

MULTI-CRITERIA DECISION MAKING PROCESS USING CUBIC SOFT MATRICES

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Abstract. A cubic soft set (CSS) is a combination of an interval-valued fuzzy set (IVFS) and a fuzzy set (FS). Cubic soft matrix (CSM) plays a significant role in analyzing modern uncertainty. In this manuscript, we discuss two novel approaches using a real-life scenario. One being, to use the combination of IVFS and FS and the other, to find a unique ranking using parameters. First, we introduce some basic operations of 'OR', 'AND', and complement of CSM. Second, we propose the concept of a weighted fuzzy vector (WFV), weighted interval-valued fuzzy vector (WIVFV), weighted cubic vector (WCV) and construction of cubic matrices to solve multi-criteria decision making (MCDM) problems. Third, we discuss the concept of the value and the score function to rank the alternatives and choose the best one. Finally, we present an MCDM problem to deal with the selection of a suitable candidate for a specific job in an organization with two case studies to illustrate the working and reliability of the proposed model.

1. Introduction

In 1965, Zadeh [24] introduced FS which formed the foundation for fuzzy mathematics. FS theory has attained great success in handling uncertainty and vagueness through numerous applications. In FS theory, the membership value of an element is a real number from the closed interval of $[0, 1]$. The concept of IVFS was later introduced by Zadeh [25] which is an extension of FS. In IVFS, the membership value of an uncertain element is specified by a closed subinterval of $[0, 1]$ instead of a real number. Later, Atanassov [5] introduced the intuitionistic fuzzy set (IFS), a concept involving the degree of membership and non-membership for each element, together with the condition that their sum does not exceed unity.

Jun et al. [22] introduced a cubic set (CS) and developed the notions of internal and external cubic sets using IVFS and FS. Molodtsov [13] proposed the concept of soft set

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(SS). Decision-makers use it extensively in many MCDM processes. Maji et al. [11] were the first to discuss the fuzzy soft set (FSS) by combining the concepts of FS and SS. Yang et al. [20] studied the notion of an interval-valued soft set (IVSS). Muhiuddin and Al-rogi [15] developed the cubic soft set (CSS) by combining IVSS and FSS. They defined some operations like AND, OR, P-orders and R-orders and investigated some properties.

Ali et al. [3] have introduced the notions of generalized cubic soft sets, generalized cubic soft left, right ideals and provided some examples. Abbas et al. [1] introduced the concept of the cubic Pythagorean fuzzy set (CPFS) in which membership degree is an interval-valued Pythagorean fuzzy set (IVPFS) and non-membership degree is a Pythagorean fuzzy set (PFS). They developed and discussed the properties of the cubic Pythagorean fuzzy weighted averaging operator (CPFVAO) and the cubic Pythagorean fuzzy weighted geometric operator (CPFVGO). Also, using CPFS solved the MCDM problems in a fuzzy environment. Qayyum et al. [16] introduced the notion of cubic soft expert sets and internal and external cubic soft expert sets. They have also given an application of cubic soft expert sets in decision making and illustrated with examples.

Adam and Hassan [2] introduced some basic operations of the Q-cubic set and Q-cubic soft multi-set with suitable examples. Jun et al. [23] introduced the notions of cubic soft closed sub-algebra and cubic soft ideals in BCK/BCI algebras and investigated some properties. The relationship between cubic soft ideals closed cubic soft sub-algebras, and cubic soft sub-algebras are studied. Saleem Abdullah et al. [18] have studied some theoretical properties of a cubic soft set and got conditions for R-intersection (P-intersection) of two cubic soft sets to be both an external cubic soft set and an internal cubic soft set. Aliya Fahmi et al. [4] defined some Einstein operations on the cubic fuzzy set and developed three arithmetic averaging operators, namely cubic fuzzy Einstein weighted averaging operator, cubic fuzzy Einstein ordered weighted averaging operator, and cubic fuzzy Einstein hybrid weighted averaging operator for aggregating the cubic fuzzy data. They have applied these operators in multiple attribute decision-making using cubic fuzzy data. They have also provided numerical examples to show the working of the method.

Matrices plays a significant role in decision making analysis. However, it is very difficult to apply classical matrix theory to deal with problems involving uncertainty. To overcome this, Yong et.al [21] introduced a fuzzy soft matrix and used it in MCDM problems. Meenakshi [12] highlighted the concept of the fuzzy matrix theory and its applications. Chinnadurai et al. [6, 7, 8, 9] introduced cubic soft matrices and studied some of their theoretical properties. MCDM process based on intuitionistic fuzzy soft matrices was studied by Rajarajeswari [17]. Later, Sarala [19] has used an interval-valued fuzzy soft matrix in decision-making problems by using a scoring matrix.

Independently, when IVFS and FS cannot rank the alternatives, the combination of IVFS and FS resulting in a CS works in solving the MCDM problems. When the decision-maker who understands the combination of interval-valued and fuzzy membership values provided by two different experts results in a greater advantage in solving the MCDM problems. Since most of the existing studies do not delineate the importance of CS with real-life scenarios, we present a novel selection process depicting the importance of CS with case studies. Also, we discuss the concept of the unique

ranking among the alternatives with the help of parameters. This approach is simple, efficient, and the computations involved are easy.

The structure of this manuscript is as below. Section 2 discusses some basic definitions required for the study. Section 3 introduces the ‘OR’, ‘AND’, complement, weight operations on cubic matrices and the construction of cubic matrices. Section 4 defines the concepts of the value function, score function, and total score to solve the MCDM process. Section 5 presents a procedure for ranking the candidates with an algorithm and flowchart. Section 6 illustrates two case studies. In the first case, the ranking of the candidates is unique, and in the second case, there is over one candidate with an equal highest total score. We get the unique ranking for the second case by following the mentioned steps with an enlarged criterion set. Section 7 ends with a conclusion and future studies.

2. Preliminaries

In this section, some basic definitions which are essential for further discussion are recalled.

2.1. Definition:Fuzzy Matrix [21]. A fuzzy matrix of order $m \times n$ is a matrix of the form $A = [a_{ij}]$ where a_{ij} is the fuzzy membership value of the $(i, j)^{th}$ element in A . The values $a_{ij} \in [0, 1] \forall i, j$.

2.2. Definition:Fuzzy Relation [12]. A fuzzy relation $R(X, Y)$, between two sets X and Y , can be represented as a matrix with entries $\mu_R(x, y)$, for $x \in X$ and $y \in Y$, where $\mu_R(x, y)$ is the degree of membership associated with the relationship between x and y . Thus $\mu_R(x, y) \in [0, 1]$ for all $x \in X, y \in Y$. A fuzzy relation from X to itself is denoted by $R(X, X)$. We use the notation μ_R to represent a fuzzy relation matrix.

Example:2.1. Let $X = \{x_1, x_2, x_3\}$ and $R(X, X)$ be the fuzzy relation defined on X as

$$R = \{0.3 \setminus (x_1, x_1) + 0.2 \setminus (x_1, x_2) + 0.5 \setminus (x_1, x_3) + 0.7 \setminus (x_2, x_2) + 0.9 \setminus (x_2, x_3) \\ + 0.6 \setminus (x_3, x_1) + 0.8 \setminus (x_3, x_3) \}$$

Then the fuzzy relation matrix is given by

$$\mu_R = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.0 & 0.7 & 0.9 \\ 0.6 & 0.0 & 0.8 \end{bmatrix}$$

2.3. Definition:Soft Set [13]. Let U be the universal set and E be a set of parameters, $P(U)$ the power set of U . A pair (F, E) is called a soft set over U where F is a mapping given by $F : E \rightarrow P(U)$. Thus a soft set is not a set, but it is a parameterized family of subsets of U .

Example: 2.2. Let $U = \{h_1, h_2, h_3, h_4\}$ be a set of houses and $E = \{e_1, e_2, e_3\}$ be the set of parameters which stand for cheap, expensive and beautiful respectively. A soft set (F, E) is a collection of subsets of U , based on some approximate description.

From Table 1, $F(\text{cheap houses}) = \{h_2, h_4\}$, $F(\text{expensive houses}) = \{h_1, h_3\}$, $F(\text{beautiful houses}) = \{h_2, h_3\}$.

TABLE 1. Representation of houses in soft set form.

U	Cheap(e_1)	Expensive(e_2)	Beautiful(e_3)
h_1	0	1	0
h_2	1	0	1
h_3	0	1	1
h_4	1	0	0

2.4. Definition:Fuzzy Soft Matrix [10]. Let $U = \{x_1, x_2, \dots, x_m\}$ be the universal set and $E = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (F, E) . The fuzzy soft set (F, A) can be represented in the form of a matrix $C = [c_{ij}]$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, l$ ($l \leq n$) where

$$c_{ij} = \begin{cases} \mu_j(x_i), & \text{if } e_j \in A \\ 0, & \text{if } e_j \notin A \end{cases}$$

$\mu_j(x_i)$ represents the membership value of the element x_i in the fuzzy set $F(e_j)$

2.5. Definition:Cubic Set [22]. Let U be the universal set. By a cubic set (CS) in U we mean a structure of the form $A = \{\langle x, \mu_A(x), \lambda_A(x) \rangle / x \in U\}$ in which μ_A is an interval valued fuzzy set (IVFS) in U and λ_A is a fuzzy set (FS) in U . A CS is also denoted by $A = \langle \mu_A, \lambda_A \rangle$, and let C^U denote the collection of all CSs defined on U .

Example:2.3. Let $U = \{a, b\}$ and $\mu(x)$ and $\lambda(x)$ be an IVFS and a FS defined on U . Then the CS $\langle \mu, \lambda \rangle$ can be represented in tabular and matrix forms as

U	$\mu(x)$	$\lambda(x)$
a	$[0.5, 0.7]$	0.5
b	$[0.1, 0.3]$	0.2

and $\begin{bmatrix} \langle [0.5, 0.7], 0.5 \rangle \\ \langle [0.1, 0.3], 0.2 \rangle \end{bmatrix}$

2.6. Definition:Cubic Soft Set [15]. Let U be the universal set and E be a set of parameters. A cubic soft set (CSS) over U is defined as a pair (F, A) where F is a mapping from A to C^U and $A \subseteq E$.

$$(F, A) = \{F(e)/e \in A\} \text{ where } F(e) = \langle \mu_{F(e)}, \lambda_{F(e)} \rangle$$

Example:2.4. Let U and E be as in Example 2.2. A CSS (F, E) describes the subset of houses approximately in terms of an IVFS and a FS as tabulated below.

TABLE 2. Tabular Representation of the CSS (F, E) .

U	Cheap(e_1)	Expensive(e_2)	Beautiful(e_3)
h_1	$\langle [0.5, 0.6], 0.3 \rangle$	$\langle [0.7, 0.8], 0.6 \rangle$	$\langle [0.8, 0.9], 0.7 \rangle$
h_2	$\langle [0.3, 0.4], 0.5 \rangle$	$\langle [0.5, 0.6], 0.4 \rangle$	$\langle [0.1, 0.2], 0.3 \rangle$
h_3	$\langle [0.6, 0.8], 0.5 \rangle$	$\langle [0.4, 0.7], 0.5 \rangle$	$\langle [0.3, 0.5], 0.6 \rangle$
h_4	$\langle [0.7, 0.8], 0.5 \rangle$	$\langle [0.3, 0.6], 0.7 \rangle$	$\langle [0.5, 0.8], 0.9 \rangle$

2.7. Definition: Cubic Soft Matrix [6, 7, 8, 9]. Let $U = \{x_1, x_2, \dots, x_m\}$ be the universal set and $E = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. Let $A \subseteq E$. Then the CSS (F, A) can be represented in matrix form as

$$C = [c_{ij}] = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1l} \\ c_{21} & c_{22} & \cdots & c_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{ml} \end{bmatrix}$$

where $[c_{ij}] = \langle \mu_{ij}, \lambda_{ij} \rangle = \langle [\mu_{ij}^L, \mu_{ij}^U], \lambda_{ij} \rangle$; $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, l$, $l \leq n$. C is an $m \times l$ cubic matrix.

Example: 2.5. The CSM for the Example 2.4 is given below

$$C = \begin{bmatrix} \langle [0.5, 0.6], 0.3 \rangle & \langle [0.7, 0.8], 0.6 \rangle & \langle [0.8, 0.9], 0.7 \rangle \\ \langle [0.3, 0.4], 0.5 \rangle & \langle [0.5, 0.6], 0.4 \rangle & \langle [0.1, 0.2], 0.3 \rangle \\ \langle [0.6, 0.8], 0.5 \rangle & \langle [0.4, 0.7], 0.5 \rangle & \langle [0.3, 0.5], 0.6 \rangle \\ \langle [0.7, 0.8], 0.5 \rangle & \langle [0.3, 0.6], 0.7 \rangle & \langle [0.5, 0.8], 0.9 \rangle \end{bmatrix}$$

3. ‘OR’, ‘AND’, Complement and weight operations on cubic matrices

In this section some operations on CSMs are defined with illustrative examples.

