

CORRIGENDUM: ON CONVOLUTION OF BOAS TRANSFORM OF WAVELETS

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Date of Receiving : 28. 03. 2020

The paper [1] requires some clarifications. In Theorem 3.3 and Theorem 3.7, we assumed $G(x) = \int_{-1-x}^{1-x} \widehat{\psi^{(1)}}(\gamma) d\gamma$. This was incorrect. The correct value is given by $G(x) = \int_{-1}^1 (1 - \frac{1}{|\gamma|}) e^{-2\pi i \gamma x} \widehat{\psi_2^{(1)}}(-\gamma) d\gamma$. For clarity we give the statements of these two theorems and also first few corrected lines of the proof of Theorem 3.3.

Theorem 3.3. *Let ψ_1, ψ_2 be wavelets such that*

- (i) $\psi_i, \hat{\psi}_i \in L^1(\mathbb{R})$ for $i = 1, 2$,
 - (ii) $\hat{\psi}_2(0) = 0$ and $\psi_2^{(1)} \in L^1(\mathbb{R})$, and
 - (iii) $\int_{\mathbb{R}} x^q G(x) dx = 0$, for $0 \leq q \leq n$, where $G(x) = \int_{-1}^1 (1 - \frac{1}{|\gamma|}) e^{-2\pi i \gamma x} \widehat{\psi_2^{(1)}}(-\gamma) d\gamma$.
- If ψ_1 and ψ_2 have m and n vanishing moments, respectively, then $\mathcal{B}\{\psi_1 * \psi_2\}$ has $(m+n)$ vanishing moments provided that $x^n \psi_2(x) \in L^2(\mathbb{R})$.*

Proof. We have

$$\begin{aligned} \int_{\mathbb{R}} x^q \mathcal{B}\{\psi_1 * \psi_2\}(x) dx &= \sum_{p=0}^q {}^q C_p \text{Mom}_p(\psi_1) \left(\text{Mom}_{q-p}(\mathcal{H}\psi_2) \right. \\ &\quad \left. - \int_{\mathbb{R}} x^{q-p} \int_{\mathbb{R}} \mathcal{F}\{\mathcal{H}T_{-x}\psi_2\}(-\gamma) \hat{g}(\gamma) d\gamma dx \right) \\ &= \sum_{p=0}^q {}^q C_p \text{Mom}_p(\psi_1) \left(\text{Mom}_{q-p}(\mathcal{H}\psi_2) \right. \\ &\quad \left. + i \int_{\mathbb{R}} x^{q-p} \int_{-1}^1 \left(1 - \frac{1}{|\gamma|}\right) \mathcal{F}\{T_{-x}\psi_2^{(1)}\}(-\gamma) d\gamma dx \right). \end{aligned}$$

2010 *Mathematics Subject Classification.* 42A38, 42C40, 44A15, 44A60.

Key words and phrases. Boas transform; wavelets; Hilbert transform; Fourier transform; vanishing moments.

Communicated by. Shiv K. Kaushik

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Since $\psi_2, \psi_2^{(1)}, \hat{\psi}_2 \in L^1(\mathbb{R})$, we have

$$\begin{aligned} \int_{\mathbb{R}} x^q \mathcal{B}\{\psi_1 * \psi_2\}(x) dx &= \sum_{p=0}^q {}^qC_p \text{Mom}_p(\psi_1) \left(\text{Mom}_{q-p}(\mathcal{H}\psi_2) \right. \\ &\quad \left. - \frac{1}{2\pi} \int_{\mathbb{R}} x^{q-p} \int_{-1}^1 \left(1 - \frac{1}{|\gamma|}\right) e^{-2\pi i \gamma x} \widehat{\psi_2^{(1)}}(-\gamma) d\gamma dx \right) \\ &= \sum_{p=0}^q {}^qC_p \text{Mom}_p(\psi_1) \left(\text{Mom}_{q-p}(\mathcal{H}\psi_2) - \frac{1}{2\pi} \int_{\mathbb{R}} x^{q-p} G(x) dx \right). \end{aligned}$$

There is no change in the remaining part of the proof. $\square$

Theorem 3.7. *Let ψ_1, ψ_2 be wavelets such that*

- (i) $\psi_i, \hat{\psi}_i \in L^1(\mathbb{R})$ for $i = 1, 2$,
 - (ii) $\hat{\psi}_2(0) = 0$ and $\psi_2^{(1)} \in L^1(\mathbb{R})$, and
 - (iii) $\int_{\mathbb{R}} x^q G(x) dx = 0$, for $0 \leq q \leq n$, where $G(x) = \int_{-1}^1 \left(1 - \frac{1}{|\gamma|}\right) e^{-2\pi i \gamma x} \widehat{\psi_2^{(1)}}(-\gamma) d\gamma$.
- If ψ_1 and ψ_2 have m and n vanishing moments, respectively, then $\mathcal{B}\{\psi_1 \star \psi_2\}$ has $(m+n)$ vanishing moments provided that $x^n \psi_2(x) \in L^2(\mathbb{R})$.*

References

- [1] N. Khanna and L. Kathuria, *On convolution of Boas transform of wavelets*, Poincare J. Anal. Appl. 2019, no. 1, 53-65.

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