

ON THE STABILITY OF MULTIPLICATIVE INVERSE CUBIC FUNCTIONAL (MICF) EQUATION IN INTUITIONISTIC FUZZY NORMED SPACES

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Abstract. The objective of this study to examine the generalized Hyers- Ulam stability (GHUS) of multiplicative inverse cubic functional (MICF) equation of the form

$$c(2l + m) + c(l + 2m) = \frac{9c(l)c(m)[c(l) + c(m) + 2c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}[c(l)^{\frac{1}{3}} + c(m)^{\frac{1}{3}}]]}{[2c(l)^{\frac{2}{3}} + 2c(m)^{\frac{2}{3}} + 5c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}]^3}$$

and using this, we give the Hyers stability, Hyers-Ulam-Rassias stability, Ulam-Gavruta-Rassias product stability and JM Rassias mixed product-sum stability results of MICF equation in intuitionistic fuzzy normed spaces (IFNS).

1. Introduction

Fuzzy set theory is a powerful hand set for modeling uncertainty and fuzziness in heterogeneous problems emerging in the different fields of engineering and science. It has applications in many fields, for example, statistical convergence, chaos control, nonlinear dynamical systems, population dynamics, nonlinear operators, etc. ([22], [30], [19]). The fuzzy topology proves to be a practical tool to deal with such circumstances where the use of classical theories ceases. The most interesting application of fuzzy topology in quantum particle physics arises in string and E-infinity theory of El Naschie ([24], [25]).

The impracticality of determining the position of particles gives the space-time a fuzzy structure[24]. Due to this fuzzy structure, position space representation of quantum mechanics ceases and therefore, a generalized normed space of quasi-position

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eigenfunction is recommended [19]. Hence, one needs to discuss on a new family of fuzzy norms. There are many situations where the norm of a vector is not possible to be found and the concept of intuitionistic fuzzy norm ([3],[13], [14]) seems to be more admissible, that is, we can deal with such situations by modeling the inexactness through the intuitionistic fuzzy norm.

The crucial point from where the concept of investigating HUS results of functional equations, differential equations, difference equations is the problem of Ulam [33]. Hyers [8] presented a partial solution to the problem of Ulam. Later, Hyers theorem was extended and generalized in various forms by many mathematicians such as Aoki [34], T. Rassias [35], J. Rassias [14] and Gavruta [29]. These results instigated many mathematicians to investigate stability of different types of functional equations in different types of spaces. For detailed review of literature on this field, one can refer ([4], [11], [12], [28], [23], [18], [9], [2], [20], [5], [32], [6], [7],[15],[16], [17]).

The authors in [21] focussed on providing evidence that the stability results are valid for the following multiplicative inverse quadratic functional equation

$$q_r(\alpha + \beta) = \frac{q_r(\alpha)q_r(\beta)}{q_r(\alpha) + q_r(\beta) + 2[q_r(\alpha)q_r(\beta)]^{\frac{1}{2}}}. \quad (1.1)$$

The stability results in the setting of intuitionistic fuzzy normed space (IFNS) for various functional equations were studied in ([1], [10], [26], [27], [30]). In 2009, M. Mursaleen [10] studied the stability of cubic functional equation in IFNS.

This paper deals with finding the generalized stability of the multiplicative inverse cubic functional (MICF) equation

$$c(2l + m) + c(l + 2m) = \frac{9c(l)c(m)[c(l) + c(m) + 2c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}[c(l)^{\frac{1}{3}} + c(m)^{\frac{1}{3}}]]}{[2c(l)^{\frac{2}{3}} + 2c(m)^{\frac{2}{3}} + 5c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}]^3} \quad (1.2)$$

2. Preliminaries

In this section, we study some basic definitions which are used throughout the paper.

Definition 2.1. [31] “A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is said to be continuous t-norm if it satisfies the following conditions:

- (i) $*$ is associative and commutative;
- (ii) $*$ is continuous;
- (iii) $a * 1 = a$ for all $a \in [0, 1]$;
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d, \in [0, 1]$.”