3.1. Definition: ‘OR’ Operation. Let $A = [a_{ij}] = \langle [\mu_{ij}^L, \mu_{ij}^U], \lambda_{ij} \rangle$ and $B = [b_{ij}] = \langle [\gamma_{ij}^L, \gamma_{ij}^U], \alpha_{ij} \rangle$ be any two elements in $CSM_{m \times n}$. The ‘OR’ operation between A and B is defined as $A \vee B = [c_{ij}]_{m \times n} = \langle [\max(\mu_{ij}^L, \gamma_{ij}^L), \max(\mu_{ij}^U, \gamma_{ij}^U)], \max(\lambda_{ij}, \alpha_{ij}) \rangle$

Example: 3.1. Consider

$$A = \begin{bmatrix} \langle [0.7, 0.9], 0.6 \rangle & \langle [0.4, 0.8], 0.1 \rangle \\ \langle [0.1, 0.6], 0.6 \rangle & \langle [0.1, 0.9], 0.2 \rangle \end{bmatrix}$$

$$B = \begin{bmatrix} \langle [0.8, 0.9], 0.5 \rangle & \langle [0.6, 0.8], 0.9 \rangle \\ \langle [0.2, 0.5], 0.8 \rangle & \langle [0.5, 0.7], 0.4 \rangle \end{bmatrix}$$

be two CSMs then ‘OR’ Operation between A and B is given by

$$A \vee B = C = \begin{bmatrix} \langle [0.8, 0.9], 0.6 \rangle & \langle [0.6, 0.8], 0.9 \rangle \\ \langle [0.2, 0.6], 0.8 \rangle & \langle [0.5, 0.9], 0.4 \rangle \end{bmatrix}$$

3.2. Definition: ‘AND’ Operation. Let A and B be as in Definition 3.1. Then the ‘AND’ operation between A and B is defined as $A \wedge B = [d_{ij}]_{m \times n}$ where $[d_{ij}]_{m \times n} = \langle [\min(\mu_{ij}^L, \gamma_{ij}^L), \min(\mu_{ij}^U, \gamma_{ij}^U)], \min(\lambda_{ij}, \alpha_{ij}) \rangle$

Example: 3.2. For the matrices A and B defined in Example 3.1 $A \wedge B$ is given by

$$A \wedge B = \begin{bmatrix} \langle [0.7, 0.9], 0.5 \rangle & \langle [0.4, 0.8], 0.1 \rangle \\ \langle [0.1, 0.5], 0.6 \rangle & \langle [0.1, 0.7], 0.2 \rangle \end{bmatrix}$$

3.3. Definition: Complement. Let $A = [a_{ij}] = \langle [\mu_{ij}^L, \mu_{ij}^U], \lambda_{ij} \rangle$ be any $CSM_{m \times n}$, then A^c , the CSM complement of A is defined as $A^c = [e_{ij}]$ where $e_{ij} = \langle [1 - \mu_{ij}^U, 1 - \mu_{ij}^L], 1 - \lambda_{ij} \rangle$

Example:3.3. Let

$$A = \begin{bmatrix} \langle [0.7, 0.9], 0.5 \rangle & \langle [0.4, 0.8], 0.1 \rangle \\ \langle [0.1, 0.6], 0.4 \rangle & \langle [0.2, 0.9], 0.2 \rangle \end{bmatrix}$$

then

$$A^c = \begin{bmatrix} \langle [0.1, 0.3], 0.5 \rangle & \langle [0.2, 0.6], 0.9 \rangle \\ \langle [0.4, 0.9], 0.4 \rangle & \langle [0.1, 0.8], 0.8 \rangle \end{bmatrix}$$

3.4. Definition:Weighted Fuzzy Vector (WFFV). Let Υ be the collection of all fuzzy membership values and $F = (f_1, f_2, \dots, f_n)$ be a fuzzy vector with components from Υ . Let $W = (w_1, w_2, \dots, w_n)$ be a weight vector associated with F . w_i can be considered as the significance or importance attached to f_i ; $i = 1, 2, \dots, n$ with $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$. Then the WFFV corresponding to F and W denoted by WF is defined as $WF = (w_1 f_1, w_2 f_2, \dots, w_n f_n)$.

Example:3.4. Let $F = (f_1, f_2, f_3, f_4) = (0.6, 0.4, 0.3, 0.5)$ and $W = (w_1, w_2, w_3, w_4) = (0.3, 0.2, 0.4, 0.1)$. Then $WF = (0.18, 0.08, 0.12, 0.05)$

3.5. Definition:Weighted Interval Valued Fuzzy Vector (WIVFV). Let I be the collection of all interval valued fuzzy membership values and $G = (g_1, g_2, \dots, g_n)$ be an interval valued fuzzy vector with components from I . Thus the components of G are $G = ([g_1^L, g_1^U], [g_2^L, g_2^U], \dots, [g_n^L, g_n^U])$. Let $W = (w_1, w_2, \dots, w_n)$ be a weight vector associated with G . w_i can be considered as the significance attached to g_i ; $i = 1, 2, \dots, n$, $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$. Then the WIVFV corresponding to G and W denoted by WG is defined as $WG = (w_1 g_1, w_2 g_2, \dots, w_n g_n) = ([w_1 g_1^L, w_1 g_1^U], [w_2 g_2^L, w_2 g_2^U], \dots, [w_n g_n^L, w_n g_n^U])$

Example:3.5. Let $G = ([0.3, 0.6], [0.6, 0.7], [0.1, 0.5], [0.2, 0.3])$ and $w = (0.1, 0.4, 0.2, 0.3)$ Then $WG = ([0.03, 0.06], [0.24, 0.28], [0.02, 0.10], [0.06, 0.09])$

3.6. Definition:Weighted Cubic Vector (WCV). Let CV denote the collection of all cubic vectors and $H = (h_1, h_2, \dots, h_n)$ be a cubic vector with components from CV . Then $h_i = \langle [\mu_i^L, \mu_i^U], \lambda_i \rangle$ for $i = 1, 2, 3, \dots, n$. Let $W_1 = (w_{11}, w_{12}, \dots, w_{1n})$ and $W_2 = (w_{21}, w_{22}, \dots, w_{2n})$ be the weight vectors associated with the interval valued fuzzy vector $([\mu_1^L, \mu_1^U], [\mu_2^L, \mu_2^U], \dots, [\mu_n^L, \mu_n^U])$ and the fuzzy vector $(\lambda_1, \lambda_2, \dots, \lambda_n)$ respectively. Then the WCV corresponding to H , W_1 and W_2 denoted by $W_1 W_2 H$ is defined as

$$\begin{aligned} W_1 W_2 H &= \left(\langle w_{11} [\mu_1^L, \mu_1^U], w_{21} \lambda_1 \rangle, \langle w_{12} [\mu_2^L, \mu_2^U], w_{22} \lambda_2 \rangle, \dots, \langle w_{1n} [\mu_n^L, \mu_n^U], w_{2n} \lambda_n \rangle \right) \\ &= \left(\langle [w_{11} \mu_1^L, w_{11} \mu_1^U], w_{21} \lambda_1 \rangle, \langle [w_{12} \mu_2^L, w_{12} \mu_2^U], w_{22} \lambda_2 \rangle, \right. \\ &\quad \left. \dots, \langle [w_{1n} \mu_n^L, w_{1n} \mu_n^U], w_{2n} \lambda_n \rangle \right) \end{aligned}$$

Example:3.6. Let $H = (\langle [0.3, 0.6], 0.6 \rangle, \langle [0.6, 0.7], 0.4 \rangle, \langle [0.1, 0.5], 0.3 \rangle, \langle [0.2, 0.3], 0.5 \rangle)$ and $W_1 = (0.3, 0.2, 0.2, 0.3)$ and $W_2 = (0.1, 0.4, 0.2, 0.3)$ be the weight vectors then

$$W_1 W_2 H = (\langle [0.09, 0.18], 0.06 \rangle, \langle [0.12, 0.14], 0.16 \rangle, \langle [0.02, 0.10], 0.06 \rangle, \langle [0.06, 0.09], 0.15 \rangle)$$

3.7. Definition: Construction of Cubic Matrices. Consider the following situation. There are m objects whose properties are to be evaluated by two different expert groups consisting of n members each. Each of the first group of experts examine every object individually and present their conclusions in the form of fuzzy membership values. These results can be presented as a fuzzy matrix of order $m \times n$

$$F_{ij} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{m1} & \lambda_{m2} & \cdots & \lambda_{mn} \end{bmatrix}$$

The second group of experts present their results in the form of an interval valued fuzzy set. In this case the results can be presented as an $m \times n$ interval valued fuzzy matrix.

$$G_{ij} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix} = \begin{bmatrix} [\mu_{11}^L, \mu_{11}^U] & [\mu_{12}^L, \mu_{12}^U] & \cdots & [\mu_{1n}^L, \mu_{1n}^U] \\ [\mu_{21}^L, \mu_{21}^U] & [\mu_{22}^L, \mu_{22}^U] & \cdots & [\mu_{2n}^L, \mu_{2n}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\mu_{m1}^L, \mu_{m1}^U] & [\mu_{m2}^L, \mu_{m2}^U] & \cdots & [\mu_{mn}^L, \mu_{mn}^U] \end{bmatrix}$$

Both the results can be combined to get the $CSM_{m \times n}$ as

$$C_{ij} = \langle G_{ij}, F_{ij} \rangle = \begin{bmatrix} \langle \mu_{11}, \lambda_{11} \rangle & \langle \mu_{12}, \lambda_{12} \rangle & \cdots & \langle \mu_{1n}, \lambda_{1n} \rangle \\ \langle \mu_{21}, \lambda_{21} \rangle & \langle \mu_{22}, \lambda_{22} \rangle & \cdots & \langle \mu_{2n}, \lambda_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \mu_{m1}, \lambda_{m1} \rangle & \langle \mu_{m2}, \lambda_{m2} \rangle & \cdots & \langle \mu_{mn}, \lambda_{mn} \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle [\mu_{11}^L, \mu_{11}^U], \lambda_{11} \rangle & \langle [\mu_{12}^L, \mu_{12}^U], \lambda_{12} \rangle & \cdots & \langle [\mu_{1n}^L, \mu_{1n}^U], \lambda_{1n} \rangle \\ \langle [\mu_{21}^L, \mu_{21}^U], \lambda_{21} \rangle & \langle [\mu_{22}^L, \mu_{22}^U], \lambda_{22} \rangle & \cdots & \langle [\mu_{2n}^L, \mu_{2n}^U], \lambda_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle [\mu_{m1}^L, \mu_{m1}^U], \lambda_{m1} \rangle & \langle [\mu_{m2}^L, \mu_{m2}^U], \lambda_{m2} \rangle & \cdots & \langle [\mu_{mn}^L, \mu_{mn}^U], \lambda_{mn} \rangle \end{bmatrix}$$

4. CSM Theory in Decision Making

In this section, we define the concepts of the value function, score function and total score for a CSM. Later these notions will be used in the MCDM process.

4.1. Definition: Value Function. The value function of a cubic matrix helps to transform the elements of a cubic matrix into real numbers to bring out the significance or importance of these fuzzy membership values. Each element of a cubic matrix is a combination of a closed subinterval of $[0, 1]$ and a real number from $[0, 1]$. As such they do not convey any useful meaning about the object. It is necessary to integrate the interval-valued membership value and the fuzzy membership value into a single real number to arrive at a numerical value that can be viewed as a measure of that property.

Let $A = [a_{ij}] = \langle [\mu_{ij}^L, \mu_{ij}^U], \lambda_{ij} \rangle$. Then the value function for the element a_{ij} is defined as $v(a_{ij}) = v_{ij} = \frac{(\mu_{ij}^L + \mu_{ij}^U)}{2} + \lambda_{ij}$ for all i, j . Thus the value function for the CSM, $A = [a_{ij}]$ is given by $V_F(A) = \left[\frac{(\mu_{ij}^L + \mu_{ij}^U)}{2} + \lambda_{ij} \right] = [v_{ij}]$. $V_F(A)$ is also an $m \times n$ matrix, having the same dimension as A and has non-negative entries.

Example:4.1. For the matrix A and B defined in Example 3.1

$$V_F(A) = \begin{bmatrix} 1.4 & 0.7 \\ 0.95 & 0.7 \end{bmatrix}, V_F(B) = \begin{bmatrix} 1.35 & 1.6 \\ 1.15 & 1.0 \end{bmatrix}$$

4.2. Definition:Score Function and Total Score. The score function is a tool to consolidate and compare the outputs generated by the value function. Each element of a row of the value function matrix of a CSM represents the unified numerical value associated with that object in respect of a particular parameter or property as assessed by the experts. Thus the sum of all the values of a row gives the total score of the object, which can be considered as a measure to facilitate the choice or selection of that object based on all the parameters or properties under consideration. Thus the total score of (S_i) of the i^{th} object is defined as the sum of all score values of the i^{th} row of the value matrix. i.e $(S_i) = \sum_{j=1}^n v_{ij}; i = 1, 2, \dots, m$.

Example:4.2. For the matrices A and B considered in Example 3.1 and 4.1, the total score for the two rows of A are $S_1 = 2.1$ and $S_2 = 1.65$. Similarly for B , $S_1 = 2.95$ and $S_2 = 2.15$.

5. Application of CSMs in multi-criteria recruitment process

In this section, we present an MCDM problem dealing with the selection of a candidate suitable for a specific job in a company. We explain the use of CSMs in the selection process and the efficiency's evaluation of candidates using the definition of value and score functions. We present a procedure for ranking candidates based on their efficiency. We develop an algorithm for the selection process and present a flowchart to show the steps of the algorithm.

