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NORMALITY VIA SOFT PRE OPEN SETS AND SOFT IDEALS

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Abstract. In this paper we introduces a weak separation axiom called soft P-normality which are strictly weaker than the axioms of soft normality using soft pre open sets. Several properties and characterizations of soft P-normal spaces have been explored. Further we extended soft P-normality to soft ideal topological spaces and present their studies.

1. Introduction

The urgent need for theories dealing with uncertainties comes from daily facing complicated problems containing data that are not always crisp. The recent mathematical tool to handle these problems is soft set which was initiated by Molodtsov [23] in 1999. The rationale of soft sets is based on the parameterization idea, which references that complex objects should be perceived from many aspects and each solo facet only provides a partial and approximate description of the whole entity. Molodtsov [23] in his pioneering work provided some applications of soft sets in different fields and elaborated its merits compared with probability theory and fuzzy sets theory which deals with vagueness or uncertainties. Afterward, Maji and his coworkers [20] investigated some operations between soft sets such as soft union and soft intersections. To overcome the shortcomings of these operations. Ali and his coworkers [5] proposed new operations such as restricted union and intersection and complement of a soft set and revealed some of their properties. Pai and Maio [24] developed information systems using soft sets. The mapping on soft classes was studied by Kharal and Ahmad [18]. In 2011, Shabir and Naz [33] as well as Çağman and his coworkers [7] employed soft sets to introduce the concept of soft topological space using two different approaches. Shabir and Naz [33] formulated soft topology on the collection of soft sets over a universal crisp set with a fixed set of parameters and on the other hand Çağman and his coworkers [7] formulated

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soft topology on the collection of soft sets over an absolute soft set with different sets of parameters that are subsets of the universal set of parameters. In this paper, we continue studying soft topology using the definition given by Shabir and Naz [33]. After the occurrence of soft topology many scholars such as Aygünoğlu and Aygün [6], Akdag and Ozkan [2], Georgiou and his coworkers [8], Hazra and his coworkers [10], Min [22], Senel and Çağman [32], Thakur and Rajput [36], Zorlutuna and his coworkers [38, 39], Hussain and his coworkers [12, 13], Hida [11], Jafari and his coworkers [14] and others explored several topological concepts via soft sets and examined the validity of some known topological results on soft topological spaces. The study of separation axioms in soft topology was initiated by Shabir and Naz [33] and further studied by Hussain [12], Georgiou, Megaritis and Petropoulos [8], Varol [37], Prasad and Thakur [25, 28, 29]. The study of soft ideal topological spaces was initiated by Kandil, Tantawy, Sheikh and Latif [17]. In the recent past many soft separation axioms such as soft $\mathcal{I}$-regularity and soft $\mathcal{I}$-normality [9], soft almost $\mathcal{I}$-regularity [26] and soft almost $\mathcal{I}$-normality [28] in soft ideal topological spaces have been introduced and studied.

In 2013, Ilango and Ravindran [16] created the concept of soft preopen sets in soft topology. Recently many soft topological concepts such as soft pre compactness and soft pre Lindelofness [3, 4], soft P-connectedness [34, 35], soft P-connectedness between sets [35], soft separation axioms [1, 30, 31, 19], and soft almost precontinuity [36, 15] have been introduced and studied utilizing soft preopen sets. In the present paper, we utilize soft preopen sets to create the axioms of soft P-normality and soft P-$\mathcal{I}$-normality and explore their studies. The axioms soft P-normality and soft P-$\mathcal{I}$-normality would be useful for the development of the theory of soft topology to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types. These axioms are an addition to strengthening the toolbox of soft topology.

2. Preliminaries

In this section, we recall some basic concepts and properties regarding soft set theory and soft topology. Let X is an initial universe set, Ω be a set of parameters, P(X) denote the power set of X and $\kappa \subseteq \Omega$.

Definition 2.1. [23] A pair (ξ, κ) is called a soft set over X, where ξ is a mapping given by $\xi : \kappa \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For all $\beta \in \kappa$, (ξ, β) may be considered as the set of β-approximate elements of the soft set (ξ, κ) .

Throughout this paper $S(X, \Omega)$ refers to the collection of all soft sets of X relative to Ω.

Definition 2.2. [20] For two soft sets (ξ_1, κ_1) and (ξ_2, κ_2) over a common universe X, we say that (ξ_1, κ_1) is a soft subset of (ξ_2, κ_2), denoted by $\xi_1(\kappa_1) \subseteq \xi_2(\kappa_2)$ if:

(a): $\kappa_1 \subseteq \kappa_2$ and

(b): $\xi_1(\beta) \subseteq \xi_2(\beta)$ for all $\beta \in \Omega$.

Definition 2.3. [20] Two soft sets (ξ_1, κ_1) and (ξ_2, κ_2) over a common universe X are said to be soft equal denoted by $(\xi_1, \kappa_1) = (\xi_2, \kappa_2)$,if $(\xi_1, \kappa_1) \subseteq (\xi_2, \kappa_2)$ and $(\xi_2, \kappa_2) \subseteq (\xi_1, \kappa_1)$.

Definition 2.4. [20] The complement of a soft set (ξ, κ) denoted by $(\xi, \kappa)^c$, is defined by $(\xi, \kappa)^c = (\xi^c, \kappa)$ where $\xi^c : \kappa \to P(X)$ is a mapping given by $\xi^c(\beta) = X - \xi(\beta)$, for all $\beta \in \Omega$.

Definition 2.5. [20] Let (ξ_1, κ_1) be a soft set over X.

(a): Null soft set denoted by ϕ if for all β, $\xi(\beta) = \phi$.

(b): Absolute soft set denoted by $\widetilde{X}$ if for each $\beta \in \phi$, $\xi(\beta) = \widetilde{X}$.

Clearly, $\widetilde{X}^c = \phi$ and $\phi^c = \widetilde{X}$.