Definition 2.2. [31] “A binary operation $\diamond$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is said to be continuous t-conorm if it satisfies the following conditions:

- (i) $\diamond$ is associative and commutative;
- (ii) $\diamond$ is continuous;
- (iii) $a \diamond 0 = a$ for all $a \in [0, 1]$;
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d, \in [0, 1]$.”

In 2006, R. Saadati and J. Park gives the concept of intuitionistic fuzzy normed space.

Definition 2.3. [31] “Let X be a vector space, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and μ, ν are fuzzy sets on $X \times (0, \infty)$. Then five tuple $(X, \mu, \nu, *, \diamond)$ is said to

be an intuitionistic fuzzy normed space (IFNS) if it gratifies the following conditions

- (i) $\mu(x, t) + \nu(x, t) \leq 1$;
 - (ii) $\mu(x, t) > 0$;
 - (iii) $\mu(x, t) = 1$ if and only if $x=0$;
 - (iv) $\mu(\alpha x, t) = \mu(x, \frac{t}{|\alpha|})$ for each $\alpha \neq 0$;
 - (v) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$;
 - (vi) $\mu(x, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous;
 - (vii) $\lim_{t \rightarrow \infty} \mu(x, t) = 1$ and $\lim_{t \rightarrow 0} \mu(x, t) = 0$;
 - (viii) $\nu(x, t) < 1$;
 - (ix) $\nu(x, t) = 0$ if and only if $x=0$;
 - (x) $\nu(\alpha x, t) = \nu(x, \frac{t}{|\alpha|})$ for each $\alpha \neq 0$;
 - (xi) $\nu(x, t) \diamond \nu(y, s) \geq \nu(x + y, t + s)$;
 - (xii) $\nu(x, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous;
 - (xiii) $\lim_{t \rightarrow \infty} \nu(x, t) = 0$ and $\lim_{t \rightarrow 0} \nu(x, t) = 1$;
- for all $x, y \in X$ and $s, t > 0$."

Example 2.4. [31] Let us assume $(\mathbf{N}, \|\cdot\|)$ to be a normed space, and $*$, $\diamond$ are continuous t- norm and continuous t- conorm described as $l * m = lm$ and $l \diamond m = \min\{l + m, 1\}$ for all $l, m \in [0, 1]$ respectively. Then $(\mathbf{N}, \mu, \nu, *, \diamond)$ is IFNS where, μ and ν are defined as

$$\mu(l, t) = \begin{cases} \frac{t}{t + \|l\|} & ; t > 0, \\ 0 & ; t \leq 0 \end{cases}$$

and

$$\nu(l, t) = \begin{cases} \frac{\|l\|}{t + \|l\|} & ; t > 0, \\ 0 & ; t \leq 0 \end{cases}$$

satisfies all the 13 properties.

3. Main Results

Throughout this section, let us assume V to be linear space and (F, μ, ν) be intuitionistic fuzzy Banach space. We consider that $l \neq 0$, $c(l) \neq 0$, $l + 2m \neq 0$, $2l + m \neq 0$, $c(l) + c(m) + 2c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}[c(l)^{\frac{1}{3}} + c(m)^{\frac{1}{3}}] \neq 0$, $2c(l)^{\frac{2}{3}} + 2c(m)^{\frac{2}{3}} + 5c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}} \neq 0$ for all $l, m \in V$. For ease, we introduce the difference operator $D_c : VXV \rightarrow F$ for given mapping $c : V \rightarrow F$, by

$$D_c(l, m) = c(2l + m) + c(l + 2m) - \frac{9c(l)c(m)[c(l) + c(m) + 2c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}[c(l)^{\frac{1}{3}} + c(m)^{\frac{1}{3}}]]}{[2c(l)^{\frac{2}{3}} + 2c(m)^{\frac{2}{3}} + 5c(l)^{\frac{1}{3}}c(m)^{\frac{1}{3}}]^3}$$

for all $l, m \in V$.