5.1. Statement of the Multi-criteria Recruitment Problem. A company wishes to appoint a suitable candidate for a specific position in its organization. We denote the set of short listed candidates by $C = (c_1, c_2, \dots, c_m)$. It has been decided to make the selection based on a ranking of the candidates by evaluating their efficiency in two independent rounds, namely Technical round and HR round. Let $E = (e_1, e_2, \dots, e_n)$ be the set of criteria based on which the candidates are to be evaluated. Assume that the teams for technical as well as HR rounds consist of an equal number of experts say $(t_1, t_2, \dots, t_l)$ and $(h_1, h_2, \dots, h_l)$. Each member of the technical expert team evaluates each candidate for their ability based on each of the n criteria and present the results in the form of an interval valued fuzzy matrix of order $m \times n$. Likewise, each member of the expert team of the HR round also evaluate every candidate with respect to each of the n criteria and present the results as a fuzzy matrix of order $m \times n$. Assume that weight-ages are assigned to each criteria for both rounds of evaluation. Let $W_1 = (\alpha_1, \alpha_2, \dots, \alpha_n)$ be the weight vector assigned to the n criteria in the technical round and $W_2 = (p_1, p_2, \dots, p_n)$ be the weight vector assigned to the n criterion in the

HR round. It is clear that $\alpha_j \geq 0, p_j \geq 0$ for $j = 1, 2, \dots, n$ and $\sum_{j=1}^n \alpha_j = \sum_{j=1}^n p_j = 1$. Combining the appropriate interval-valued fuzzy matrix with the corresponding fuzzy matrix, we form l cubic matrices, one for each pair of the expert team consisting of one member from each of the two groups. Thus the entire data can be presented in the form of l cubic matrices $A^1, A^2, \dots, A^l$. The problem is to rank the candidates based on their efficiency with appropriate criteria weight-age and to select the candidate best suited for the position.

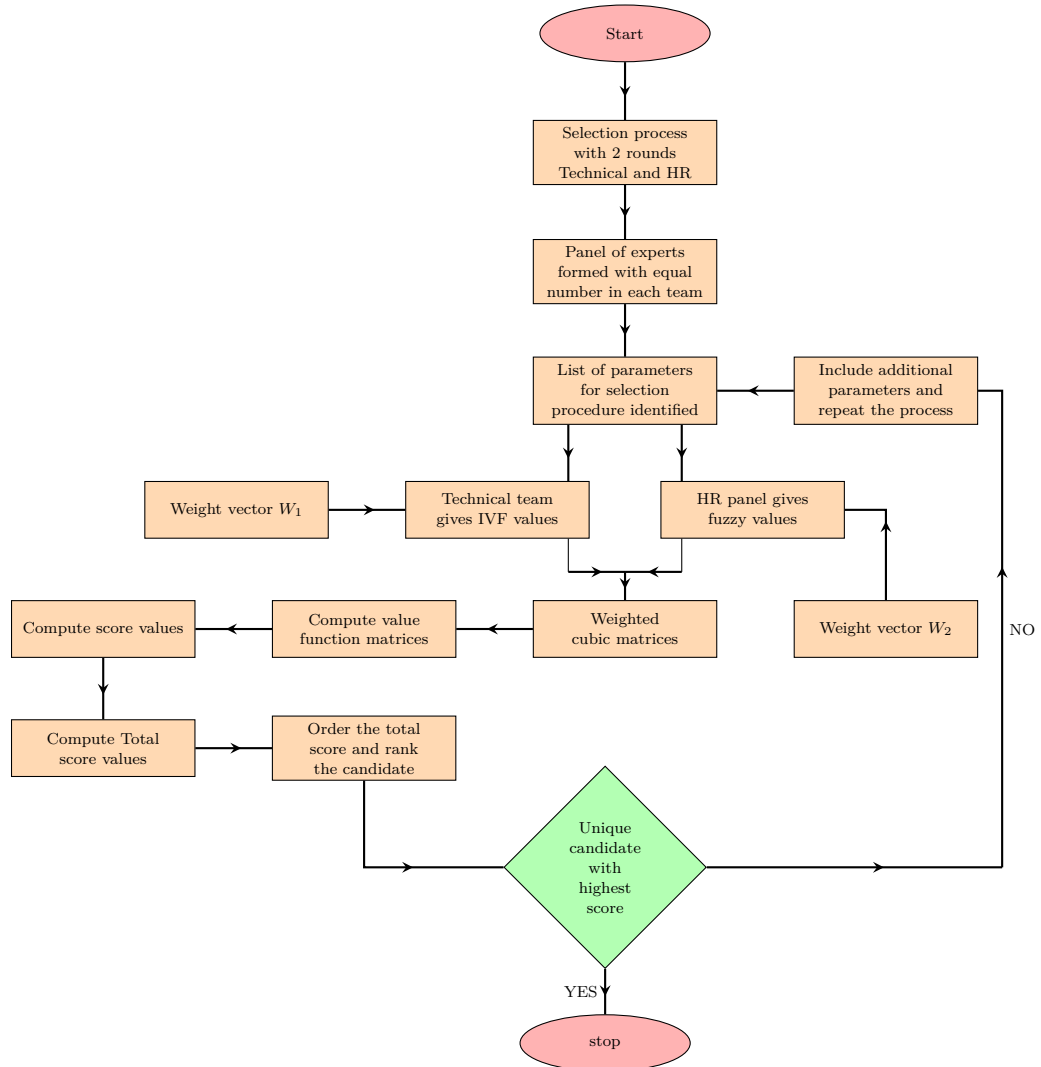
5.2. Methodology for ranking the candidates. Two groups of experts $(t_1, t_2, \dots, t_l)$ and $(h_1, h_2, \dots, h_l)$ are paired so that the pairs $(t_1, h_1), (t_2, h_2), \dots, (t_l, h_l)$ are formed. It is assumed that the experts evaluate the candidates independently without any bias. The data collected from each pair are combined using the method explained in Section 3.7 to form l cubic matrices. Using Definition 3.6 and weight vectors W_1 and W_2 , the WCMs are calculated. The resultant WCMs are denoted by $B^1, B^2, \dots, B^l$. i.e $B^k = W_1 W_2 A^k = [b_{ij}^k] = \langle \mu_{ij}^k, \lambda_{ij}^k \rangle$ where $\mu_{ij}^k = [\mu_{ij}^{kL}, \mu_{ij}^{kU}]$, $k = 1, 2, \dots, l$. For each of the WCM B^k , construct the value function matrix as $V_F [B^k] = [\nu_{ij}^k] = \left[\frac{(\mu_{ij}^{kL} + \mu_{ij}^{kU})}{2} + \lambda_{ij}^k \right]$.

The score function for the i^{th} candidate as evaluated by the k^{th} pair of experts (t_k, h_k) is calculated by adding the values of the i^{th} row of the value function matrix corresponding to the pair (t_k, h_k) , i.e., the i^{th} row of the WCM B^k . Let us denote this sum by the symbol S_i^k . The total score TS_i for the i^{th} candidate is obtained by summing S_i^k over k . i.e., The total score for the i^{th} candidate $TS_i = \sum_{k=1}^l S_i^k = S_i^1 + S_i^2 + \dots + S_i^l$. The total score is evaluated for all the candidates $i = 1, 2, \dots, m$. By arranging the total score values in descending order a complete ranking of the candidates can be achieved. The candidate with the highest total score value is the best suited one for the position. In case when there is more than one candidate with equal highest total score value, the entire procedure may be repeated by adding one or more criteria, until a single candidate with the highest total score is identified.

5.3. Algorithm for Recruitment Process using Cubic soft Matrices.

- Step 1:** Short list the candidates.
 - Step 2:** Identify the set of criteria for selection.
 - Step 3:** Form two teams of expert of equal member size.
 - Step 4:** Collect data from each member of the two teams and choose the weight vectors - Form the pairs of experts.
 - Step 5:** Form the weighted cubic matrices, one for each pair of experts.
 - Step 6:** Evaluate the value function matrix for each weighted cubic matrix and the score value for each candidate.
 - Step 7:** Find the total score for each candidate and rank based on total score value. The candidate with highest total score is the best candidate for the job.
- If two or more candidates exist with equal highest total score, go to step 4 and repeat the process with added criteria list.

5.4. Flowchart.



6. Case Studies

In this section, we present two case studies to illustrate the working of the algorithm. In 6.1 we present an example where the ranking of the candidates is unique and the selection is done based on a subset of the criteria set. In 6.2 an example is given where the initially selected set of criteria do not provide unique ranking and there is more than one candidate with an equal highest total score. The addition of another criterion yields a clear ranking and the selection is performed by repeating some of the steps with enlarged criteria set.

6.1. Case Study I. A company is in the process of appointing a suitable candidate for a specific position in its organization.

1. Let $C = (c_1, c_2, \dots, c_{10})$ be the set of short listed candidates. The selection process is based on two independent rounds, namely Technical round and HR round.

2. Let $E = (e_1, e_2, e_3, e_4)$ be the set of parameters which form the criteria for selection. Here, e_1 = domain knowledge, e_2 = communication skill, e_3 = computer skill and e_4 = previous experience.

3. Let (t_1, t_2, t_3) and (h_1, h_2, h_3) be the set of experts in the technical group and HR group respectively.

The three-member technical team and the three-member HR team evaluate each candidate individually based only on the first three criteria $A = (e_1, e_2, e_3) \subset E$ and present the results in the form of matrices as

$$t_1 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.37, 0.60] \\ [0.57, 0.65] \\ [0.40, 0.70] \\ [0.56, 0.76] \\ [0.56, 0.69] \\ [0.56, 0.67] \\ [0.74, 0.85] \\ [0.65, 0.78] \\ [0.56, 0.75] \\ [0.56, 0.78] \end{bmatrix} & \begin{bmatrix} [0.27, 0.65] \\ [0.55, 0.60] \\ [0.50, 0.70] \\ [0.45, 0.57] \\ [0.56, 0.78] \\ [0.56, 0.67] \\ [0.72, 0.82] \\ [0.68, 0.78] \\ [0.45, 0.69] \\ [0.58, 0.67] \end{bmatrix} & \begin{bmatrix} [0.47, 0.67] \\ [0.70, 0.80] \\ [0.55, 0.70] \\ [0.74, 0.84] \\ [0.55, 0.68] \\ [0.67, 0.76] \\ [0.56, 0.77] \\ [0.68, 0.76] \\ [0.45, 0.54] \\ [0.49, 0.67] \end{bmatrix} \end{matrix}$$

$$t_2 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.24, 0.35] \\ [0.52, 0.68] \\ [0.47, 0.67] \\ [0.66, 0.78] \\ [0.67, 0.77] \\ [0.40, 0.50] \\ [0.35, 0.46] \\ [0.67, 0.77] \\ [0.38, 0.52] \\ [0.75, 0.83] \end{bmatrix} & \begin{bmatrix} [0.57, 0.70] \\ [0.32, 0.47] \\ [0.75, 0.83] \\ [0.46, 0.67] \\ [0.45, 0.67] \\ [0.57, 0.65] \\ [0.45, 0.65] \\ [0.55, 0.69] \\ [0.49, 0.72] \\ [0.78, 0.89] \end{bmatrix} & \begin{bmatrix} [0.58, 0.79] \\ [0.46, 0.56] \\ [0.45, 0.46] \\ [0.72, 0.92] \\ [0.45, 0.67] \\ [0.72, 0.82] \\ [0.57, 0.65] \\ [0.68, 0.83] \\ [0.57, 0.68] \\ [0.58, 0.67] \end{bmatrix} \end{matrix}$$

$$t_3 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.20, 0.40] \\ [0.68, 0.70] \\ [0.20, 0.40] \\ [0.68, 0.78] \\ [0.45, 0.56] \\ [0.52, 0.68] \\ [0.45, 0.54] \\ [0.73, 0.89] \\ [0.58, 0.79] \\ [0.46, 0.56] \end{bmatrix} & \begin{bmatrix} [0.60, 0.80] \\ [0.57, 0.78] \\ [0.70, 0.80] \\ [0.69, 0.77] \\ [0.68, 0.78] \\ [0.20, 0.40] \\ [0.47, 0.67] \\ [0.66, 0.78] \\ [0.67, 0.79] \\ [0.43, 0.78] \end{bmatrix} & \begin{bmatrix} [0.70, 0.80] \\ [0.76, 0.88] \\ [0.48, 0.68] \\ [0.84, 0.92] \\ [0.77, 0.88] \\ [0.55, 0.70] \\ [0.46, 0.57] \\ [0.67, 0.74] \\ [0.65, 0.78] \\ [0.41, 0.56] \end{bmatrix} \end{matrix}$$

$$h_1 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.50 & 0.90 & 0.87 \\ 0.60 & 0.70 & 0.67 \\ 0.40 & 0.70 & 0.90 \\ 0.67 & 0.74 & 0.87 \\ 0.62 & 0.65 & 0.77 \\ 0.78 & 0.67 & 0.87 \\ 0.54 & 0.62 & 0.87 \\ 0.78 & 0.67 & 0.62 \\ 0.66 & 0.67 & 0.62 \\ 0.67 & 0.62 & 0.45 \end{bmatrix} \end{matrix} \quad h_2 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.45 & 0.75 & 0.46 \\ 0.46 & 0.69 & 0.63 \\ 0.43 & 0.85 & 0.26 \\ 0.59 & 0.55 & 0.78 \\ 0.72 & 0.54 & 0.43 \\ 0.70 & 0.58 & 0.61 \\ 0.36 & 0.90 & 0.87 \\ 0.75 & 0.77 & 0.73 \\ 0.69 & 0.87 & 0.46 \\ 0.85 & 0.75 & 0.72 \end{bmatrix} \end{matrix}$$

$$h_3 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.50 & 0.90 & 0.75 \\ 0.60 & 0.60 & 0.46 \\ 0.50 & 0.90 & 0.56 \\ 0.84 & 0.67 & 0.69 \\ 0.46 & 0.83 & 0.59 \\ 0.56 & 0.50 & 0.66 \\ 0.62 & 0.69 & 0.37 \\ 0.75 & 0.56 & 0.57 \\ 0.46 & 0.84 & 0.52 \\ 0.63 & 0.81 & 0.62 \end{bmatrix} \end{matrix}$$