Definition 2.6. [20] Union of two soft sets (ξ_1, κ_1) and (ξ_2, κ_2) over the common universe X, is the soft set (ξ_3, κ_3), where $\kappa_3 = \kappa_1 \cup \kappa_2$ and for all $\beta \in \kappa_3$,

Definition 2.7. [20] Intersection of two soft sets (ξ_1, κ_1) and (ξ_2, κ_2) over the common universe X, is the soft set (ξ_3, κ_3), where $\kappa_3 = \kappa_1 \cap \kappa_2$ and $\xi_3(\beta) = \xi_1(\beta) \cap \xi_2(\beta)$ for all $\beta \in \kappa$

Definition 2.8. [21] If (ξ, κ) be a soft set over X, then

(a): $(\xi, \kappa) \cup (\xi, \kappa) = (\xi, \kappa)$.

(b): $(\xi, \kappa) \cap (\xi, \kappa) = (\xi, \kappa)$.

(c): $(\xi, \kappa) \cup \phi = (\xi, \kappa)$.

(d): $(\xi, \kappa) \cap \phi = \phi$.

(e): $(\xi, \kappa) \cup \tilde{X} = \tilde{X}$ where $\tilde{X}$ is the absolute soft set.

Definition 2.9. [33] A family $\Gamma \subseteq S(X, \Omega)$ is called a soft topology on X, if:

(1) $\widetilde{\phi}, \widetilde{X} \in \Gamma$.

(2) If $(\lambda_i, \Omega) \in \Gamma$ for each $i \in \Lambda$ then $\cup_{i \in \Lambda}(\lambda_i, \Omega) \in \Gamma$.

(3) If $(\lambda_1, \Omega), (\lambda_2, \Omega) \in \Gamma$ then $(\lambda_1, \Omega) \cap (\lambda_2, \Omega) \in \Gamma$.

The triplet (X, Γ, Ω) is called a soft topological space over X and is denoted by STS(X, Γ, Ω). The members of Γ are known as soft open sets in X.

Definition 2.10. [33] A soft set (ξ, Ω) of a STS(X, Γ, Ω) is said to be soft closed if $(\xi, \Omega)^c \in \Gamma$. The collection of all soft closed sets in a STS(X, Γ, Ω) is denoted by $SC(X, \Omega)$.

Definition 2.11. [33, 38] Let (X, Γ, Ω) be a STS and $(\xi, \Omega) \in S(X, \Omega)$. Then the closure and interior of (ξ, Ω) are defined as follows:
$Cl(\xi, \Omega) = \cap\{(\sigma, \Omega) : (\sigma, \Omega) \in SC(X, \Omega) \text{ and } (\xi, \Omega) \subseteq (\sigma, \Omega)\}$.
$Int(\xi, \Omega) = \cup\{(\sigma, \Omega) : (\sigma, \Omega) \in \Gamma \text{ and } (\sigma, \Omega) \subseteq (\xi, \Omega)\}$.

Lemma 2.12. [7, 33, 38] *Let (X, Γ, Ω) be a STS and $(\xi, \Omega), (\sigma, \Omega) \in S(X, \Omega)$. Then:*

(1) $(\xi, \Omega) \in SC(X, \Omega) \Leftrightarrow (\xi, \Omega) = Cl(\xi, \Omega)$

(2) $(\xi, \Omega) \subseteq (\sigma, \Omega) \Rightarrow Cl(\xi, \Omega) \subseteq Cl(\sigma, \Omega)$.

(3) $(\xi, \Omega) \in \Gamma \Leftrightarrow (\xi, \Omega) = Int(\xi, \Omega)$.

(4) $(\xi, \Omega) \subseteq (\sigma, \Omega) \Rightarrow Int(\xi, \Omega) \subseteq Int(\sigma, \Omega)$.

(5) $(Cl(\xi, \Omega))^c = Int((\xi, \Omega)^c)$.

(6) $(Int(\xi, \Omega))^c = Cl((\xi, \Omega)^c)$.

Definition 2.13. [12] Let (X, Γ, Ω) be a STS and $Y \subseteq X$. Then $\Gamma_Y = \{(\xi_Y, \Omega) : (\xi, \Omega) \in \Gamma\}$ is said to be the soft relative topology on Y and (Y, Γ_Y, Ω) is called a soft subspace of (X, Γ, Ω).

Lemma 2.14. [12] *Let (Y, Γ_Y, Ω) be a soft subspace of a STS(X, Γ, Ω) and $(\xi, \Omega) \in \Gamma_Y$. If $\tilde{Y} \in \Gamma$ then $(\xi, \Omega) \in \Gamma$.*

Lemma 2.15. [33] *Let (Y, Γ_Y, Ω) be a soft subspace of a STS(X, Γ, Ω) and $(\xi, \Omega) \in S(X, \Omega)$, then:*
(1) $(\xi, \Omega) \in \Gamma_Y \Leftrightarrow (\xi, \Omega) = \tilde{Y} \cap (\sigma, \Omega)$ *for some* $(\sigma, \Omega) \in \Gamma$.
(2) $(\xi, \Omega) \in SC(Y, \Omega) \Leftrightarrow (\xi, \Omega) = \tilde{Y} \cap (\sigma, \Omega)$ *for some* $(\sigma, \Omega) \in SC(X, \Omega)$.

Lemma 2.16. [32] *Let (Y, Γ_Y, Ω) be a soft subspace of a STS(X, Γ, Ω). Then a soft set $(\xi_Y, \Omega) \in SC(X, \Omega) \Leftrightarrow \tilde{Y} \in SC(X, \Omega)$.*

Definition 2.17. [12] A soft set (ξ, Ω) in a STS(X, Γ, Ω) is said to be soft regular open (resp. soft regular closed) if $(\xi, \Omega)) = Int(Cl(\xi, \Omega))$ (resp. $(\xi, \Omega) = Cl(Int(\xi, \Omega)))$.

The collection of all soft regular open (resp. soft regular closed) sets over (X, Ω) will be denoted by $SRO(X, \Omega)$ (resp. $SRC(X, \Omega)$).