Theorem 3.1. Suppose $\varphi : VXV \rightarrow [0, \infty)$ is a function such that

$$\chi(l, m) = \sum_{k=0}^{\infty} 27^k \varphi(3^k l, 3^k m) < \infty \quad (3.1)$$

for all $l, m \in V$. Let $c : V \rightarrow F$ be a function such that

$$\lim_{t \rightarrow \infty} \mu(D_c(l, m), t\varphi(l, m)) = 1 \quad \lim_{t \rightarrow \infty} \nu(D_c(l, m), t\varphi(l, m)) = 0 \quad (3.2)$$

uniformly in VXV .

Then $R_c(l) = (\mu, \nu)\text{-}\lim_{n \rightarrow \infty} 27^n c(3^n l)$ for each $l \in V$ occurs and defines a multiplicative inverse-cubic (MIC) mapping $R_c : V \rightarrow F$ such that if for some $\delta > 0$, $\alpha > 0$ and all $l, m \in V$,

$$\mu(D_c(l, m), \delta\varphi(l, m)) > \alpha \quad \nu(D_c(l, m), \delta\varphi(l, m)) < 1 - \alpha \quad (3.3)$$

then

$$\begin{aligned} \mu(R_c(l) - c(l), \frac{27\delta}{2}\chi(l, l)) &> \alpha \\ \nu(R_c(l) - c(l), \frac{27\delta}{2}\chi(l, l)) &< 1 - \alpha \end{aligned}$$

Also, multiplicative inverse cubic mapping R_c is unique such that

$$\lim_{n \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27t}{2}\chi(l, l)) = 1, \quad \lim_{n \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27t}{2}\chi(l, l)) = 0 \quad (3.4)$$

uniformly in V .

Proof. For given $\varepsilon > 0$ and using (3.2), one can find some $t_0 > 0$ such that

$$\mu(D_c(l, m), t\varphi(l, m)) \geq 1 - \varepsilon, \quad \nu(D_c(l, m), t\varphi(l, m)) \leq \varepsilon \quad (3.5)$$

for all $l, m \in V$ and all $t \geq t_0$. Replacing m by l in (3.5), we obtain

$$\mu(27c(3l) - c(l), \frac{27t}{2}\varphi(l, l)) \geq 1 - \varepsilon, \quad \nu(27c(3l) - c(l), \frac{27t}{2}\varphi(l, l)) \leq \varepsilon \quad (3.6)$$

for all $l \in V$ and all $t \geq t_0$. Now, substitute $3l$ in place of l in (3.6), we get

$$\mu(27^2c(3^2l) - 27c(3l), \frac{27^2t}{2}\varphi(3l, 3l)) > 1 - \varepsilon, \quad (3.7)$$

$$\nu(27^2c(3^2l) - 27c(3l), \frac{27^2t}{2}\varphi(3l, 3l)) < \varepsilon \quad (3.8)$$

for all $l \in V$ and all $t \geq t_0$. After merging (3.6) and (3.7), we get

$$\begin{aligned} &\mu(27^2c(3^2l) - c(l), \frac{27t}{2} \sum_{k=0}^1 27^k \varphi(3^k l, 3^k l)) \\ &\geq \mu(27^2c(3^2l) - 27c(3l), \frac{27^2t}{2}\varphi(3l, 3l)) * \mu(27c(3l) - c(l), \frac{27t}{2}\varphi(l, l)) \\ &\geq (1 - \varepsilon) * (1 - \varepsilon) = 1 - \varepsilon \end{aligned}$$

and

$$\begin{aligned} &\nu(27^2c(3^2l) - c(l), \frac{27t}{2} \sum_{k=0}^1 27^k \varphi(3^k l, 3^k l)) \\ &\leq \nu(27^2c(3^2l) - 27c(3l), \frac{27^2t}{2}\varphi(3l, 3l)) \diamond \nu(27c(3l) - c(l), \frac{27t}{2}\varphi(l, l)) \leq \varepsilon \diamond \varepsilon = \varepsilon \end{aligned}$$

for all $l \in V$ and all $t \geq t_0$. Now, proceeding in same manner and using induction on a positive integer n , we obtain