Using Definition 3.7, and combining $(t_1, h_1), (t_2, h_2), (t_3, h_3)$ we get the cubic matrices A^1, A^2, A^3 as

$$A^1 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.37, 0.60], 0.50 \rangle & \langle [0.27, 0.65], 0.90 \rangle & \langle [0.47, 0.67], 0.87 \rangle \\ \langle [0.57, 0.65], 0.60 \rangle & \langle [0.55, 0.60], 0.70 \rangle & \langle [0.70, 0.80], 0.67 \rangle \\ \langle [0.40, 0.70], 0.40 \rangle & \langle [0.50, 0.70], 0.70 \rangle & \langle [0.55, 0.70], 0.90 \rangle \\ \langle [0.56, 0.76], 0.67 \rangle & \langle [0.45, 0.57], 0.74 \rangle & \langle [0.74, 0.84], 0.87 \rangle \\ \langle [0.56, 0.69], 0.62 \rangle & \langle [0.56, 0.78], 0.65 \rangle & \langle [0.55, 0.68], 0.77 \rangle \\ \langle [0.56, 0.67], 0.78 \rangle & \langle [0.56, 0.67], 0.67 \rangle & \langle [0.67, 0.76], 0.87 \rangle \\ \langle [0.74, 0.85], 0.54 \rangle & \langle [0.72, 0.82], 0.62 \rangle & \langle [0.56, 0.77], 0.87 \rangle \\ \langle [0.65, 0.78], 0.78 \rangle & \langle [0.68, 0.78], 0.67 \rangle & \langle [0.68, 0.76], 0.62 \rangle \\ \langle [0.56, 0.75], 0.66 \rangle & \langle [0.45, 0.69], 0.67 \rangle & \langle [0.45, 0.54], 0.62 \rangle \\ \langle [0.56, 0.78], 0.67 \rangle & \langle [0.58, 0.67], 0.62 \rangle & \langle [0.49, 0.67], 0.45 \rangle \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.24, 0.35], 0.45 \rangle & \langle [0.57, 0.70], 0.75 \rangle & \langle [0.58, 0.79], 0.46 \rangle \\ \langle [0.52, 0.68], 0.46 \rangle & \langle [0.32, 0.47], 0.69 \rangle & \langle [0.46, 0.56], 0.63 \rangle \\ \langle [0.47, 0.67], 0.43 \rangle & \langle [0.75, 0.83], 0.85 \rangle & \langle [0.45, 0.46], 0.26 \rangle \\ \langle [0.66, 0.78], 0.59 \rangle & \langle [0.46, 0.67], 0.55 \rangle & \langle [0.72, 0.92], 0.78 \rangle \\ \langle [0.67, 0.77], 0.72 \rangle & \langle [0.45, 0.67], 0.54 \rangle & \langle [0.45, 0.67], 0.43 \rangle \\ \langle [0.40, 0.50], 0.70 \rangle & \langle [0.57, 0.65], 0.58 \rangle & \langle [0.72, 0.82], 0.61 \rangle \\ \langle [0.35, 0.46], 0.36 \rangle & \langle [0.45, 0.65], 0.90 \rangle & \langle [0.57, 0.65], 0.87 \rangle \\ \langle [0.67, 0.77], 0.75 \rangle & \langle [0.55, 0.69], 0.77 \rangle & \langle [0.68, 0.83], 0.73 \rangle \\ \langle [0.38, 0.52], 0.69 \rangle & \langle [0.49, 0.72], 0.87 \rangle & \langle [0.57, 0.68], 0.46 \rangle \\ \langle [0.75, 0.83], 0.85 \rangle & \langle [0.78, 0.89], 0.75 \rangle & \langle [0.58, 0.67], 0.72 \rangle \end{bmatrix} \end{matrix}$$

$$A^3 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.20, 0.40], 0.50 \rangle & \langle [0.60, 0.80], 0.90 \rangle & \langle [0.70, 0.80], 0.75 \rangle \\ \langle [0.68, 0.70], 0.60 \rangle & \langle [0.57, 0.78], 0.60 \rangle & \langle [0.76, 0.88], 0.46 \rangle \\ \langle [0.20, 0.40], 0.50 \rangle & \langle [0.70, 0.80], 0.90 \rangle & \langle [0.48, 0.68], 0.56 \rangle \\ \langle [0.68, 0.78], 0.84 \rangle & \langle [0.69, 0.77], 0.67 \rangle & \langle [0.84, 0.92], 0.69 \rangle \\ \langle [0.45, 0.56], 0.46 \rangle & \langle [0.68, 0.78], 0.83 \rangle & \langle [0.77, 0.88], 0.59 \rangle \\ \langle [0.52, 0.68], 0.56 \rangle & \langle [0.20, 0.40], 0.50 \rangle & \langle [0.55, 0.70], 0.66 \rangle \\ \langle [0.45, 0.54], 0.62 \rangle & \langle [0.47, 0.67], 0.69 \rangle & \langle [0.46, 0.57], 0.37 \rangle \\ \langle [0.73, 0.89], 0.75 \rangle & \langle [0.66, 0.78], 0.56 \rangle & \langle [0.67, 0.74], 0.57 \rangle \\ \langle [0.58, 0.79], 0.46 \rangle & \langle [0.67, 0.79], 0.84 \rangle & \langle [0.65, 0.78], 0.52 \rangle \\ \langle [0.46, 0.56], 0.63 \rangle & \langle [0.43, 0.78], 0.81 \rangle & \langle [0.41, 0.56], 0.62 \rangle \end{bmatrix} \end{matrix}$$

4. Let $W_1 = (0.3, 0.3, 0.4)$ be the weight vector assigned by the management to the criteria in technical round and $W_2 = (0.2, 0.4, 0.4)$ be the weight vector assigned to these parameters in the HR round.

5. Applying Definition 3.6 the resultant WCMs B^1, B^2, B^3 are obtained as

$$B^1 = \begin{array}{c} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{array} \begin{array}{ccc} e_1 & e_2 & e_3 \\ \left[\begin{array}{ccc} \langle [0.11, 0.18], 0.10 \rangle & \langle [0.08, 0.20], 0.36 \rangle & \langle [0.19, 0.27], 0.35 \rangle \\ \langle [0.17, 0.20], 0.12 \rangle & \langle [0.17, 0.18], 0.28 \rangle & \langle [0.28, 0.32], 0.27 \rangle \\ \langle [0.12, 0.21], 0.08 \rangle & \langle [0.15, 0.21], 0.28 \rangle & \langle [0.22, 0.28], 0.36 \rangle \\ \langle [0.17, 0.23], 0.20 \rangle & \langle [0.14, 0.17], 0.22 \rangle & \langle [0.22, 0.25], 0.26 \rangle \\ \langle [0.17, 0.21], 0.12 \rangle & \langle [0.17, 0.23], 0.26 \rangle & \langle [0.22, 0.27], 0.31 \rangle \\ \langle [0.17, 0.20], 0.16 \rangle & \langle [0.17, 0.20], 0.27 \rangle & \langle [0.27, 0.30], 0.35 \rangle \\ \langle [0.22, 0.26], 0.11 \rangle & \langle [0.22, 0.25], 0.25 \rangle & \langle [0.22, 0.31], 0.35 \rangle \\ \langle [0.20, 0.23], 0.23 \rangle & \langle [0.20, 0.23], 0.20 \rangle & \langle [0.20, 0.23], 0.19 \rangle \\ \langle [0.17, 0.23], 0.23 \rangle & \langle [0.14, 0.21], 0.27 \rangle & \langle [0.18, 0.22], 0.25 \rangle \\ \langle [0.17, 0.23], 0.13 \rangle & \langle [0.17, 0.20], 0.25 \rangle & \langle [0.20, 0.27], 0.18 \rangle \end{array} \right] \end{array}$$

$$B^2 = \begin{array}{c} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{array} \begin{array}{ccc} e_1 & e_2 & e_3 \\ \left[\begin{array}{ccc} \langle [0.07, 0.10], 0.09 \rangle & \langle [0.17, 0.21], 0.30 \rangle & \langle [0.23, 0.31], 0.18 \rangle \\ \langle [0.16, 0.20], 0.09 \rangle & \langle [0.10, 0.14], 0.28 \rangle & \langle [0.18, 0.22], 0.25 \rangle \\ \langle [0.14, 0.20], 0.09 \rangle & \langle [0.22, 0.25], 0.34 \rangle & \langle [0.18, 0.18], 0.10 \rangle \\ \langle [0.20, 0.24], 0.18 \rangle & \langle [0.14, 0.20], 0.17 \rangle & \langle [0.22, 0.28], 0.23 \rangle \\ \langle [0.20, 0.23], 0.14 \rangle & \langle [0.14, 0.20], 0.22 \rangle & \langle [0.18, 0.27], 0.17 \rangle \\ \langle [0.12, 0.15], 0.14 \rangle & \langle [0.17, 0.20], 0.23 \rangle & \langle [0.29, 0.33], 0.24 \rangle \\ \langle [0.11, 0.14], 0.07 \rangle & \langle [0.14, 0.20], 0.36 \rangle & \langle [0.23, 0.26], 0.35 \rangle \\ \langle [0.20, 0.23], 0.22 \rangle & \langle [0.17, 0.21], 0.23 \rangle & \langle [0.20, 0.25], 0.22 \rangle \\ \langle [0.11, 0.16], 0.14 \rangle & \langle [0.15, 0.22], 0.35 \rangle & \langle [0.23, 0.27], 0.18 \rangle \\ \langle [0.22, 0.25], 0.17 \rangle & \langle [0.23, 0.27], 0.30 \rangle & \langle [0.23, 0.27], 0.29 \rangle \end{array} \right] \end{array}$$

$$B^3 = \begin{array}{c} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{array} \begin{array}{ccc} e_1 & e_2 & e_3 \\ \left[\begin{array}{ccc} \langle [0.06, 0.12], 0.10 \rangle & \langle [0.18, 0.24], 0.36 \rangle & \langle [0.28, 0.32], 0.30 \rangle \\ \langle [0.20, 0.21], 0.12 \rangle & \langle [0.17, 0.23], 0.24 \rangle & \langle [0.30, 0.35], 0.18 \rangle \\ \langle [0.06, 0.12], 0.10 \rangle & \langle [0.21, 0.24], 0.36 \rangle & \langle [0.19, 0.27], 0.22 \rangle \\ \langle [0.20, 0.23], 0.25 \rangle & \langle [0.21, 0.23], 0.20 \rangle & \langle [0.25, 0.28], 0.21 \rangle \\ \langle [0.14, 0.17], 0.09 \rangle & \langle [0.20, 0.23], 0.33 \rangle & \langle [0.31, 0.35], 0.24 \rangle \\ \langle [0.16, 0.20], 0.11 \rangle & \langle [0.06, 0.12], 0.20 \rangle & \langle [0.22, 0.28], 0.26 \rangle \\ \langle [0.14, 0.16], 0.12 \rangle & \langle [0.14, 0.20], 0.28 \rangle & \langle [0.18, 0.23], 0.15 \rangle \\ \langle [0.22, 0.27], 0.23 \rangle & \langle [0.20, 0.24], 0.17 \rangle & \langle [0.20, 0.22], 0.17 \rangle \\ \langle [0.17, 0.24], 0.09 \rangle & \langle [0.20, 0.24], 0.34 \rangle & \langle [0.26, 0.31], 0.21 \rangle \\ \langle [0.14, 0.17], 0.13 \rangle & \langle [0.13, 0.23], 0.32 \rangle & \langle [0.16, 0.22], 0.25 \rangle \end{array} \right] \end{array}$$

6. Using Definition 4.1 the value function matrices are obtained as

$$V_F(B^1) = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.25 & 0.50 & 0.58 \\ 0.30 & 0.45 & 0.57 \\ 0.25 & 0.46 & 0.61 \\ 0.40 & 0.38 & 0.50 \\ 0.31 & 0.46 & 0.55 \\ 0.34 & 0.45 & 0.63 \\ 0.35 & 0.48 & 0.61 \\ 0.45 & 0.42 & 0.40 \\ 0.33 & 0.44 & 0.45 \\ 0.34 & 0.44 & 0.41 \end{bmatrix} \end{matrix} \quad V_F(B^2) = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.18 & 0.49 & 0.46 \\ 0.27 & 0.39 & 0.46 \\ 0.26 & 0.58 & 0.28 \\ 0.39 & 0.33 & 0.48 \\ 0.36 & 0.38 & 0.40 \\ 0.28 & 0.42 & 0.55 \\ 0.19 & 0.53 & 0.59 \\ 0.44 & 0.42 & 0.45 \\ 0.27 & 0.53 & 0.43 \\ 0.41 & 0.55 & 0.54 \end{bmatrix} \end{matrix}$$

$$V_F(B^3) = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.19 & 0.57 & 0.60 \\ 0.33 & 0.44 & 0.51 \\ 0.19 & 0.59 & 0.46 \\ 0.47 & 0.42 & 0.47 \\ 0.24 & 0.55 & 0.57 \\ 0.29 & 0.29 & 0.51 \\ 0.27 & 0.45 & 0.35 \\ 0.47 & 0.38 & 0.38 \\ 0.30 & 0.56 & 0.49 \\ 0.28 & 0.51 & 0.44 \end{bmatrix} \end{matrix}$$

Applying Definition 4.2 the score functions are calculated as

$$S_i^1 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.32 \\ 1.32 \\ 1.32 \\ 1.27 \\ 1.33 \\ 1.43 \\ 1.44 \\ 1.27 \\ 1.21 \\ 1.18 \end{bmatrix} \quad S_i^2 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.12 \\ 1.12 \\ 1.12 \\ 1.21 \\ 1.14 \\ 1.24 \\ 1.31 \\ 1.30 \\ 1.23 \\ 1.50 \end{bmatrix} \quad S_i^3 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.36 \\ 1.28 \\ 1.23 \\ 1.36 \\ 1.36 \\ 1.10 \\ 1.07 \\ 1.23 \\ 1.35 \\ 1.23 \end{bmatrix}$$

7. The total score value for each candidate is calculated and presented as

$$TS_i = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 3.80 \\ 3.73 \\ 3.66 \\ 3.84 \\ 3.83 \\ 3.76 \\ 3.82 \\ 3.81 \\ 3.79 \\ 3.90 \end{bmatrix}$$

Arranging the candidates according to their total score values we obtain the ranking of the candidates as

TABLE 3. Tabular representation of candidate's total score values.