Definition 2.18. [16, 1] A soft set (ξ, Ω) of a STS(X, Γ, Ω) is called soft preopen if $(\xi, \Omega) \subseteq Cl(Int(\xi, \Omega))$. The collection of all soft pre open sets is denoted by $SPO(X, \Omega)$.

Definition 2.19. [16, 1] A soft set (ξ, Ω) of a STS(X, Γ, Ω) is said to be soft preclosed if $(\xi, \Omega)^c \in SPO(X, \Omega)$. The collection of all soft preclosed sets is denoted by $SPC(X, \Omega)$.

Definition 2.20. [2] A soft set (ξ, Ω) of a STS(X, Γ, Ω) is called soft α-open if $(\xi, \Omega) \subseteq Int(Cl(Int(\xi, \Omega)))$. The collection of all soft α-open sets over X will be denoted by $S\alpha O(X, \Omega)$.

Definition 2.21. [2] The complement of a soft α-open set is called soft α-closed. The collection of all soft α-closed sets over (X, Ω) will be denoted by $S\alpha C(X, \Omega)$.

Remark 2.22. [1, 2] In a STS(X, Γ, Ω) the following statements are true:
(a) $SRO(X, \Omega) \subseteq \Gamma \subseteq S\alpha O(X, \Omega) \subseteq SPO(X, \Omega)$.
(b) $SRC(X, \Omega) \subseteq SC(X, \Omega) \subseteq S\alpha C(X, \Omega) \subseteq SPC(X, \Omega)$.
The reverse containments may be false.

Definition 2.23. [1] Let (X, Γ, Ω) be a STS and $(\xi, \Omega) \in S(X, \Omega)$. Then the pre closure of (ξ, Ω) (denoted by $pCl(\xi, \Omega)$) and pre interior of (ξ, Ω) (denoted by $pInt(\xi, \Omega)$) are defined as follows:
(a) $pCl(\xi, \Omega) = \cap\{(\sigma, \Omega) : (\sigma, \Omega) \in SPC(X, \Omega)$ and $(\xi, \Omega) \subseteq (\sigma, \Omega)\}$.
(b) $pInt(\xi, \Omega) = \cup\{(\sigma, \Omega) : (\sigma, \Omega) \in SPO(X, \Omega)$ and $(\sigma, \Omega) \subseteq (\xi, \Omega)\}$.

Definition 2.24. [18] Let $S(X, \Omega)$ and $S(Y, \Psi)$ be families of soft sets over X and Y. Let $u : X \to Y$ and $p : \Omega \to \Psi$ be mappings. Then a mapping $f_{pu} : S(X, \Omega) \to S(Y, \Psi)$ is defined as:

(a): Let (ξ, Ω) be a soft set in $S(X, \Omega)$. The image of (ξ, Ω) under f_{pu} is written as $f_{pu}(\xi, \Omega) = (f_{pu}(\xi), p(\Omega))$ is a soft set in $S(Y, \Psi)$ such that

$$f_{pu}(\xi)(\alpha) = \begin{cases} \bigcup_{\beta \in p^{-1}(\alpha) \cap \Omega} u(\xi(\beta)), & p^{-1}(\alpha) \cap \Omega \neq \phi, \\ \phi, & p^{-1}(\alpha) \cap \Omega = \phi. \end{cases}$$

For all $\alpha \in \Psi$.

(b): Let (μ,Ψ) be a soft set in $S(Y,\Psi)$. The inverse image of (μ,Ψ) under f_{pu} is written as

$$f_{pu}^{-1}(\mu)(\beta)=\begin{cases}u^{-1}\mu(p(\beta)), & p(\beta)\in\Psi,\\ \phi, & otherwise.\end{cases}$$

For all $\beta\in\Omega$.

Definition 2.25. [22] Let $f_{pu}:S(X,\Omega)\to S(Y,\Psi)$ be a mapping and $u:X\to Y$ and $p:\Omega\to\Psi$ be mappings. Then f_{pu} is soft injective (respectively surjective, bijective) if $u:X\to Y$ and $p:\Omega\to\Psi$ are injective (respectively, surjective, bijective).

Definition 2.26. [1, 2, 37, 38, 39] A soft mapping $f_{pu}:(X,\Gamma,\Omega)\to(Y,\eta,\Psi)$ is called:
(1) Soft continuous if $f_{pu}^{-1}(\mu,\Psi)\in\Gamma$ for all soft sets $(\mu,\Psi)\in\eta$.
(2) Soft open if $f_{pu}(\xi,\Omega)\in\eta$ for all soft sets $(\xi,\Omega)\in\Gamma$.
(3) Soft precontinuous if $f_{pu}^{-1}(\mu,\Psi)\in SPO(X,\Omega)$ for all soft sets $(\mu,\Psi)\in\eta$
(4) Soft α-open if $f_{pu}(\xi,\Omega)\in S\alpha O(Y,\Psi)$ for all soft sets $(\xi,\Omega)\in\Gamma$.
(5) Soft homeomorphism if f_{pu} is soft continuous, soft open and soft bijective.

Remark 2.27. [1, 2] The class of all soft precontinuous (resp. soft α-open) mappings contains the class of all soft continuous (resp. soft open)mappings.

Lemma 2.28. [2] *Let $f_{pu}:(X,\Gamma,\Omega)\to(Y,\eta,\Psi)$ be a soft precontinuous and soft α-open mapping. If $(\sigma,\Omega)\in SPO(Y,\Omega)$ then $f_{pu}^{-1}(\sigma,\Omega)\in SPO(X,\Omega)$.*

Definition 2.29. [38] The soft set $(\xi,\Omega)\in S(X,\Omega)$ is called a soft point if, $\exists\ x\in X$ and $\beta\in\Omega$ such that $\xi(\beta)=\{x\}$ and $\xi(\beta^c)=\phi$ for each $\beta^c\in\Omega-\{\beta\}$ and the soft point (ξ,Ω) is denoted by x_β. We denote the family of all soft points over X by SP(X,Ω).

Definition 2.30. [38] The soft point $x_\beta\in(\sigma,\Omega)$, if $x_\beta\subseteq(\sigma,\Omega)$.