$$\mu(27^n c(3^n l) - c(l), \frac{27t}{2} \sum_{k=0}^{n-1} 27^k \varphi(3^k l, 3^k l)) \geq 1 - \varepsilon, \quad (3.9)$$

$$\nu(27^n c(3^n l) - c(l), \frac{27t}{2} \sum_{k=0}^{n-1} 27^k \varphi(3^k l, 3^k l)) \leq \varepsilon \quad (3.10)$$

for all $l \in V$ and all $t \geq t_0$. For proving the convergence of the sequence $\{27^n c(3^n l)\}$, letting $t = t_0$ and changing (l, m) by $(3^m l, 3^m m)$ in (3.9), we find that for $n > m > 0$

$$\mu(27^{n+m} c(3^{n+m} l) - 27^m c(3^m l), \frac{27t_0}{2} \sum_{k=0}^{n-1} 27^{k+m} \varphi(3^{k+m} l, 3^{k+m} l)) \geq 1 - \varepsilon \quad (3.11)$$

$$\nu(27^{n+m} c(3^{n+m} l) - 27^m c(3^m l), \frac{27t_0}{2} \sum_{k=0}^{n-1} 27^{k+m} \varphi(3^{k+m} l, 3^{k+m} l)) \leq \varepsilon \quad (3.12)$$

The convergence of (3.1) and

$$\frac{27}{2} \sum_{k=0}^{n-1} 27^{k+m} \varphi(3^{k+m} l, 3^{k+m} l) = \frac{27}{2} \sum_{k=m}^{m+n-1} 27^k \varphi(3^k l, 3^k l)$$

imply that for given $\delta > 0$ there is $n_0 \in F$ such that

$$\frac{27t_0}{2} \sum_{k=m}^{m+n-1} 27^k \varphi(3^k l, 3^k l) < \delta,$$

for all $m \geq n_0$ and all $n > 0$. From (3.11) we deduce that

$$\begin{aligned} & \mu(27^{n+m} c(3^{n+m} l) - 27^m c(3^m l), \delta) \\ & \geq \mu(27^{n+m} c(3^{n+m} l) - 27^m c(3^m l), \frac{27t_0}{2} \sum_{k=0}^{n-1} 27^{k+m} \varphi(3^{k+m} l, 3^{k+m} l)) \geq 1 - \varepsilon \end{aligned}$$

$$\text{and } \nu(27^{n+m} c(3^{n+m} l) - 27^m c(3^m l), \delta)$$

$$\leq \nu(27^{n+m} c(3^{n+m} l) - 27^m c(3^m l), \frac{27t_0}{2} \sum_{k=0}^{n-1} 27^{k+m} \varphi(3^{k+m} l, 3^{k+m} l)) \leq \varepsilon$$

for all $m \geq n_0$ and all $n > 0$. Hence $\{27^n c(3^n l)\}$ is a Cauchy sequence in F . Also, F is intuitionistic fuzzy Banach space, therefore $\{27^n c(3^n l)\}$ converges to $R_c(l) \in F$ (say). Hence we can define a mapping $R_c : V \rightarrow F$ such that $R_c(l) = (\mu, \nu) - \lim_{n \rightarrow \infty} 27^n c(3^n l)$ namely, for each $l \in V$ and $t > 0$,

$$\begin{aligned} & \mu(R_c(l) - 27^n c(3^n l), t) = 1 \\ \text{and} \quad & \nu(R_c(l) - 27^n c(3^n l), t) = 0 \end{aligned}$$

Taking the limit $n \rightarrow \infty$ in (3.9), we see that the existence of (3.4) uniformly in V . Now, let $l, m \in V$, choose any fixed value of $t > 0$, and $\varepsilon \in (0, 1)$. For convenience, Let us