TS_i	c_i	Score	Rank
TS_{10}	c_{10}	3.90	1
TS_4	c_4	3.84	2
TS_5	c_5	3.83	3
TS_7	c_7	3.82	4
TS_8	c_8	3.81	5
TS_1	c_1	3.80	6
TS_9	c_9	3.79	7
TS_6	c_6	3.76	8
TS_2	c_2	3.73	9
TS_3	c_3	3.66	10

From Table 3, the candidate C_{10} ranks first and he is the most suitable candidate for the position.

6.2. Case Study II. An organization wishes to select a candidate for a specific job in one of its industry.

1. Let $C = (c_1, c_2, \dots, c_{10})$ be the set of short listed candidates. The selection process is based on two independent rounds, namely Technical round and HR round.

2. Let $E = (e_1, e_2, e_3, e_4)$ be the set of parameters which form the criteria for selection. Here, e_1 = domain knowledge, e_2 = communication skill, e_3 = computer skill and e_4 = previous experience.

3. Let (t_1, t_2, t_3) and (h_1, h_2, h_3) be the set of experts in the technical group and HR group respectively. The three-member technical team and the three-member HR team evaluate each candidate individually based only on the first three criteria $A = (e_1, e_2, e_3) \subset E$ and present the results in the form of matrices as

$$t_1 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.37, 0.60] \\ [0.57, 0.65] \\ [0.40, 0.70] \\ [0.56, 0.76] \\ [0.56, 0.69] \\ [0.56, 0.67] \\ [0.74, 0.85] \\ [0.65, 0.78] \\ [0.56, 0.75] \\ [0.56, 0.78] \end{bmatrix} & \begin{bmatrix} [0.27, 0.65] \\ [0.55, 0.60] \\ [0.50, 0.70] \\ [0.45, 0.57] \\ [0.56, 0.78] \\ [0.56, 0.67] \\ [0.72, 0.82] \\ [0.68, 0.78] \\ [0.45, 0.69] \\ [0.58, 0.67] \end{bmatrix} & \begin{bmatrix} [0.47, 0.67] \\ [0.70, 0.80] \\ [0.55, 0.70] \\ [0.74, 0.84] \\ [0.55, 0.68] \\ [0.67, 0.76] \\ [0.56, 0.77] \\ [0.68, 0.76] \\ [0.45, 0.54] \\ [0.49, 0.67] \end{bmatrix} \end{matrix}$$

$$t_2 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.24, 0.35] \\ [0.52, 0.68] \\ [0.47, 0.67] \\ [0.66, 0.78] \\ [0.67, 0.77] \\ [0.40, 0.50] \\ [0.35, 0.46] \\ [0.67, 0.77] \\ [0.38, 0.52] \\ [0.75, 0.83] \end{bmatrix} & \begin{bmatrix} [0.57, 0.70] \\ [0.32, 0.47] \\ [0.75, 0.83] \\ [0.46, 0.67] \\ [0.45, 0.67] \\ [0.57, 0.65] \\ [0.45, 0.65] \\ [0.55, 0.69] \\ [0.49, 0.72] \\ [0.78, 0.89] \end{bmatrix} & \begin{bmatrix} [0.58, 0.79] \\ [0.46, 0.56] \\ [0.45, 0.46] \\ [0.72, 0.92] \\ [0.45, 0.67] \\ [0.72, 0.82] \\ [0.57, 0.65] \\ [0.68, 0.83] \\ [0.57, 0.68] \\ [0.58, 0.67] \end{bmatrix} \end{matrix}$$

$$t_3 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.20, 0.40] \\ [0.68, 0.70] \\ [0.20, 0.40] \\ [0.68, 0.78] \\ [0.45, 0.56] \\ [0.52, 0.68] \\ [0.45, 0.54] \\ [0.73, 0.89] \\ [0.58, 0.79] \\ [0.46, 0.56] \end{bmatrix} & \begin{bmatrix} [0.60, 0.80] \\ [0.57, 0.78] \\ [0.70, 0.80] \\ [0.69, 0.77] \\ [0.68, 0.78] \\ [0.20, 0.40] \\ [0.47, 0.67] \\ [0.66, 0.78] \\ [0.67, 0.79] \\ [0.43, 0.78] \end{bmatrix} & \begin{bmatrix} [0.70, 0.80] \\ [0.76, 0.88] \\ [0.48, 0.68] \\ [0.84, 0.92] \\ [0.77, 0.88] \\ [0.55, 0.70] \\ [0.46, 0.57] \\ [0.67, 0.74] \\ [0.65, 0.78] \\ [0.41, 0.56] \end{bmatrix} \end{matrix}$$

$$h_1 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.50 & 0.90 & 0.87 \\ 0.60 & 0.70 & 0.67 \\ 0.40 & 0.70 & 0.90 \\ 0.67 & 0.74 & 0.87 \\ 0.62 & 0.65 & 0.77 \\ 0.78 & 0.67 & 0.87 \\ 0.54 & 0.62 & 0.87 \\ 0.78 & 0.67 & 0.62 \\ 0.66 & 0.67 & 0.62 \\ 0.67 & 0.62 & 0.45 \end{bmatrix} \end{matrix} \quad h_2 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.45 & 0.75 & 0.46 \\ 0.46 & 0.69 & 0.63 \\ 0.43 & 0.85 & 0.26 \\ 0.59 & 0.55 & 0.78 \\ 0.72 & 0.54 & 0.43 \\ 0.70 & 0.58 & 0.61 \\ 0.36 & 0.90 & 0.87 \\ 0.75 & 0.77 & 0.73 \\ 0.69 & 0.87 & 0.46 \\ 0.85 & 0.75 & 0.72 \end{bmatrix} \end{matrix}$$

$$h_3 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.50 & 0.90 & 0.75 \\ 0.60 & 0.60 & 0.46 \\ 0.50 & 0.90 & 0.56 \\ 0.84 & 0.67 & 0.74 \\ 0.46 & 0.83 & 0.59 \\ 0.56 & 0.50 & 0.66 \\ 0.62 & 0.69 & 0.37 \\ 0.75 & 0.56 & 0.74 \\ 0.46 & 0.84 & 0.52 \\ 0.63 & 0.81 & 0.52 \end{bmatrix} \end{matrix}$$

Using Definition 3.7 and combining (t_1, h_1) , (t_2, h_2) , (t_3, h_3) we get the cubic matrices A^1, A^2, A^3 as

$$A^1 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.37, 0.60], 0.50 \rangle & \langle [0.27, 0.65], 0.90 \rangle & \langle [0.47, 0.67], 0.87 \rangle \\ \langle [0.57, 0.65], 0.60 \rangle & \langle [0.55, 0.60], 0.70 \rangle & \langle [0.70, 0.80], 0.67 \rangle \\ \langle [0.40, 0.70], 0.40 \rangle & \langle [0.50, 0.70], 0.70 \rangle & \langle [0.55, 0.70], 0.90 \rangle \\ \langle [0.56, 0.76], 0.67 \rangle & \langle [0.45, 0.57], 0.74 \rangle & \langle [0.74, 0.84], 0.87 \rangle \\ \langle [0.56, 0.69], 0.62 \rangle & \langle [0.56, 0.78], 0.65 \rangle & \langle [0.55, 0.68], 0.77 \rangle \\ \langle [0.56, 0.67], 0.78 \rangle & \langle [0.56, 0.67], 0.67 \rangle & \langle [0.67, 0.76], 0.87 \rangle \\ \langle [0.74, 0.85], 0.54 \rangle & \langle [0.72, 0.82], 0.62 \rangle & \langle [0.56, 0.77], 0.87 \rangle \\ \langle [0.65, 0.78], 0.78 \rangle & \langle [0.68, 0.78], 0.67 \rangle & \langle [0.68, 0.76], 0.62 \rangle \\ \langle [0.56, 0.75], 0.66 \rangle & \langle [0.45, 0.69], 0.67 \rangle & \langle [0.45, 0.54], 0.62 \rangle \\ \langle [0.56, 0.78], 0.67 \rangle & \langle [0.58, 0.67], 0.62 \rangle & \langle [0.49, 0.67], 0.45 \rangle \end{bmatrix} \end{matrix}$$

$$A^2 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.24, 0.35], 0.45 \rangle & \langle [0.57, 0.70], 0.75 \rangle & \langle [0.58, 0.79], 0.46 \rangle \\ \langle [0.52, 0.68], 0.46 \rangle & \langle [0.32, 0.47], 0.69 \rangle & \langle [0.46, 0.56], 0.63 \rangle \\ \langle [0.47, 0.67], 0.43 \rangle & \langle [0.75, 0.83], 0.85 \rangle & \langle [0.45, 0.46], 0.26 \rangle \\ \langle [0.66, 0.78], 0.59 \rangle & \langle [0.46, 0.67], 0.55 \rangle & \langle [0.72, 0.92], 0.78 \rangle \\ \langle [0.67, 0.77], 0.72 \rangle & \langle [0.45, 0.67], 0.54 \rangle & \langle [0.45, 0.67], 0.43 \rangle \\ \langle [0.40, 0.50], 0.70 \rangle & \langle [0.57, 0.65], 0.58 \rangle & \langle [0.72, 0.82], 0.61 \rangle \\ \langle [0.35, 0.46], 0.36 \rangle & \langle [0.45, 0.65], 0.90 \rangle & \langle [0.57, 0.65], 0.87 \rangle \\ \langle [0.67, 0.77], 0.75 \rangle & \langle [0.55, 0.69], 0.77 \rangle & \langle [0.68, 0.83], 0.73 \rangle \\ \langle [0.38, 0.52], 0.69 \rangle & \langle [0.49, 0.72], 0.87 \rangle & \langle [0.57, 0.68], 0.46 \rangle \\ \langle [0.75, 0.83], 0.85 \rangle & \langle [0.78, 0.89], 0.75 \rangle & \langle [0.58, 0.67], 0.72 \rangle \end{bmatrix} \end{matrix}$$

$$A^3 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.20, 0.40], 0.50 \rangle & \langle [0.60, 0.80], 0.90 \rangle & \langle [0.70, 0.80], 0.75 \rangle \\ \langle [0.68, 0.70], 0.60 \rangle & \langle [0.57, 0.78], 0.60 \rangle & \langle [0.76, 0.88], 0.46 \rangle \\ \langle [0.20, 0.40], 0.50 \rangle & \langle [0.70, 0.80], 0.90 \rangle & \langle [0.48, 0.68], 0.56 \rangle \\ \langle [0.68, 0.78], 0.84 \rangle & \langle [0.69, 0.77], 0.67 \rangle & \langle [0.84, 0.92], 0.74 \rangle \\ \langle [0.45, 0.56], 0.46 \rangle & \langle [0.68, 0.78], 0.83 \rangle & \langle [0.77, 0.88], 0.59 \rangle \\ \langle [0.52, 0.68], 0.56 \rangle & \langle [0.20, 0.40], 0.50 \rangle & \langle [0.55, 0.70], 0.66 \rangle \\ \langle [0.45, 0.54], 0.62 \rangle & \langle [0.47, 0.67], 0.69 \rangle & \langle [0.46, 0.57], 0.37 \rangle \\ \langle [0.73, 0.89], 0.75 \rangle & \langle [0.66, 0.78], 0.56 \rangle & \langle [0.67, 0.74], 0.74 \rangle \\ \langle [0.58, 0.79], 0.46 \rangle & \langle [0.67, 0.79], 0.84 \rangle & \langle [0.65, 0.78], 0.52 \rangle \\ \langle [0.46, 0.56], 0.63 \rangle & \langle [0.43, 0.78], 0.81 \rangle & \langle [0.41, 0.56], 0.52 \rangle \end{bmatrix} \end{matrix}$$

4. Let $W_1 = (0.3, 0.3, 0.4)$ be the weight vector assigned by the management to the criteria in technical round and $W_2 = (0.2, 0.4, 0.4)$ be the weight vector assigned to these parameters in the HR round.