Definition 2.31. [17] A non-empty collection of soft sets $\Gamma\subseteq S(X,\Omega)$ is said to be a soft ideal on X if,
(1) $(\xi,\Omega)\in\mathcal{I}$ and $(\sigma,\Omega)\in\mathcal{I}$ implies $(\xi,\Omega)\cup(\sigma,\Omega)\in\mathcal{I}$.
(2) $(\xi,\Omega)\in\mathcal{I}$ and $(\sigma,\Omega)\subseteq(\xi,\Omega)$ implies $(\sigma,\Omega)\in\mathcal{I}$. A soft topological space (X,Γ,Ω) with a soft ideal $\mathcal{I}$ is called soft ideal topological space and is denoted by SITS $(X,\Gamma,\Omega,\mathcal{I})$.

Definition 2.32. [8, 27] A STS(X,Γ,Ω) is said to be soft normal (resp. soft almost normal) if for every $(\sigma_1,\Omega),(\sigma_2,\Omega)\in$ SC (X,Ω) (resp. $(\sigma_1,\Omega)\in SC(X,\Omega)$ and $(\sigma_2,\Omega)\in SRC(X,\Omega)$) such that $(\sigma_1,\Omega)\cap(\sigma_2,\Omega)=\phi$, $\exists$ $(\xi_1,\Omega),(\xi_2,\Omega)\in\Gamma$ such that $(\sigma_1,\Omega)\subseteq(\xi_1,\Omega),(\sigma_2,\Omega)\subseteq(\xi_2,\Omega)$ and $(\xi_1,\Omega)\cap(\xi_2,\Omega)=\phi$.

Definition 2.33. [9] A SITS$(X,\Gamma,\Omega,\mathcal{I})$ is called a soft $\mathcal{I}$-normal (resp. soft almost $\mathcal{I}$-normal), if for every $(\xi,\Omega),(\sigma,\Omega)\in$ SC(X,Ω) (resp.$(\xi,\Omega)\in$SC(X,Ω) and $(\sigma,\Omega)\in$SRC(X,Ω)) such that $(\xi,\Omega)\cap(\sigma,\Omega)=\phi$, $\exists$ disjoint soft sets $(\mu,\Omega),(\lambda,\Omega)\in\Gamma$ and such that $(\xi,\Omega)-(\mu,\Omega)\in\mathcal{I}$ and $(\sigma,\Omega)-(\lambda,\Omega)\in\mathcal{I}$.

3. Soft P-Normal Spaces

The axiom of normality is very important to study the covering axioms in general topology. A topological space is normal, if every pair of disjoint closed sets are separated by pair of open sets. After the introduction of preopen sets the axiom P-normality was created by replacing open sets by preopen sets in the definition of normality. The axiom of P-normality is strictly weaker than the axiom of normality. The present section extended the axiom of P-normality to soft sets and present their studies in soft topological spaces.

Definition 3.1. A STS(X, Γ, Ω) is called soft P-normal if for every $(\xi, \Omega), (\sigma, \Omega) \in$ SC(X, Ω) such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$, $\exists$ $(\mu, \Omega), (\lambda, \Omega) \in$ SPO(X, Ω) such that $(\xi, \Omega) \subseteq (\mu, \Omega), (\sigma, \Omega) \subseteq (\lambda, \Omega)$ and $(\mu, \Omega) \cap (\lambda, \Omega) = \phi$.

Remark 3.2. The axiom soft normality implies soft P-normality However, a soft P-normal space may fails to be soft normal. For,

Example 3.3. Let $X = \{l_1, l_2\}, \Omega = \{\beta_1, \beta_2\}$ and the soft sets (λ, Ω), (μ, Ω) and (ν, Ω) are defined as follows:
$(\lambda, \Omega) = \{(\beta_1, \{l_1, l_2\}), (\beta_2, \{l_2\})\}$,
$(\mu, \Omega) = \{(\beta_1, \{l_1\}), (\beta_2, \{l_1, l_2\})\}$,
$(\nu, \Omega) = \{(\beta_1, \{l_1\}), (\beta_2, \{l_2\})\}$,

Then (X, Γ, Ω) where $\Gamma = \{\phi, X, (\lambda, \Omega), (\mu, \Omega), (\nu, \Omega)\}$ is soft P-normal but not soft normal.

Remark 3.4. The axiom soft P-normality is independent to the axiom soft almost normality. For,

Example 3.5. Let $X = \{l_1, l_2, l_3\}$, $\Omega = \{\beta_1, \beta_2\}$ and the soft sets (λ, Ω), (μ, Ω) and (ν, Ω) are defined as follows:
$(\lambda, \Omega) = \{(\beta_1, \{l_1\}),\ (\beta_2, \{l_1\})\}$,
$(\mu, \Omega) = \{(\beta_1, \{l_1, l_2\}),\ (\beta_2, \{l_1, l_2\})\}$,
$(\nu, \Omega) = \{(\beta_1, \{l_1, l_3\}), (\beta_2, \{l_1, l_3\})\}$.

Let $\Gamma = \{\phi, X, (\lambda, \Omega), (\mu, \Omega), (\nu, \Omega), (\kappa, \Omega), (\chi, \Omega), (\psi, \Omega)\}$ be a soft topology on X with the set of parameters Ω. Then STS(X, Γ, Ω) is soft P-normal but not soft almost normal.

