\kappa \in \mathcal{K}$. By Theorem 2.14, $\zeta_{/\mathcal{K}_\kappa}(\mathcal{D}) \in \omega_p O(X)$. That is, $\zeta^{-1}(\mathcal{D}) = \bigcup_{\kappa \in \mathcal{K}} \zeta^{-1}_{/\mathcal{K}_\kappa}(\mathcal{D}) \in \omega_p O(X)$. Therefore, $\zeta^{-1}(\mathcal{D}) \in \omega_p O(X)$, by part (2) of Theorem 5.2, ζ is $a.c.\omega_p.c.$ □

Theorem 5.13. *If ζ is a function from a submaximal space X into any space Y, then ζ is almost contra pre$-$continuous if and only if its $a.c.\omega_p.c.$*

Proof. Let ζ be a an almost contra pre$-$continuous function and $\mathcal{D} \in RC(Y)$. Then, $\zeta^{-1}(\mathcal{D}) \in PO(X)$. Since, X is submaximal, by Proposition 3.5, $\zeta^{-1}(\mathcal{D}) \in \omega_p O(X)$. Thus, ζ is $a.c.\omega_p.c.$ Conversely, it follows part (2) of Lemma 2.10, the fact that every ω_p-open is pre$-$open. □

Theorem 5.14. *Let (X, τ) be an anti locally-countable space, if a function $\zeta : (X, \tau) \longrightarrow (Y, \sigma)$ is $a.c.\omega.c.$, then ζ is $a.c.\omega_p.c.$*

Proof. Let $\mathcal{D} \in RC(Y)$. By hypothesis, $\zeta^{-1}(\mathcal{D}) \in \tau^\omega O(X)$. Since, X is anti locally-countable, then by Proposition 3.3, $\zeta^{-1}(\mathcal{D})$ is $\omega_p O(X)$. Hence, ζ is $a.c.\omega_p.c.$ □

Theorem 5.15. *Let ζ be a function from be a locally countable space X to any space Y. If ζ is $a.c.\omega_p.c.$, then ζ is almost contra pre$-$continuous and almost contra $\omega-$continuous.*

Proof. By Theorem 3.1, in locally-countable space every ω_p-open set is open, so, every open set is pre$-$open and $\omega-$open. □

Theorem 5.16. *Let $\zeta : X \longrightarrow Y$ be a function. Then, the followings are equivalent:*

(1) ζ is almost perfectly continuous;

(2) ζ is almost continuous and $a.c.\omega_p.c.$

Proof. It follows from Theorem 2.17. □

References

[1] H. M. Darwesh, Between preopen and open sets in topological spaces, Thai J. Math., 11 (2013), no. 1, 143–155.

[2] K. Dontchev, Contra-continuous functions and strongly S-closed spaces, Int. J. Math. Math. Sci., 19 (1996), 303–310.

[3] J. Dontchev and T. Noiri, Contra-semicontinuous functions, Math. Pannon., 10 (1999), 159–168.

[4] M. Caldas and S. Jafari, Some properties of contra β-continuous functions, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 22 (2001), 19–28.

[5] S. Jafari and T. Noiri, On contra-precontinuous functions, Bull. Malays. Math. Sci. Soc., 25 (2002), 115–128.

[6] S. Jafari and T. Noiri, Contra α-continuous functions between topological spaces, Iran. Int. J. Sci., 2 (2001), 153–167.

[7] E. Ekici, Almost contra-precontinuous functions, Bull. Malays. Math. Sci. Soc., 27 (2004), 53–65.

[8] T. Noiri and V. Popa, Some properties of almost contra-precontinuous functions, Bull. Malays. Math. Sci. Soc. (2), 28 (2005), no. 2, 107–116.

[9] A. A. Nasef, Some properties of contra-γ-continuous functions, Chaos Solitons Fractals., 24 (2005), no. 2, 471–477.

[10] A. Al-Omari and M. Noorani, Contra-ω-continuous and almost contra-ω-continuous, Int. J. Math. Math. Sci., (2007), Art. ID 40469, 13 pp.

[11] H. H. Aljarrah, M. Noorani and T. Noiri, Contra $\omega\beta$-continuity, Bol. Soc. Parana. Mat. (3), 32 (2014), no. 2, 9–22.

[12] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt., 53 (1982), 47–53.

[13] N. Levine, Semi open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36–41.

[14] O. Njastad, On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961–970.

[15] M.E.Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mapping, Bull. Fac. Sci. Assiut Univ. A, 12 (1983), no. 1, 77–90.

[16] M. Stone, Applications of the theory of boolean ring to general topology, Trans. Amer. Math. Soc., 41 (1937), 375–381.

[17] E. Ekici and M. Caldas, Slightly continuous γ- functions, Bol. Soc. Parana. Mat., 22 (2004), no. 2, 63–74.

[18] H. Z. Hdeib, ω-closed mapping, Revista Colombiana de Mathematics., 16 (1982), 65–78.

[19] H. H. Aljarrah, M. Noorani and T. Noiri, On $\omega\beta$-open sets, Malays. J. Math. Sci., 8 (2014), no. 2, 325–340.

[20] T. Noiri, A. Al-Omari, M. S. M. Noorani, Weak forms of ω-open sets and decompositions of continuity, Eur. J. Pure Appl. Math., 2 (2009), no. 1, 73–84.

[21] H. Z. Hdeib, ω-continuous functions, Dirasat J., 16 (1988), no. 2, 65–78.

[22] T. Noiri, Super-continuity and some strong forms of continuity, Indian J. Pure Appl. Math., 15 (1984), 241–250.

[23] M. K. Singal and A. R. Singal, Almost-continuous mappings, Yokohama Math. J., 16 (1968), 63–73.

[24] D. Carnahan, Some properties related to compactness in topological spaces, Thesis (Ph.D.)-University of Arkansas ProQuest LLC, Ann Arbor, MI, 1973, 67 pp.

[25] T. M. Nour, Almost ω-continuous functions, European J. Sci. Res., 8 (2005), no. 1, 43–47.

[26] J. Dontchev, M. Ganster and I. Reilly, More on almost s-continuity, Indian J. Math., 41 (1999), 139–146.

[27] J. Dontchev, On door spaces, Indian J. Pure Appl. Math., 26 (1995), 873–881.

[28] J. Foran and P. Liebnitz, A characterization of almost resolvable spaces, Rend. Circ. Mat. Palermo., 40 (1991), 136–141.

[29] I. L. Reilly and M. K. Vamanamurthy, On some questions concerning preopen sets, Kyungpook Math. J., 30 (1990), 87–93.

[30] K. Al-Zoubi and B. Al-Nashef, The topology of ω-open subsets, Al-Manarah J., 9 (2003), 169–179.

[31] H. M Darwesh, Some types of separation axioms and dimension functions in topological spaces, Thesis (Ph.D.)-Sulaimani University, Sulaimani, Kurdistan Region, 2009.

[32] J. N. Sharma, *Krishna's Topology*, Krishna Prakashan Media 1st ed., India, 1979.

(Halgwrd M. Darwesh) Department of Mathematics, College of Science, University of Sulaimani, Sulaymaniyah, Kurdistan Region-Iraq

E-mail address: halgwrd.darwesh@univsul.edu.iq

(Shagull H. Mahmood) Department of Mathematics, College of Science, University of Sulaimani, Sulaymaniyah, Kurdistan Region-Iraq

E-mail address: shagull.mahmood7@gmail.com