5. Applying Definition 3.6 the resultant WCMs B^1, B^2, B^3 are obtained as

$$B^1 = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.11, 0.18], 0.10 \rangle & \langle [0.08, 0.20], 0.36 \rangle & \langle [0.19, 0.27], 0.35 \rangle \\ \langle [0.17, 0.20], 0.12 \rangle & \langle [0.17, 0.18], 0.28 \rangle & \langle [0.28, 0.32], 0.27 \rangle \\ \langle [0.12, 0.21], 0.08 \rangle & \langle [0.15, 0.21], 0.28 \rangle & \langle [0.22, 0.28], 0.36 \rangle \\ \langle [0.17, 0.23], 0.20 \rangle & \langle [0.14, 0.17], 0.22 \rangle & \langle [0.22, 0.25], 0.26 \rangle \\ \langle [0.17, 0.21], 0.12 \rangle & \langle [0.17, 0.23], 0.26 \rangle & \langle [0.22, 0.27], 0.31 \rangle \\ \langle [0.17, 0.20], 0.16 \rangle & \langle [0.17, 0.20], 0.27 \rangle & \langle [0.27, 0.30], 0.35 \rangle \\ \langle [0.22, 0.26], 0.11 \rangle & \langle [0.22, 0.25], 0.25 \rangle & \langle [0.22, 0.31], 0.35 \rangle \\ \langle [0.20, 0.23], 0.23 \rangle & \langle [0.20, 0.23], 0.20 \rangle & \langle [0.20, 0.23], 0.19 \rangle \\ \langle [0.17, 0.23], 0.23 \rangle & \langle [0.14, 0.21], 0.27 \rangle & \langle [0.18, 0.22], 0.25 \rangle \\ \langle [0.17, 0.23], 0.13 \rangle & \langle [0.17, 0.20], 0.25 \rangle & \langle [0.20, 0.27], 0.18 \rangle \end{bmatrix} \end{matrix}$$

$$B^2 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.07, 0.10], 0.09 \rangle \\ \langle [0.16, 0.20], 0.09 \rangle \\ \langle [0.14, 0.20], 0.09 \rangle \\ \langle [0.20, 0.24], 0.18 \rangle \\ \langle [0.20, 0.23], 0.14 \rangle \\ \langle [0.12, 0.15], 0.14 \rangle \\ \langle [0.11, 0.14], 0.07 \rangle \\ \langle [0.20, 0.23], 0.22 \rangle \\ \langle [0.11, 0.16], 0.14 \rangle \\ \langle [0.22, 0.25], 0.17 \rangle \end{bmatrix} & \begin{bmatrix} \langle [0.17, 0.21], 0.30 \rangle \\ \langle [0.10, 0.14], 0.28 \rangle \\ \langle [0.22, 0.25], 0.34 \rangle \\ \langle [0.14, 0.20], 0.17 \rangle \\ \langle [0.14, 0.20], 0.22 \rangle \\ \langle [0.17, 0.20], 0.23 \rangle \\ \langle [0.14, 0.20], 0.36 \rangle \\ \langle [0.17, 0.21], 0.23 \rangle \\ \langle [0.15, 0.22], 0.35 \rangle \\ \langle [0.23, 0.27], 0.30 \rangle \end{bmatrix} & \begin{bmatrix} \langle [0.23, 0.31], 0.18 \rangle \\ \langle [0.18, 0.22], 0.25 \rangle \\ \langle [0.18, 0.18], 0.10 \rangle \\ \langle [0.22, 0.28], 0.23 \rangle \\ \langle [0.18, 0.27], 0.17 \rangle \\ \langle [0.29, 0.33], 0.24 \rangle \\ \langle [0.23, 0.26], 0.35 \rangle \\ \langle [0.20, 0.25], 0.22 \rangle \\ \langle [0.23, 0.27], 0.18 \rangle \\ \langle [0.23, 0.27], 0.29 \rangle \end{bmatrix} \end{matrix}$$

$$B^3 = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} \langle [0.06, 0.12], 0.10 \rangle \\ \langle [0.20, 0.21], 0.12 \rangle \\ \langle [0.06, 0.12], 0.10 \rangle \\ \langle [0.20, 0.23], 0.25 \rangle \\ \langle [0.14, 0.17], 0.09 \rangle \\ \langle [0.16, 0.20], 0.11 \rangle \\ \langle [0.14, 0.16], 0.12 \rangle \\ \langle [0.22, 0.27], 0.23 \rangle \\ \langle [0.17, 0.24], 0.09 \rangle \\ \langle [0.14, 0.17], 0.13 \rangle \end{bmatrix} & \begin{bmatrix} \langle [0.18, 0.24], 0.36 \rangle \\ \langle [0.17, 0.23], 0.24 \rangle \\ \langle [0.21, 0.24], 0.36 \rangle \\ \langle [0.21, 0.23], 0.20 \rangle \\ \langle [0.20, 0.23], 0.33 \rangle \\ \langle [0.06, 0.12], 0.20 \rangle \\ \langle [0.14, 0.20], 0.28 \rangle \\ \langle [0.20, 0.24], 0.17 \rangle \\ \langle [0.20, 0.24], 0.34 \rangle \\ \langle [0.13, 0.23], 0.32 \rangle \end{bmatrix} & \begin{bmatrix} \langle [0.28, 0.32], 0.30 \rangle \\ \langle [0.30, 0.35], 0.18 \rangle \\ \langle [0.19, 0.27], 0.22 \rangle \\ \langle [0.25, 0.28], 0.22 \rangle \\ \langle [0.31, 0.35], 0.24 \rangle \\ \langle [0.22, 0.28], 0.26 \rangle \\ \langle [0.18, 0.23], 0.15 \rangle \\ \langle [0.20, 0.22], 0.22 \rangle \\ \langle [0.26, 0.31], 0.21 \rangle \\ \langle [0.16, 0.22], 0.21 \rangle \end{bmatrix} \end{matrix}$$

6. Using Definition 4.1 the value function matrices are obtained as

$$V_F(B^1) = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.25 & 0.50 & 0.58 \\ 0.30 & 0.45 & 0.57 \\ 0.25 & 0.46 & 0.61 \\ 0.40 & 0.38 & 0.50 \\ 0.31 & 0.46 & 0.55 \\ 0.34 & 0.45 & 0.63 \\ 0.35 & 0.48 & 0.61 \\ 0.45 & 0.42 & 0.40 \\ 0.33 & 0.44 & 0.45 \\ 0.34 & 0.44 & 0.41 \end{bmatrix} \end{matrix} \quad V_F(B^2) = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.18 & 0.49 & 0.46 \\ 0.27 & 0.39 & 0.46 \\ 0.26 & 0.58 & 0.28 \\ 0.39 & 0.33 & 0.48 \\ 0.36 & 0.38 & 0.40 \\ 0.28 & 0.42 & 0.55 \\ 0.19 & 0.53 & 0.59 \\ 0.44 & 0.42 & 0.45 \\ 0.27 & 0.53 & 0.43 \\ 0.41 & 0.55 & 0.54 \end{bmatrix} \end{matrix}$$

$$V_F(B^3) = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.19 & 0.57 & 0.60 \\ 0.33 & 0.44 & 0.51 \\ 0.19 & 0.59 & 0.46 \\ 0.47 & 0.42 & 0.49 \\ 0.24 & 0.55 & 0.57 \\ 0.29 & 0.29 & 0.51 \\ 0.27 & 0.45 & 0.35 \\ 0.47 & 0.38 & 0.43 \\ 0.30 & 0.56 & 0.49 \\ 0.28 & 0.51 & 0.40 \end{bmatrix} \end{matrix}$$

Applying Definition 4.2, the score functions are calculated as

$$S_i^1 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.32 \\ 1.32 \\ 1.32 \\ 1.27 \\ 1.33 \\ 1.43 \\ 1.44 \\ 1.27 \\ 1.21 \\ 1.18 \end{bmatrix} \quad S_i^2 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.12 \\ 1.12 \\ 1.12 \\ 1.21 \\ 1.14 \\ 1.24 \\ 1.31 \\ 1.30 \\ 1.23 \\ 1.50 \end{bmatrix} \quad S_i^3 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.36 \\ 1.28 \\ 1.23 \\ 1.38 \\ 1.36 \\ 1.10 \\ 1.07 \\ 1.29 \\ 1.35 \\ 1.19 \end{bmatrix}$$

7. The total score value for each candidate is calculated and presented as

$$TS_i = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 3.80 \\ 3.73 \\ 3.66 \\ 3.86 \\ 3.83 \\ 3.76 \\ 3.82 \\ 3.86 \\ 3.79 \\ 3.86 \end{bmatrix}$$

Arranging the candidates according to their total score we assign the ranking for the candidates.

TABLE 4. Tabular representation of candidate's total score values.

TS_i	c_i	Score	Rank
TS_{10}	c_{10}	3.86	1
TS_8	c_8	3.86	1
TS_4	c_4	3.86	1
TS_5	c_5	3.83	4
TS_7	c_7	3.82	5
TS_1	c_1	3.80	6
TS_9	c_9	3.79	7
TS_6	c_6	3.76	8
TS_2	c_2	3.73	9
TS_3	c_3	3.66	10

From Table 4, as there are more than one candidate with the same ranking we add one more parameter e_4 in the evaluation list and repeat the calculation process.

$$t_1 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.37, 0.60] \\ [0.57, 0.65] \\ [0.40, 0.70] \\ [0.56, 0.76] \\ [0.56, 0.69] \\ [0.56, 0.67] \\ [0.74, 0.85] \\ [0.65, 0.78] \\ [0.56, 0.75] \\ [0.56, 0.78] \end{bmatrix} & \begin{bmatrix} [0.27, 0.65] \\ [0.55, 0.60] \\ [0.50, 0.70] \\ [0.45, 0.57] \\ [0.56, 0.78] \\ [0.56, 0.67] \\ [0.72, 0.82] \\ [0.68, 0.78] \\ [0.45, 0.69] \\ [0.58, 0.67] \end{bmatrix} & \begin{bmatrix} [0.47, 0.67] \\ [0.70, 0.80] \\ [0.55, 0.70] \\ [0.74, 0.84] \\ [0.55, 0.68] \\ [0.67, 0.76] \\ [0.56, 0.77] \\ [0.68, 0.76] \\ [0.45, 0.54] \\ [0.58, 0.67] \end{bmatrix} & \begin{bmatrix} [0.47, 0.67] \\ [0.70, 0.80] \\ [0.55, 0.70] \\ [0.74, 0.84] \\ [0.55, 0.68] \\ [0.67, 0.76] \\ [0.56, 0.77] \\ [0.68, 0.76] \\ [0.45, 0.54] \\ [0.49, 0.67] \end{bmatrix} \end{matrix}$$

$$t_2 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.24, 0.35] \\ [0.52, 0.68] \\ [0.47, 0.67] \\ [0.66, 0.78] \\ [0.67, 0.77] \\ [0.40, 0.50] \\ [0.35, 0.46] \\ [0.67, 0.77] \\ [0.38, 0.52] \\ [0.75, 0.83] \end{bmatrix} & \begin{bmatrix} [0.57, 0.70] \\ [0.32, 0.47] \\ [0.75, 0.83] \\ [0.46, 0.67] \\ [0.45, 0.67] \\ [0.57, 0.65] \\ [0.45, 0.65] \\ [0.55, 0.69] \\ [0.49, 0.72] \\ [0.78, 0.89] \end{bmatrix} & \begin{bmatrix} [0.58, 0.79] \\ [0.46, 0.56] \\ [0.45, 0.46] \\ [0.72, 0.92] \\ [0.45, 0.67] \\ [0.72, 0.82] \\ [0.57, 0.65] \\ [0.68, 0.83] \\ [0.57, 0.68] \\ [0.58, 0.67] \end{bmatrix} & \begin{bmatrix} [0.32, 0.47] \\ [0.75, 0.83] \\ [0.66, 0.78] \\ [0.67, 0.77] \\ [0.40, 0.50] \\ [0.57, 0.65] \\ [0.68, 0.83] \\ [0.45, 0.65] \\ [0.75, 0.83] \\ [0.72, 0.92] \end{bmatrix} \end{matrix}$$

$$t_3 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} [0.20, 0.40] \\ [0.68, 0.70] \\ [0.20, 0.40] \\ [0.68, 0.78] \\ [0.45, 0.56] \\ [0.52, 0.68] \\ [0.45, 0.54] \\ [0.73, 0.89] \\ [0.58, 0.79] \\ [0.46, 0.56] \end{bmatrix} & \begin{bmatrix} [0.60, 0.80] \\ [0.57, 0.78] \\ [0.70, 0.80] \\ [0.69, 0.77] \\ [0.68, 0.78] \\ [0.20, 0.40] \\ [0.47, 0.67] \\ [0.66, 0.78] \\ [0.67, 0.79] \\ [0.43, 0.78] \end{bmatrix} & \begin{bmatrix} [0.70, 0.80] \\ [0.76, 0.88] \\ [0.48, 0.68] \\ [0.84, 0.92] \\ [0.77, 0.88] \\ [0.55, 0.70] \\ [0.46, 0.57] \\ [0.67, 0.74] \\ [0.65, 0.78] \\ [0.41, 0.56] \end{bmatrix} & \begin{bmatrix} [0.20, 0.40] \\ [0.68, 0.78] \\ [0.45, 0.56] \\ [0.69, 0.88] \\ [0.68, 0.78] \\ [0.20, 0.40] \\ [0.46, 0.57] \\ [0.67, 0.74] \\ [0.65, 0.78] \\ [0.60, 0.80] \end{bmatrix} \end{matrix}$$