Theorem 3.6. *A STS(X, Γ, Ω) is soft P-normal if and only if $\forall$ $(\xi, \Omega), (\sigma, \Omega) \in \Gamma$ such that $(\xi, \Omega) \cup (\sigma, \Omega) = \tilde{X}$, $\exists$ $(\rho, \Omega), (\delta, \Omega) \in$ SPC(X, Ω) such that $(\xi, \Omega) \subseteq (\rho, \Omega)$ and $(\sigma, \Omega) \subseteq (\delta, \Omega)$ and $(\rho, \Omega) \cup (\delta, \Omega) = \tilde{X}$.*

Proof. Obvious □

Theorem 3.7. *A STS(X, Γ, Ω) is soft P-normal if and only if for every $(\xi, \Omega) \in$ SC(X, Ω) and each $(\sigma, \Omega) \in \Gamma$ containing (ξ, Ω) $\exists$, $(\rho, \Omega) \in$ SPO(X, Ω) such that $(\xi, \Omega) \subseteq (\rho, \Omega) \subseteq pCl(\rho, \Omega) \subseteq (\sigma, \Omega)$.*

Proof. **Necessity:** Let $(\xi, \Omega) \in$SC(X, Ω) and $(\sigma, \Omega) \in \Gamma$ containing (ξ, Ω). Then, $(\xi, \Omega), (\sigma, \Omega)^c \in$ SC(X, Ω) such that $(\xi, \Omega) \cap (\sigma, \Omega)^c = \phi$. Since (X, Γ, Ω) is soft P-normal, $\exists$ disjoint $(\rho, \Omega), (\lambda, \Omega) \in$ SPO(X, Ω) such that $(\xi, \Omega) \subseteq (\rho, \Omega)$ and $(\sigma, \Omega)^c \subseteq (\lambda, \Omega)$. Thus $(\xi, \Omega) \subseteq (\rho, \Omega)$ and $(\lambda, \Omega)^c \subseteq (\sigma, \Omega)$. Now $(\lambda, \Omega)^c \in SPC(X, \Omega)$, it follows that $(\xi, \Omega) \subseteq (\rho, \Omega)$, $sCl(\rho, \Omega) \subseteq (\lambda, \Omega)^c$ and $(\lambda, \Omega)^c \subseteq (\sigma, \Omega)$.
Sufficiency: Let $(\xi, \Omega), (\sigma, \Omega) \in$SC$(X, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$. So, $(\xi, \Omega) \subseteq (\sigma, \Omega)^c$. Now, $(\sigma, \Omega)^c \in \Gamma$, $\exists$ $(\rho, \Omega) \in$ SPO(X, Ω), such that $(\xi, \Omega) \subseteq (\rho, \Omega)$ and

$\mathrm{pCl}(\rho,\Omega)\subseteq(\sigma,\Omega)^c$. Now let $(\lambda,\Omega)=(\mathrm{pCl}(\rho,\Omega))^c=\mathrm{pInt}((\rho,\Omega)^c)$. Therefore, (λ,Ω) is the largest soft pre open set contained in $(\rho,\Omega)^c$. Also, $(\sigma,\Omega)\subseteq(\lambda,\Omega)$. Thus $(\xi,\Omega)\subseteq(\rho,\Omega)$, $(\sigma,\Omega)\subseteq(\lambda,\Omega)$ and $(\rho,\Omega)\cap(\lambda,\Omega)=\phi$. Hence, (X,Γ,Ω) is soft P-normal. □

Theorem 3.8. *Let $f_{pu}:(X,\Gamma,\Omega)\to(Y,\eta,\Psi)$ be a soft precontinuous, soft α-open and bijective mapping. If $(\sigma,\Omega)\in SPC(X,\Omega)$ then $f_{pu}(\sigma,\Omega)\in SPC(Y,\Omega)$.*

Proof. Since $(\sigma,\Omega)^c\in\mathrm{SPO}(X,\Omega)$ and f_{pu} is soft α-open and soft precontinuous, $f_{pu}((\sigma,\Omega)^c)\in\mathrm{SPO}(Y,\Omega)$ by Lemma 2.28. Also f_{pu} being one-one and onto, $f_{pu}((\sigma,\Omega)^c)=(f_{pu}(\sigma,\Omega))^c$. Consequently, $f_{pu}(\sigma,\Omega)\in\mathrm{SPC}(Y,\Omega)$. □

Theorem 3.9. [35] *Let (X,Γ,Ω) be a STS and $(\xi,\Omega)\in S(X,\Omega)$. Then a soft point $x_\beta\in pCl(\xi,\Omega)$ if and only if $(\xi,\Omega)\cap(\sigma,\Omega)\neq\phi$ for all soft set $(\sigma,\Omega)\in SPO(X,\Omega)$ containing x_β*

Theorem 3.10. *If $f_{pu}:(X,\Gamma,\Omega)\to(Y,\eta,\Psi)$ is a soft precontinuous, soft α-open and bijective mapping then $f_{pu}(pCl(\xi,\Omega))=pCl(f_{pu}(\xi,\Omega))$ for each soft set (ξ,Ω) of (X,Γ,Ω).*

Proof. Let, $y_\beta\in f_{pu}(pCl(\xi,\Omega))$. Then, $\exists$ a soft point $x_\beta\in pCl(\xi,\Omega)$ such that $f_{pu}(x_\beta)=y_\beta$. Let $(\mu,\Omega)\in\mathrm{SPO}(X,\Omega)$ containing x_β. And so by Lemma 2.28, $f_{pu}^{-1}(\mu,\Omega)\in\mathrm{SPO}(X,\Omega)$ and contains y_β. Therefore $f_{pu}^{-1}(\mu,\Omega)\cap(\xi,\Omega)\neq\phi$, for $x_\beta\in pCl(\xi,\Omega)$. Let $y_\beta\in f_{pu}^{-1}(\mu,\Omega)\cap(\xi,\Omega)$. This gives $f_{pu}(y_\beta)\in(\mu,\Omega)\cap f_{pu}(\xi,\Omega)$. Thus each $(\mu,\Omega)\in\mathrm{SPO}(X,\Omega)$ containing x_β meets $f_{pu}(\xi,\Omega)$. Hence, $x_\beta\in pCl(\xi,\Omega)$. Now $(\xi,\Omega)\subseteq pCl(\xi,\Omega)$ implies $f_{pu}(\xi,\Omega)\subseteq f_{pu}(pCl(\xi,\Omega))$. It follows that, $pCl(\xi,\Omega)\subseteq pCl(f_{pu}(pCl(\xi,\Omega)))=f_{pu}(pCl(\xi,\Omega))$ for $pCl(\xi,\Omega)\in\mathrm{SPC}(X,\Omega)$ by Theorem 3.8, $f_{pu}(pCl(\xi,\Omega))\in\mathrm{SPC}(X,\Omega)$. Consequently, $f_{pu}(pCl(\xi,\Omega))=pCl(f_{pu}(\xi,\Omega))$. □