assume

$$\begin{aligned} A(l, m) &= 9R_c(l)R_c(m)[R_c(l) + R_c(m) + 2R_c(l)^{\frac{1}{3}}R_c(m)^{\frac{1}{3}}[R_c(l)^{\frac{1}{3}} + R_c(m)^{\frac{1}{3}}]] \\ B(l, m) &= [2(27)^n c(3^n l)^{\frac{2}{3}} + 2(27)^n c(3^n m)^{\frac{2}{3}} + 5(27)^n c(3^n l)^{\frac{1}{3}}(27)^n c(3^n m)^{\frac{1}{3}}]^3 \\ C(l, m) &= (27)^n c(3^n l)(27)^n c(3^n m)[(27)^n c(3^n l) + (27)^n c(3^n m) + 2(27)^n c(3^n l)^{\frac{1}{3}} \\ &\quad (27)^n c(3^n m)^{\frac{1}{3}}[(27)^n c(3^n l)^{\frac{1}{3}} + (27)^n c(3^n m)^{\frac{1}{3}}]] \end{aligned}$$

Since $\lim_{n \rightarrow \infty} 27^n \varphi(3^n l, 3^n m) = 0$, there exists $n_1 \geq n_0$ such that $t_0 \varphi(3^n l, 3^n m) < \frac{t}{4 \cdot 27^n}$ for all $n > n_1$. Hence for each $n \geq n_1$, we have

$$\begin{aligned} \mu(D_{R_c}(l, m), t) &\geq \mu(R_c(2l + m) - (27)^n c(3^n(2l + m)), \frac{t}{4}) * \mu(R_c(l + 2m) - (27)^n \\ &\quad c(3^n(l + 2m)), \frac{t}{4}) * \mu\left(\frac{A(l, m)}{[2R_c(l)^{\frac{2}{3}} + 2R_c(m)^{\frac{2}{3}} + 5R_c(l)^{\frac{1}{3}}R_c(m)^{\frac{1}{3}}]^3} - \frac{C(l, m)}{B(l, m)}, \frac{t}{4}\right) \\ &\quad * \mu(D_c(3^n l, 3^n m), \frac{t}{4 \cdot 27^n}) \end{aligned} \quad (3.13)$$

and also

$$\mu(D_c(3^n l, 3^n m), \frac{t}{4 \cdot 27^n}) \geq \mu(D_c(3^n l, 3^n m), t_0 \varphi(3^n l, 3^n m)) \quad (3.14)$$

Letting $n \rightarrow \infty$ in (3.13) and using (3.5) and (3.14), we get $\mu(D_{R_c}(l, m), t) \geq 1 - \varepsilon$ for all $t > 0$ and $\varepsilon \in (0, 1)$. Similarly, we obtain $\nu(D_{R_c}(l, m), t) \leq \varepsilon$ for all $t > 0$ and $\varepsilon \in (0, 1)$. It follows that $\mu(D_{R_c}(l, m), t) = 1$ and $\nu(D_{R_c}(l, m), t) = 0$ for all $t > 0$. Hence R_c satisfies equation (1.2), which proves that R_c is MIC mapping. Next, suppose that for some positive δ and α (3.3) holds and

$$\varphi_n(l, m) = \frac{27}{2} \sum_{i=0}^{n-1} (27)^i \varphi(3^i l, 3^i m), \text{ for all } l, m \in V.$$

Using the same arguments as starting of the proof we conclude from (3.3)

$$\mu((27)^n c(3^n l) - c(l), \frac{27\delta}{2} \sum_{i=0}^{n-1} (27)^i \varphi(3^i l, 3^i m)) \geq \alpha \quad (3.15)$$

$$\nu((27)^n c(3^n l) - c(l), \frac{27\delta}{2} \sum_{i=0}^{n-1} (27)^i \varphi(3^i l, 3^i m)) \leq 1 - \alpha \quad (3.16)$$

for all positive integers n . For $k > 0$ we have

$$\begin{aligned} \mu(R_c(l) - c(l), \delta \varphi_n(l, l) + k) &\geq \mu((27)^n c(3^n l) - c(l), \delta \varphi_n(l, l)) \\ &\quad * \mu(R_c(l) - (27)^n c(3^n l), k) \end{aligned} \quad (3.17)$$

$$\begin{aligned} \nu(R_c(l) - c(l), \delta \varphi_n(l, l) + k) &\leq \nu((27)^n c(3^n l) - c(l), \delta \varphi_n(l, l)) \\ &\quad \diamond \nu(R_c(l) - (27)^n c(3^n l), k) \end{aligned} \quad (3.18)$$