$$h_1 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.50 & 0.90 & 0.87 & 0.67 \\ 0.60 & 0.70 & 0.67 & 0.62 \\ 0.40 & 0.70 & 0.90 & 0.78 \\ 0.67 & 0.74 & 0.87 & 0.90 \\ 0.62 & 0.65 & 0.77 & 0.70 \\ 0.78 & 0.67 & 0.87 & 0.70 \\ 0.54 & 0.62 & 0.87 & 0.62 \\ 0.78 & 0.67 & 0.62 & 0.62 \\ 0.66 & 0.67 & 0.62 & 0.54 \\ 0.67 & 0.62 & 0.45 & 0.78 \end{bmatrix} & h_2 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.45 & 0.75 & 0.46 & 0.69 \\ 0.46 & 0.69 & 0.63 & 0.85 \\ 0.43 & 0.85 & 0.26 & 0.59 \\ 0.59 & 0.55 & 0.78 & 0.72 \\ 0.72 & 0.54 & 0.43 & 0.70 \\ 0.70 & 0.58 & 0.61 & 0.87 \\ 0.36 & 0.90 & 0.87 & 0.73 \\ 0.75 & 0.77 & 0.73 & 0.90 \\ 0.69 & 0.87 & 0.46 & 0.85 \\ 0.85 & 0.75 & 0.72 & 0.78 \end{bmatrix} \end{matrix}$$

$$h_3 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.50 & 0.90 & 0.75 & 0.50 \\ 0.60 & 0.60 & 0.46 & 0.84 \\ 0.50 & 0.90 & 0.56 & 0.46 \\ 0.84 & 0.67 & 0.74 & 0.67 \\ 0.46 & 0.83 & 0.59 & 0.83 \\ 0.56 & 0.50 & 0.66 & 0.50 \\ 0.62 & 0.69 & 0.37 & 0.46 \\ 0.75 & 0.56 & 0.74 & 0.74 \\ 0.46 & 0.84 & 0.52 & 0.69 \\ 0.63 & 0.81 & 0.52 & 0.90 \end{bmatrix} \end{matrix}$$

The combination of cubic matrices A^1 , A^2 and A^3 are as below

$$A^1 =$$

$$\begin{array}{c}
\begin{array}{cccc}
e_1 & e_2 & e_3 & e_4 \\
c_1 & \langle [0.37, 0.60], 0.50 \rangle & \langle [0.27, 0.65], 0.90 \rangle & \langle [0.47, 0.67], 0.87 \rangle & \langle [0.56, 0.76], 0.67 \rangle \\
c_2 & \langle [0.57, 0.65], 0.60 \rangle & \langle [0.55, 0.60], 0.70 \rangle & \langle [0.70, 0.80], 0.67 \rangle & \langle [0.56, 0.69], 0.62 \rangle \\
c_3 & \langle [0.40, 0.70], 0.40 \rangle & \langle [0.50, 0.70], 0.70 \rangle & \langle [0.55, 0.70], 0.90 \rangle & \langle [0.56, 0.67], 0.78 \rangle \\
c_4 & \langle [0.56, 0.76], 0.67 \rangle & \langle [0.45, 0.57], 0.74 \rangle & \langle [0.74, 0.84], 0.87 \rangle & \langle [0.47, 0.65], 0.90 \rangle \\
c_5 & \langle [0.56, 0.69], 0.62 \rangle & \langle [0.56, 0.78], 0.65 \rangle & \langle [0.55, 0.68], 0.77 \rangle & \langle [0.55, 0.60], 0.70 \rangle \\
c_6 & \langle [0.56, 0.67], 0.78 \rangle & \langle [0.56, 0.67], 0.67 \rangle & \langle [0.67, 0.76], 0.87 \rangle & \langle [0.50, 0.70], 0.70 \rangle \\
c_7 & \langle [0.74, 0.85], 0.54 \rangle & \langle [0.72, 0.82], 0.62 \rangle & \langle [0.56, 0.77], 0.87 \rangle & \langle [0.68, 0.76], 0.62 \rangle \\
c_8 & \langle [0.65, 0.78], 0.78 \rangle & \langle [0.68, 0.78], 0.67 \rangle & \langle [0.68, 0.76], 0.62 \rangle & \langle [0.45, 0.54], 0.62 \rangle \\
c_9 & \langle [0.56, 0.75], 0.66 \rangle & \langle [0.45, 0.69], 0.67 \rangle & \langle [0.45, 0.54], 0.62 \rangle & \langle [0.74, 0.85], 0.54 \rangle \\
c_{10} & \langle [0.56, 0.78], 0.67 \rangle & \langle [0.58, 0.67], 0.62 \rangle & \langle [0.49, 0.67], 0.45 \rangle & \langle [0.65, 0.78], 0.78 \rangle
\end{array}
\end{array}$$

$$A^2 =$$

$$\begin{array}{c}
\begin{array}{cccc}
e_1 & e_2 & e_3 & e_4 \\
c_1 & \langle [0.24, 0.35], 0.45 \rangle & \langle [0.57, 0.70], 0.75 \rangle & \langle [0.58, 0.79], 0.46 \rangle & \langle [0.32, 0.47], 0.69 \rangle \\
c_2 & \langle [0.52, 0.68], 0.46 \rangle & \langle [0.32, 0.47], 0.69 \rangle & \langle [0.46, 0.56], 0.63 \rangle & \langle [0.75, 0.83], 0.85 \rangle \\
c_3 & \langle [0.47, 0.67], 0.43 \rangle & \langle [0.75, 0.83], 0.85 \rangle & \langle [0.45, 0.46], 0.26 \rangle & \langle [0.66, 0.78], 0.59 \rangle \\
c_4 & \langle [0.66, 0.78], 0.59 \rangle & \langle [0.46, 0.67], 0.55 \rangle & \langle [0.72, 0.92], 0.78 \rangle & \langle [0.67, 0.77], 0.72 \rangle \\
c_5 & \langle [0.67, 0.77], 0.72 \rangle & \langle [0.45, 0.67], 0.54 \rangle & \langle [0.45, 0.67], 0.43 \rangle & \langle [0.40, 0.50], 0.70 \rangle \\
c_6 & \langle [0.40, 0.50], 0.70 \rangle & \langle [0.57, 0.65], 0.58 \rangle & \langle [0.72, 0.82], 0.61 \rangle & \langle [0.57, 0.65], 0.87 \rangle \\
c_7 & \langle [0.35, 0.46], 0.36 \rangle & \langle [0.45, 0.65], 0.90 \rangle & \langle [0.57, 0.65], 0.87 \rangle & \langle [0.68, 0.83], 0.73 \rangle \\
c_8 & \langle [0.67, 0.77], 0.75 \rangle & \langle [0.55, 0.69], 0.77 \rangle & \langle [0.68, 0.83], 0.73 \rangle & \langle [0.45, 0.65], 0.90 \rangle \\
c_9 & \langle [0.38, 0.52], 0.69 \rangle & \langle [0.49, 0.72], 0.87 \rangle & \langle [0.57, 0.68], 0.46 \rangle & \langle [0.75, 0.83], 0.85 \rangle \\
c_{10} & \langle [0.75, 0.83], 0.85 \rangle & \langle [0.78, 0.89], 0.75 \rangle & \langle [0.58, 0.67], 0.72 \rangle & \langle [0.72, 0.92], 0.78 \rangle
\end{array}
\end{array}$$

$$A^3 =$$

$$\begin{array}{c}
\begin{array}{cccc}
e_1 & e_2 & e_3 & e_4 \\
c_1 & \langle [0.20, 0.40], 0.50 \rangle & \langle [0.60, 0.80], 0.90 \rangle & \langle [0.70, 0.80], 0.90 \rangle & \langle [0.20, 0.40], 0.50 \rangle \\
c_2 & \langle [0.68, 0.70], 0.60 \rangle & \langle [0.57, 0.78], 0.60 \rangle & \langle [0.76, 0.88], 0.80 \rangle & \langle [0.68, 0.78], 0.84 \rangle \\
c_3 & \langle [0.20, 0.40], 0.50 \rangle & \langle [0.70, 0.80], 0.90 \rangle & \langle [0.60, 0.80], 0.90 \rangle & \langle [0.45, 0.56], 0.42 \rangle \\
c_4 & \langle [0.68, 0.78], 0.84 \rangle & \langle [0.69, 0.77], 0.67 \rangle & \langle [0.84, 0.92], 0.74 \rangle & \langle [0.69, 0.88], 0.67 \rangle \\
c_5 & \langle [0.45, 0.56], 0.46 \rangle & \langle [0.68, 0.78], 0.83 \rangle & \langle [0.77, 0.88], 0.67 \rangle & \langle [0.68, 0.78], 0.83 \rangle \\
c_6 & \langle [0.52, 0.68], 0.56 \rangle & \langle [0.20, 0.40], 0.50 \rangle & \langle [0.55, 0.70], 0.90 \rangle & \langle [0.20, 0.40], 0.50 \rangle \\
c_7 & \langle [0.45, 0.54], 0.62 \rangle & \langle [0.47, 0.67], 0.69 \rangle & \langle [0.46, 0.57], 0.46 \rangle & \langle [0.46, 0.57], 0.46 \rangle \\
c_8 & \langle [0.73, 0.89], 0.75 \rangle & \langle [0.66, 0.78], 0.56 \rangle & \langle [0.67, 0.74], 0.74 \rangle & \langle [0.67, 0.74], 0.74 \rangle \\
c_9 & \langle [0.58, 0.79], 0.46 \rangle & \langle [0.67, 0.79], 0.84 \rangle & \langle [0.65, 0.78], 0.69 \rangle & \langle [0.65, 0.78], 0.69 \rangle \\
c_{10} & \langle [0.46, 0.56], 0.63 \rangle & \langle [0.43, 0.78], 0.81 \rangle & \langle [0.41, 0.56], 0.52 \rangle & \langle [0.60, 0.80], 0.90 \rangle
\end{array}
\end{array}$$

Let $W_1 = (0.3, 0.3, 0.2, 0.2)$ be the weight vector assigned in the technical round and $W_2 = (0.2, 0.3, 0.2, 0.3)$ be the weight vector assigned in the HR round. The resultant cubic matrices B^1 , B^2 and B^3 are as below.

$$B^1 =$$

$$\begin{array}{c}
\begin{array}{cccc}
e_1 & e_2 & e_3 & e_4
\end{array} \\
\begin{array}{l}
c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10}
\end{array}
\end{array}
\begin{bmatrix}
\langle [0.11, 0.18], 0.10 \rangle & \langle [0.08, 0.20], 0.27 \rangle & \langle [0.09, 0.13], 0.17 \rangle & \langle [0.11, 0.15], 0.20 \rangle \\
\langle [0.17, 0.20], 0.12 \rangle & \langle [0.17, 0.18], 0.21 \rangle & \langle [0.14, 0.16], 0.13 \rangle & \langle [0.11, 0.14], 0.19 \rangle \\
\langle [0.12, 0.21], 0.08 \rangle & \langle [0.15, 0.21], 0.21 \rangle & \langle [0.11, 0.14], 0.18 \rangle & \langle [0.11, 0.13], 0.23 \rangle \\
\langle [0.17, 0.23], 0.13 \rangle & \langle [0.14, 0.17], 0.22 \rangle & \langle [0.15, 0.17], 0.17 \rangle & \langle [0.09, 0.13], 0.27 \rangle \\
\langle [0.17, 0.21], 0.12 \rangle & \langle [0.17, 0.23], 0.20 \rangle & \langle [0.11, 0.14], 0.15 \rangle & \langle [0.11, 0.12], 0.21 \rangle \\
\langle [0.17, 0.20], 0.16 \rangle & \langle [0.17, 0.20], 0.20 \rangle & \langle [0.13, 0.15], 0.17 \rangle & \langle [0.10, 0.14], 0.21 \rangle \\
\langle [0.22, 0.26], 0.11 \rangle & \langle [0.22, 0.25], 0.10 \rangle & \langle [0.11, 0.15], 0.17 \rangle & \langle [0.14, 0.15], 0.19 \rangle \\
\langle [0.20, 0.23], 0.16 \rangle & \langle [0.20, 0.23], 0.20 \rangle & \langle [0.14, 0.15], 0.12 \rangle & \langle [0.09, 0.11], 0.19 \rangle \\
\langle [0.17, 0.23], 0.13 \rangle & \langle [0.14, 0.21], 0.20 \rangle & \langle [0.09, 0.11], 0.12 \rangle & \langle [0.15, 0.17], 0.16 \rangle \\
\langle [0.17, 0.23], 0.13 \rangle & \langle [0.17, 0.20], 0.19 \rangle & \langle [0.10, 0.13], 0.09 \rangle & \langle [0.13, 0.16], 0.23 \rangle
\end{bmatrix}$$