Theorem 3.11. *Let $f_{pu}:(X,\Gamma,\Omega)\to(Y,\eta,\Psi)$ be a soft precontinuous, soft α-open and bijective mapping. If (X,Γ,Ω) is soft P-normal then so is (Y,η,Ψ).*

Proof. Let (X,Γ,Ω) be soft P-normal and f_{pu} be a soft precontinuous soft α-open mapping of (X,Γ,Ω) onto a STS (Y,η,Ψ). Let $(\xi,\Omega)\in\mathrm{SC}(Y,\Omega)$ and (σ,Ω) be soft open in (Y,η,Ψ) such that, $(\xi,\Omega)\subseteq(\sigma,\Omega)$. Then $f_{pu}^{-1}(\xi,\Omega)\in\mathrm{SC}(X,\Omega)$ contained in the soft open set $f_{pu}^{-1}(\sigma,\Omega)$. Since (X,Γ,Ω) is soft P-normal, by Theorem 3.7, $\exists$ $(\mu,\Omega)\in SPO(X,\Omega)$ such that $f_{pu}^{-1}(\xi,\Omega)\subseteq(\mu,\Omega)\subseteq\mathrm{pCl}(\mu,\Omega)\subseteq f_{pu}^{-1}(\sigma,\Omega)$. Since f_{pu} is onto, we get, $(\xi,\Omega)\subseteq f_{pu}(\mu,\Omega)\subseteq f_{pu}(pCl(\mu,\Omega))\subseteq(\sigma,\Omega)$. Now by Theorem 3.10 , this reduces to $(\xi,\Omega)\subseteq f_{pu}(\mu,\Omega)\subseteq\mathrm{pCl}(f_{pu}(\mu,\Omega))\subseteq(\sigma,\Omega)$. Hence by Theorem 3.7, (Y,η,Ψ) is soft P-normal since $f_{pu}(\mu,\Omega)\in\mathrm{SPO}(X,\Omega)$. □

Corollary 3.12. Let $f_{pu}:(X,\Gamma,\Omega)\to(Y,\eta,\Psi)$ be a soft homeomorphism. If (X,Γ,Ω) is soft P-normal then so is (Y,η,Ψ).

Theorem 3.13. *Let (Y,Γ_Y,Ω) be a soft subspace of a STS (X,Γ,Ω) and $(\xi,\Omega)\subset\tilde{Y}$. If (ξ,Ω) is a soft preopen set over X, then it is a soft preopen set over Y.*

Theorem 3.14. *Let (Y,Γ_Y,Ω) be a soft open subspace of a STS (X,Γ,Ω) and (ξ,Ω) be a soft preopen set over X, then $(\xi,\Omega)\cap(Y,\Omega)$ is soft preopen in Y.*

Theorem 3.15. *Let (Y, Ω) be a soft closed soft open subset of a STS (X, Γ, Ω). If (X, Γ, Ω) is soft P-normal then so is the soft subspace (Y, Γ_Y, Ω).*

Proof. Let (Y, Γ_Y, Ω) be a soft closed open soft subspace of (X, Γ, Ω). Suppose $(\xi, \Omega), (\sigma, \Omega) \in \mathrm{SC}(Y, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$. Then $(\xi, \Omega), (\sigma, \Omega) \in \mathrm{SC}(X, \Omega)$ because (Y, Ω) is a soft closed set in (X, Γ, Ω). Now (X, Γ, Ω) being soft P-normal, $\exists\ (\mu, \Omega), (\lambda, \Omega) \in \mathrm{SPO}(X, \Omega)$ such that $(\xi, \Omega) \subseteq (\mu, \Omega)$, $(\sigma, \Omega) \subseteq (\lambda, \Omega)$ and $(\mu, \Omega) \cap (\lambda, \Omega) = \phi$. Since (Y, Ω) is a soft open set in (X, Γ, Ω), it follows from Theorem 3.13 and Theorem 3.14 that $(\mu, \Omega) \cap (Y, \Omega)$ and $(\lambda, \Omega) \cap (Y, \Omega)$ are disjoint soft pre open sets in (Y, Γ_Y, Ω) such that $(\xi, \Omega) \subseteq (\mu, \Omega) \cap (Y, \Omega)$, $(\sigma, \Omega) \subseteq (\lambda, \Omega) \cap (Y, \Omega)$. Hence (Y, Γ_Y, Ω) is soft P-normal. $\square$

4. Soft P-$\mathcal{I}$-Normal Spaces

The axiom of soft $\mathcal{I}$-normality in soft ideal topological spaces was studied by Guler and his coworkers [9]. Recently authors of this paper [28] created the axiom of soft almost $\mathcal{I}$-normality which generalizes the axiom of soft $\mathcal{I}$-normality. In this section, we introduce a new soft separation axiom called soft P-$\mathcal{I}$-normality which is weaker than the axiom of soft $\mathcal{I}$-normality and independent to the axiom of soft $\mathcal{I}$-normality and obtain some of their characterizations and properties in soft ideal topological spaces.

Definition 4.1. A $\mathrm{SITS}(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal, if $\forall\ (\xi, \Omega), (\sigma, \Omega) \in \mathrm{SC}(X, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$, $\exists\ (\mu, \Omega), (\lambda, \Omega) \in \mathrm{SPO}(X, \Omega)$ such that $(\xi, \Omega) - (\mu, \Omega) \in \mathcal{I}$, $(\sigma, \Omega) - (\lambda, \Omega) \in \mathcal{I}$ and $(\mu, \Omega) \cap (\lambda, \Omega) = \phi$.

