

CORRIGENDUM: ON A NEW KIND OF EINSTEIN WARPED PRODUCT (POLJ)-MANIFOLD

A. PIGAZZINI, C. ÖZEL[†], P. LINKER AND S. JAFARI

Date of Receiving : 27. 03. 2020

The paper [1] requires some corrections in our calculations, in particular we were wrong to consider the skew symmetry of the Riemann tensor, so the equations (2.5) and (2.7) in [1] become:

$$\begin{cases} \Delta f - \frac{c}{m} f^2 = 0 \\ (m+1)|\nabla f|^2 + \frac{c}{m} f^3 = 0 \end{cases}$$

We wrote $(m+1)$ instead $(m-1)$ because the skew symmetry of the Riemann curvature tensor causes one the terms in the trace to vanish, and since m must be negative, the terms is eliminated by adding 1. In this scenario the $(2, -2)$ -POLJ manifold does not exist, but exists the $(3, -3)$ -POLJ manifold. In case where $n = 3$, $m = -3$, $\lambda = \mu = 0$, $R_B = R_{f_B}$, $h \neq 0$ and by setting $x = (2/h)f$, the equations (2.8) and (2.9) in [1] become: (1) $\Delta x - 2x^2 = 0$ and (2) $|\nabla x|^2 - x^3 = 0$. The technique used remains the same as [1] and by performing the new calculations we obtain: (3) $\omega_2^2 + \omega_3^2 = x^{(1/2)}(E dy^2 + 2F dy dz + G dz^2)$, for some functions E, F and G on the domain of the coordinate chart (x, y, z) that satisfy $EG - F^2 = 1$. Now assume that our metric has the form: (4) $g = x^{-3} dx^2 + (x^a dy)^2 + (x^b dz)^2$, then this is equivalent to having: (5) $2a + 2b - 1 = 0$ and the condition that $R_{f_B} + 6x = 0$ (i.e. the f -curvature-Base [2]), since the scalar curvature is: $R_B = -[2(a^2 + ab + b^2) + a + b]x$. Hence the equation becomes: (6) $6 - 2(a^2 + ab + b^2) - a - b = 0$. Since these two equations ((5) and (6)) are satisfied only for $a = 1$, $b = -1/2$, or $a = -1/2$, $b = 1$, then we have the following metric: $g = x^{-3} dx^2 + x^2 dy^2 + x^{-1} dz^2$. Then our $(3, -3)$ -POLJ manifold metric is: $ds^2 = x^{-3} dx^2 + x^2 dy^2 + x^{-1} dz^2 - ((cx)^2/36)(d\xi^2 + d\varphi^2 + d\rho^2)_{-3}$. We underline that the fiber-manifold (derived-manifold with negative dimension) is intended as $\mathbb{R}^d$ + bundle of obstruction ([3]), therefore on F we consider the valid Riemannian geometry defined on $\mathbb{R}^d$ paying attention to the dimension.

2010 *Mathematics Subject Classification.* 53C25.

Key words and phrases. Einstein warped product, derived-differentials, negative dimensional manifolds, invisible-manifolds, point-manifolds, derived-manifold.

Communicated by: Shiv.K.Kaushik

[†]Corresponding author.

References

- [1] A. Pigazzini, C. Özel, P. Linker, S. Jafari - *On a new kind of Einstein warped product (POLJ)-manifold* - Poincare journal of analysis and applications, 2 (2019).
- [2] Alexander Pigazzini, *On the $(2+2)$ -Einstein Warped Product Manifolds with f -curvature-Base*, Italian journal of pure and applied mathematics, 44 (2020).
- [3] D. Joyce, *Kuranishi spaces and Symplectic Geometry, Volume II*, (2017)

(Alexander Pigazzini) IT-IMPRESA SRL, 20900 MONZA, ITALY.

E-mail address: `pigazzinialexander18@gmail.com`

(Cenap Ozel) KING ABDULAZIZ UNIVERSITY, DEPARTMENT OF MATHEMATICS, 21589 JEDDAH KSA.

E-mail address: `cenap.ozel@gmail.com`

(Patrick Linker) DEPARTMENT OF MATERIALS TESTING, UNIVERSITY OF STUTTGART, 70569 STUTTGART, GERMANY.

E-mail address: `Mrpatricklinker@gmail.com`

(Saeid Jafari) COLLEGE OF VESTSJAELLAND SOUTH, HERRESTRAEDE 11, 4200 SLAGELSE, DENMARK.

E-mail address: `jafaripersia@gmail.com`