$$B^2 =$$

$$\begin{array}{c}
\begin{array}{cccc}
e_1 & e_2 & e_3 & e_4
\end{array} \\
\begin{array}{l}
c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10}
\end{array}
\end{array}
\begin{bmatrix}
\langle [0.07, 0.10], 0.09 \rangle & \langle [0.17, 0.21], 0.22 \rangle & \langle [0.12, 0.16], 0.09 \rangle & \langle [0.06, 0.09], 0.21 \rangle \\
\langle [0.16, 0.20], 0.09 \rangle & \langle [0.10, 0.14], 0.21 \rangle & \langle [0.09, 0.11], 0.13 \rangle & \langle [0.15, 0.17], 0.26 \rangle \\
\langle [0.14, 0.20], 0.09 \rangle & \langle [0.22, 0.25], 0.26 \rangle & \langle [0.09, 0.09], 0.05 \rangle & \langle [0.13, 0.16], 0.18 \rangle \\
\langle [0.20, 0.24], 0.12 \rangle & \langle [0.14, 0.20], 0.17 \rangle & \langle [0.14, 0.18], 0.16 \rangle & \langle [0.13, 0.15], 0.22 \rangle \\
\langle [0.20, 0.23], 0.14 \rangle & \langle [0.14, 0.20], 0.16 \rangle & \langle [0.09, 0.13], 0.09 \rangle & \langle [0.08, 0.10], 0.21 \rangle \\
\langle [0.12, 0.15], 0.14 \rangle & \langle [0.17, 0.20], 0.17 \rangle & \langle [0.14, 0.16], 0.12 \rangle & \langle [0.11, 0.13], 0.26 \rangle \\
\langle [0.11, 0.14], 0.07 \rangle & \langle [0.14, 0.20], 0.27 \rangle & \langle [0.11, 0.13], 0.17 \rangle & \langle [0.14, 0.17], 0.22 \rangle \\
\langle [0.20, 0.23], 0.15 \rangle & \langle [0.17, 0.21], 0.23 \rangle & \langle [0.14, 0.17], 0.15 \rangle & \langle [0.09, 0.13], 0.27 \rangle \\
\langle [0.11, 0.16], 0.14 \rangle & \langle [0.15, 0.22], 0.26 \rangle & \langle [0.11, 0.14], 0.09 \rangle & \langle [0.15, 0.17], 0.26 \rangle \\
\langle [0.22, 0.25], 0.17 \rangle & \langle [0.23, 0.27], 0.23 \rangle & \langle [0.12, 0.13], 0.14 \rangle & \langle [0.14, 0.18], 0.23 \rangle
\end{bmatrix}$$

$$B^3 =$$

$$\begin{array}{c}
\begin{array}{cccc}
e_1 & e_2 & e_3 & e_4
\end{array} \\
\begin{array}{l}
c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10}
\end{array}
\end{array}
\begin{bmatrix}
\langle [0.06, 0.12], 0.10 \rangle & \langle [0.18, 0.24], 0.27 \rangle & \langle [0.14, 0.16], 0.15 \rangle & \langle [0.04, 0.08], 0.15 \rangle \\
\langle [0.20, 0.21], 0.12 \rangle & \langle [0.17, 0.23], 0.18 \rangle & \langle [0.15, 0.18], 0.09 \rangle & \langle [0.14, 0.16], 0.25 \rangle \\
\langle [0.06, 0.12], 0.10 \rangle & \langle [0.21, 0.24], 0.27 \rangle & \langle [0.10, 0.14], 0.11 \rangle & \langle [0.09, 0.11], 0.13 \rangle \\
\langle [0.20, 0.23], 0.25 \rangle & \langle [0.21, 0.23], 0.20 \rangle & \langle [0.17, 0.18], 0.15 \rangle & \langle [0.14, 0.18], 0.20 \rangle \\
\langle [0.14, 0.17], 0.09 \rangle & \langle [0.20, 0.23], 0.25 \rangle & \langle [0.15, 0.18], 0.12 \rangle & \langle [0.14, 0.16], 0.25 \rangle \\
\langle [0.16, 0.20], 0.11 \rangle & \langle [0.06, 0.12], 0.15 \rangle & \langle [0.11, 0.14], 0.13 \rangle & \langle [0.04, 0.08], 0.15 \rangle \\
\langle [0.14, 0.16], 0.12 \rangle & \langle [0.14, 0.20], 0.21 \rangle & \langle [0.09, 0.11], 0.07 \rangle & \langle [0.09, 0.11], 0.14 \rangle \\
\langle [0.22, 0.27], 0.23 \rangle & \langle [0.20, 0.24], 0.17 \rangle & \langle [0.13, 0.15], 0.15 \rangle & \langle [0.13, 0.15], 0.22 \rangle \\
\langle [0.17, 0.24], 0.09 \rangle & \langle [0.20, 0.24], 0.25 \rangle & \langle [0.13, 0.16], 0.10 \rangle & \langle [0.13, 0.16], 0.21 \rangle \\
\langle [0.14, 0.17], 0.13 \rangle & \langle [0.13, 0.23], 0.24 \rangle & \langle [0.08, 0.11], 0.10 \rangle & \langle [0.12, 0.16], 0.27 \rangle
\end{bmatrix}$$

The value function matrices are obtained as

$$V_F(B^1) = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.25 & 0.41 & 0.29 & 0.33 \\ 0.30 & 0.38 & 0.28 & 0.31 \\ 0.25 & 0.39 & 0.31 & 0.36 \\ 0.33 & 0.38 & 0.33 & 0.38 \\ 0.31 & 0.40 & 0.28 & 0.33 \\ 0.34 & 0.39 & 0.32 & 0.33 \\ 0.35 & 0.42 & 0.31 & 0.33 \\ 0.37 & 0.42 & 0.27 & 0.29 \\ 0.33 & 0.37 & 0.22 & 0.32 \\ 0.34 & 0.37 & 0.21 & 0.38 \end{bmatrix} \end{matrix} \quad V_F(B^2) = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.18 & 0.41 & 0.23 & 0.29 \\ 0.27 & 0.32 & 0.23 & 0.41 \\ 0.26 & 0.49 & 0.14 & 0.32 \\ 0.33 & 0.33 & 0.32 & 0.36 \\ 0.36 & 0.33 & 0.20 & 0.30 \\ 0.28 & 0.36 & 0.28 & 0.38 \\ 0.19 & 0.44 & 0.30 & 0.37 \\ 0.37 & 0.42 & 0.30 & 0.38 \\ 0.27 & 0.44 & 0.22 & 0.41 \\ 0.41 & 0.48 & 0.27 & 0.40 \end{bmatrix} \end{matrix}$$

$$V_F(B^3) = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} & \begin{bmatrix} 0.19 & 0.48 & 0.30 & 0.21 \\ 0.33 & 0.38 & 0.26 & 0.40 \\ 0.19 & 0.50 & 0.23 & 0.23 \\ 0.39 & 0.42 & 0.32 & 0.36 \\ 0.24 & 0.47 & 0.28 & 0.40 \\ 0.29 & 0.24 & 0.26 & 0.21 \\ 0.27 & 0.38 & 0.18 & 0.24 \\ 0.39 & 0.38 & 0.29 & 0.36 \\ 0.30 & 0.47 & 0.25 & 0.35 \\ 0.28 & 0.42 & 0.20 & 0.41 \end{bmatrix} \end{matrix}$$

The score functions are calculated as

$$S_i^1 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.27 \\ 1.28 \\ 1.30 \\ 1.42 \\ 1.31 \\ 1.37 \\ 1.40 \\ 1.34 \\ 1.24 \\ 1.29 \end{bmatrix} \quad S_i^2 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.10 \\ 1.24 \\ 1.21 \\ 1.35 \\ 1.19 \\ 1.29 \\ 1.29 \\ 1.46 \\ 1.34 \\ 1.55 \end{bmatrix} \quad S_i^3 = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 1.18 \\ 1.36 \\ 1.14 \\ 1.49 \\ 1.39 \\ 1.00 \\ 1.07 \\ 1.43 \\ 1.36 \\ 1.31 \end{bmatrix}$$

The total score values are calculated as.

$$TS_i = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{matrix} \begin{bmatrix} 3.56 \\ 3.88 \\ 3.65 \\ 4.26 \\ 3.89 \\ 3.66 \\ 3.76 \\ 4.23 \\ 3.95 \\ 4.16 \end{bmatrix}$$

Arranging the candidates according to their total score values we obtain the ranking of the candidates as

TABLE 5. Tabular representation of candidate's total score values.

TS_i	c_i	Score	Rank
TS_4	c_4	4.26	1
TS_8	c_8	4.23	2
TS_{10}	c_{10}	4.16	3
TS_9	c_9	3.95	4
TS_5	c_5	3.89	5
TS_2	c_2	3.88	6
TS_7	c_7	3.76	7
TS_6	c_6	3.66	8
TS_3	c_3	3.65	9
TS_1	c_1	3.56	10

From Table 5, the candidate c_4 ranks first and he is the most suitable candidate for the position.

7. Conclusion

In this manuscript, we use CSM to solve MCDM problems in an ambiguous situation. We define some underlying operations like 'OR', 'AND', complement, WFV, WIVFV, WCV and the construction of cubic matrices. This model is more reliable and provides an upper notch for the decision-maker who understands the nuance of the CS structure in solving MCDM problems. We develop an algorithm and discuss two case studies to show the reliability and effectiveness of the proposed approach. In the first case, the ranking of the candidates is unique and in the second case the ranking of the candidates is not unique and we illustrate the selection by repeating the steps with expanded criteria set. The advantage of this study is that it brings out the importance of the combination

of cubic structure when two different experts provide the membership values in interval-valued and fuzzy. Future, we aim to study CSM with different aggregation operators, pattern recognition, and distance measures.

References

- [1] S.Z.Abbas, M.S.Ali Khan, S.Abdullah, H.Sun, F.Hussain, Cubic Pythagorean fuzzy sets and their application to multi-attribute decision making with unknown weight information, *Journal of Intelligent and Fuzzy Systems*, **37** (2019), 1529-1544.
- [2] F.Adam, N.Hassan, Q-Cubic soft multi set Theory, *International Journal of Mathematics and Computer Science*, **14** (2019), 103-119.
- [3] A. Ali, Y.B.Jun, M.Khan, F.Shi, S. Anis, Generalized cubic soft sets and their applications to algebraic structures *Italian Journal of Pure and Applied Mathematics* , **35** (2015), 393-414.
- [4] A.Fahmi, F.Amin, S.Abdullah, A.Ali, Cubic fuzzy Einstein aggregation operators and its application to decision-making, *International journal of systems science* **49**(2018), 2385-2397.
- [5] K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **20**, No.1 (1986), 87-96.
- [6] V.Chinnadurai, S.Barkavi, Cubic soft matrices, *International Journal For Research In Mathematics And Statistics*, **3**, (2017), 01-22.
- [7] V.Chinnadurai, S.Barkavi, Operations on P-Order and R-Order of internal cubic soft matrices, *International Journal of Scientific Research and Modern Education*, **2**, (2017), 122-145.
- [8] V.Chinnadurai, S.Barkavi, Operations on P-Order and R-Order of external cubic soft matrices, *International Journal of Current Research and Modern Education*, **2**, (2017), 132-155.
- [9] V.Chinnadurai, S.Barkavi, S. Kadalarasi, Operations on internal and external cubic soft matrices, *International Journal of Engineering Research and Modern Education*, **2**, (2017), 121-131.
- [10] M.J.Borah, T.J.Neog, D.K.Sut, Fuzzy soft matrix theory and its decision making, *International Journal of Modern Engineering Research*, **2**, No.2 (2012), 121-127.
- [11] P.K.Maji, R.Biswas, A.R.Roy, Fuzzy soft sets, *Journal of Fuzzy Mathematics*, **9**, No.3 (2001), 589-602.
- [12] A.R.Meenakshi, Fuzzy matrix theory and applications, *MJP Publishers*, India (2008).
- [13] D.A.Molodstov, Soft set theory-first results, *Computers and Mathematics with Applications* **37**, No.4 (1999), 19-31.
- [14] G.Muhiuddin, A.M.Al-roqi, Cubic soft sets with applications in BCK/BCL-algebras, *Annals of Fuzzy Mathematics and Informatics*, **8**, No.2 (2014), 291-304.
- [15] G.Muhiuddin, F.Feng, Y.B.Jun, Subalgebras of BCK/BCI-Algebras based on cubic soft sets, *The scientific world journal*, **3**,(2014),438-458.
- [16] A.Qayyum, S.Abdullah , M.Aslam, Cubic soft expert sets and their application in decision making, *Journal of Intelligent and Fuzzy Systems*, **31** (2016), 1585-1596.
- [17] P.Rajarajeswari, P.Dhanalakshmi, Interval valued fuzzy soft matrix theory, *Annals of Pure and Applied Mathematics*, **7**, No.2 (2014), 61-72.
- [18] S.Abdullah, I.Khan, M.Aslam, A new approach to soft set through applications of cubic set, *arXiv:1210.6517*.
- [19] N.Sarala, M.Prabhavathi, An application of interval valued fuzzy soft matrix in medical diagnosis, *IOSR Journal of Mathematics*, **11**, No.1 (2015), 01-06.
- [20] X.B.Yang, T.Y.Lin, J.Y.Yang, Combination of interval valued fuzzy set and soft set, *Computers and Mathematics with Applications*, **58**, No.3 (2009), 521-527.
- [21] Yong yang, Chenli Ji, Fuzzy soft matrices and their applications, *Artificial Intelligence and Computational Intelligence*, **7002**, (2011), 618-627.

- [22] Y.B.Jun, C.S.Kim, K.O.Yang, Cubic sets, *Annals of fuzzy Mathematics and Informatics*, **4**, No.1 (2012), 83-98.
- [23] Y.B.Jun, S.Z.Song, S.J.Kim, Cubic interval-valued intuitionistic fuzzy sets and their application in BCK/BCI- algebras, *Axioms*, **7**, (2018).
- [24] L.A.Zadeh, Fuzzy sets, *Information and Control*, **8**, (1965), 338-353.
- [25] L.A.Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, *Information Science*, **8**, (1975), 199-249.

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