Remark 4.2. The axiom soft $\mathcal{I}$-normality implies soft P-$\mathcal{I}$-normality However, a soft P-$\mathcal{I}$-normal space may fail to be soft $\mathcal{I}$-normal. For,

Example 4.3. Let $X = \{l_1, l_2\}, \Omega = \{\beta_1, \beta_2\}$ and the soft sets (λ, Ω), (μ, Ω) and (ν, Ω) are defined as follows:
$(\lambda, \Omega) = \{(\beta_1, \{l_1, l_2\}), (\beta_2, \{l_2\})\}$,
$(\mu, \Omega) = \{(\beta_1, \{l_1\}), (\beta_2, \{l_1, l_2\})\}$,
$(\nu, \Omega) = \{(\beta_1, \{l_1\}), (\beta_2, \{l_2\})\}$.

Let $\mathcal{I} = \{\phi\}$ be a soft ideal over (X, Ω). Then the $\mathrm{SITS}(X, \Gamma, \Omega, \mathcal{I})$ where $\Gamma = \{\phi, X, (\lambda, \Omega), (\mu, \Omega), (\nu, \Omega), (\kappa, \Omega), (\chi, \Omega)\}$ is soft P-$\mathcal{I}$-normal but not soft $\mathcal{I}$-normal.

Remark 4.4. The axiom soft P-$\mathcal{I}$-normality is independent of the axiom soft almost $\mathcal{I}$-normality. For,

Example 4.5. Let $X = \{l_1, l_2, l_3, l_4\}, \Omega = \{\beta_1, \beta_2\}$ and the soft sets (λ, Ω), (μ, Ω), (ν, Ω) , (κ, Ω), (χ, Ω) and (ψ, Ω) be defined as follows:
$(\lambda, \Omega) = \{(\beta_1, \{l_1, l_2, l_3\}),\ (\beta_2, \{l_1, l_2, l_3\})\}$,
$(\mu, \Omega) = \{(\beta_1, \{l_1, l_2\}),\ (\beta_2, \{l_1, l_2\})\}$,
$(\nu, \Omega) = \{(\beta_1, \{l_2, l_3\}),\ (\beta_2, \{l_2, l_3\})\}$,
$(\kappa, \Omega) = \{(\beta_1, \{l_2\}),\ (\beta_2, \{l_2\})\}$,
$(\chi, \Omega) = \{(\beta_1, \{l_2, l_3, l_4\}),\ (\beta_2, \{l_2, l_3, l_4\})\}$,
$(\psi, \Omega) = \{(\beta_1, \{l_3\}),\ (\beta_2, \{l_3\})\}$.

Let $\mathcal{I} = \{\phi\}$ be a soft ideal over (X, Ω). Then the SITS$(X, \Gamma, \Omega, \mathcal{I})$ where $\Gamma = \{\phi, X, (\lambda, \Omega), (\mu, \Omega), (\nu, \Omega), (\kappa, \Omega), (\chi, \Omega), (\psi, \Omega)\}$ is soft P-$\mathcal{I}$-normal but it is not soft almost $\mathcal{I}$-normal.

Example 4.6. Let $X = \{l_1, l_2, l_3\}$, $\Omega = \{\beta_1, \beta_2\}$ and the soft sets (λ, Ω), (μ, Ω) and (ν, Ω) be defined as follows:
$(\lambda, \Omega) = \{(\beta_1, \{l_1\}),\ (\beta_2, \{l_1\})\}$,
$(\mu, \Omega) = \{(\beta_1, \{l_1, l_2\}),\ (\beta_2, \{l_1, l_2\})\}$,
$(\nu, \Omega) = \{(\beta_1, \{l_1, l_3\}), (\beta_2, \{l_1, l_3\})\}$.

Let $\mathcal{I} = \{\phi\}$ be a soft ideal over (X, Ω). The SITS$(X, \Gamma, \Omega, \mathcal{I})$ where $\Gamma = \{\phi, X, (\lambda, \Omega), (\mu, \Omega), (\nu, \Omega)\}$ is soft almost $\mathcal{I}$-normal but not soft P-$\mathcal{I}$-normal.

Theorem 4.7. *A SITS$(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal if and only if $\forall$ $(\xi, \Omega) \in SC(X, \Omega)$ and each $(\sigma, \Omega) \in \Gamma$ containing (ξ, Ω), $\exists$ $(\rho, \Omega) \in SPO(X, \Omega)$ such that $(\xi, \Omega) - (\rho, \Omega) \in \mathcal{I}$ and $pCl(\rho, \Omega) - (\sigma, \Omega) \in \mathcal{I}$.*

Proof. **Necessity:** Let $(\xi, \Omega) \in \mathrm{SC}(X, \Omega)$ and $(\sigma, \Omega) \in \Gamma$ containing (ξ, Ω). Then, $(\xi, \Omega), (\sigma, \Omega)^c \in \mathrm{SC}(X, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega)^c = \phi$. Since $(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal $\exists$ disjoint $(\rho, \Omega), (\lambda, \Omega) \in \mathrm{SPO}(X, \Omega)$ such that $(\xi, \Omega) - (\rho, \Omega) \in \mathcal{I}$ and $(\sigma, \Omega)^c - (\lambda, \Omega) \in \mathcal{I}$. Thus $(\xi, \Omega) - (\rho, \Omega) \in \mathcal{I}$ and $(\lambda, \Omega)^c - (\sigma, \Omega) \in \mathcal{I}$. Now $(\lambda, \Omega)^c \in \mathrm{SPC}(X, \Omega)$, it follows that $(\xi, \Omega) - (\rho, \Omega) \in \mathcal{I}$, $\mathrm{sCl}(\rho, \Omega) - (\lambda, \Omega)^c \in \mathcal{I}$ and $(\lambda, \Omega)^c - (\sigma, \Omega) \in \mathcal{I}$.

Sufficiency: Let $(\xi, \Omega), (\sigma, \Omega) \in \mathrm{SC}(X, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$. So, $(\xi, \Omega) - (\sigma, \Omega)^c \in \mathcal{I}$. Now, $(\sigma, \Omega)^c \in \Gamma$, $\exists$ $(\rho, \Omega) \in \mathrm{SPO}(X, \Omega)$, such that $(\xi, \Omega) - (\rho, \Omega) \in \mathcal{I}$ and $\mathrm{pCl}(\rho, \Omega) - (\sigma, \Omega)^c \in \mathcal{I}$. Now let $(\lambda, \Omega) = (\mathrm{pCl}(\rho, \Omega))^c = \mathrm{pInt}((\rho, \Omega)^c)$. Therefore, (λ, Ω) is the largest soft pre open set contained in $(\rho, \Omega)^c$. Also, $(\sigma, \Omega) - (\lambda, \Omega) \in \mathcal{I}$. Thus $(\xi, \Omega) - (\rho, \Omega) \in \mathcal{I}$, $(\sigma, \Omega) - (\lambda, \Omega) \in \mathcal{I}$ and $(\rho, \Omega) \cap (\lambda, \Omega) = \phi$. Hence, $(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal. □

Theorem 4.8. *Let $f_{pu} : (X, \Gamma, \Omega, \mathcal{I}) \to (Y, \eta, \Psi, J)$ be a soft precontinuous soft α-open bijection. If $(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal then so is (Y, η, Ψ, J).*

Proof. Let $(X, \Gamma, \Omega, \mathcal{I})$ be soft P-$\mathcal{I}$-normal and f_{pu} be a soft precontinuous soft α-open mapping of $(X, \Gamma, \Omega, \mathcal{I})$ onto a SITS (Y, η, Ψ, J). Let $(\xi, \Omega) \in \mathrm{SC}(Y, \Omega)$ and (σ, Ω) be soft open in (Y, η, Ψ, J) such that $(\sigma, \Omega) - (\xi, \Omega) \in \mathcal{I}$. Then $f_{pu}^{-1}(\xi, \Omega) \in \mathrm{SC}(X, \Omega)$ contained in the soft open set $f_{pu}^{-1}(\sigma, \Omega)$. Since $(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal, by Theorem 4.7, $\exists$ a soft set $(\mu, \Omega) \in \mathrm{SPO}(X, \Omega)$, such that $f_{pu}^{-1}(\xi, \Omega) - (\mu, \Omega) \in \mathcal{I}$ and $\mathrm{pCl}(\mu, \Omega) - f_{pu}^{-1}(\sigma, \Omega) \in \mathcal{I}$. Since f_{pu} is onto, we get, $(\xi, \Omega) - f_{pu}(\mu, \Omega) \in J$ and $f_{pu}(pCl(\mu, \Omega)) - (\sigma, \Omega) \in J$. Now by Theorem 3.10, this reduces to $(\xi, \Omega) - f_{pu}(\mu, \Omega) \in J$. and $(\mathrm{pCl}(f_{pu}(\mu, \Omega))) - (\sigma, \Omega) \in J$. Hence (Y, η, Ψ, J) is soft p-J-normal by Theorem 4.7, since $f_{pu}(\mu, \Omega) \in \mathrm{SPO}(X, \Omega)$. □

Corollary 4.9. Let $f_{pu} : (X, \Gamma, \Omega, \mathcal{I}) \to (Y, \eta, \Psi, J)$ be a soft homeomorphism. If $(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal then so is (Y, η, Ψ, J).

Theorem 4.10. *Let (Y, Ω) be a soft closed soft open subset of a SITS$(X, \Gamma, \Omega, \mathcal{I})$. If $(X, \Gamma, \Omega, \mathcal{I})$ is soft P-$\mathcal{I}$-normal then the soft subspace $(Y, \Gamma_Y, \Omega, \mathcal{I}_Y)$ is soft P-$\mathcal{I}_Y$-normal.*

Proof. Let $(Y, \Gamma_Y, \Omega, \mathcal{I}_Y)$ be a soft subspace of $(X, \Gamma, \Omega, \mathcal{I})$ and (Y, Ω) be a soft closed soft open subset of $(X, \Gamma, \Omega, \mathcal{I})$. Suppose $(\xi, \Omega), (\sigma, \Omega) \in \mathrm{SC}(Y, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$, then they are also soft closed in $(X, \Gamma, \Omega, \mathcal{I})$. $(X, \Gamma, \Omega, \mathcal{I})$ being soft P-$\mathcal{I}$-normal $\exists$ $(\mu, \Omega), (\lambda, \Omega) \in \mathrm{SPO}(X, \Omega)$, such that $(\xi, \Omega) - (\mu, \Omega) \in \mathcal{I}$, $(\sigma, \Omega) - (\lambda, \Omega) \in \mathcal{I}$ and $(\mu, \Omega) \cap (\lambda, \Omega) = \phi$. It follows from Theorem 3.13 and Theorem 3.14 that $(\mu, \Omega) \cap (Y, \Omega), (\lambda, \Omega) \cap (Y, \Omega) \in SPO(Y, \Gamma_Y, \Omega, \mathcal{I}_Y)$ and $((\mu, \Omega) \cap (Y, \Omega)) \cap ((\lambda, \Omega) \cap (Y, \Omega)) = \phi$ such that $(\xi, \Omega) - ((\mu, \Omega) \cap (Y, \Omega)) \in \mathcal{I}_Y$ and $(\sigma, \Omega) - ((\lambda, \Omega) \cap (Y, \Omega)) \in \mathcal{I}_Y$. Hence, $(Y, \Gamma_Y, \Omega, \mathcal{I}_Y)$ is soft P-$\mathcal{I}_Y$-normal. □

5. Conclusion

In this work, we introduced soft P-normal spaces and soft P-$\mathcal{I}$-normal spaces. It has been shown with the examples that the axiom soft P-normality (resp. soft P-$\mathcal{I}$-normality) is strictly weaker than the axiom of soft normality (resp. soft $\mathcal{I}$-normality) and independent to the axiom of soft almost normality (resp. soft almost $\mathcal{I}$ normality). Several theorem related to the characterizations, hereditary properties of soft P-normal and soft P-$\mathcal{I}$-normal spaces have been established. The axioms soft P-normality and soft P-$\mathcal{I}$-normality would be useful for the development of the theory of soft topology to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types. These axioms are the addition for strengthening the toolbox of soft topology.

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