Using (3.15), (3.17) and (3.18) and using

$$\lim_{n \rightarrow \infty} \mu(R_c(l) - (27)^n c(3^n l), k) = 1, \quad \lim_{n \rightarrow \infty} \nu(R_c(l) - (27)^n c(3^n l), k) = 0$$

Hence we obtain

$$\mu(R_c(l) - c(l), \delta\varphi_n(l, l) + k) \geq \alpha, \quad \nu(R_c(l) - c(l), \delta\varphi_n(l, l) + k) \leq 1 - \alpha$$

for n to be very large. From the (upper- semi) continuity of real functions $\mu(R_c(l) - c(l), \cdot)$ and $\nu(R_c(l) - c(l), \cdot)$, we have

$$\mu(R_c(l) - c(l), \frac{27\delta}{2}\chi(l, l) + k) \geq \alpha, \quad \nu(R_c(l) - c(l), \frac{27\delta}{2}\chi(l, l) + k) \leq 1 - \alpha.$$

Taking the limit $k \rightarrow \infty$, we obtain

$$\mu(R_c(l) - c(l), \frac{27\delta}{2}\chi(l, l)) \geq \alpha, \quad \nu(R_c(l) - c(l), \frac{27\delta}{2}\chi(l, l)) \leq 1 - \alpha.$$

Now, for proving uniqueness of function R_c , Let us assume another multiplicative inverse cubic mapping R_C satisfying (3.4). Choose any fixed value of $c > 0$. Given $\varepsilon > 0$, there is some $t_0 > 0$ such that (3.4) for R_c and R_C

$$\mu(R_c(l) - c(l), \frac{27t}{2}\chi(l, l)) \geq 1 - \varepsilon, \quad \mu(R_C(l) - c(l), \frac{27t}{2}\chi(l, l)) \geq 1 - \varepsilon$$

$$\nu(R_c(l) - c(l), \frac{27t}{2}\chi(l, l)) \leq \varepsilon, \quad \nu(R_C(l) - c(l), \frac{27t}{2}\chi(l, l)) \leq \varepsilon$$

for all $l \in V$, and all $t \geq t_0$. For some $l \in V$, we can find some integer n_0 such that

$$t_0 \sum_{i=n}^{\infty} (27)^i \varphi(3^i l, 3^i l) < \frac{c}{2}, \quad \text{for all } n \geq n_0.$$

$$\begin{aligned} \text{Since } \sum_{i=n}^{\infty} (27)^i \varphi(3^i l, 3^i l) &= (27)^n \sum_{i=n}^{\infty} (27)^{i-n} \varphi(3^{i-n}(3^n l), 3^{i-n}(3^n l)) \\ &= (27)^n \sum_{i=0}^{\infty} (27)^i \varphi(3^i(3^n l), 3^i(3^n l)) = (27)^n \chi(3^n l, 3^n l), \end{aligned}$$

we have

$$\begin{aligned} \mu(R_c(l) - R_C(l), c) &\geq \mu(R_c(l) - (27)^n c(3^n l), \frac{c}{2}) * \mu(R_C(l) - (27)^n c(3^n l), \frac{c}{2}) \\ &\geq \mu(R_c(3^n l) - c(3^n l), \frac{c}{2 \cdot (27)^n}) * \mu(R_C(3^n l) - c(3^n l), \frac{c}{2 \cdot (27)^n}) \\ &\geq \mu(R_c(3^n l) - c(3^n l), \frac{t_0}{(27)^n} \sum_{i=n}^{\infty} (27)^i \varphi(3^i l, 3^i l)) * \mu(R_C(3^n l) - c(3^n l), \frac{t_0}{(27)^n} \sum_{i=n}^{\infty} (27)^i \varphi(3^i l, 3^i l)) \\ &\geq \mu(R_c(3^n l) - c(3^n l), t_0 \chi(3^n l, 3^n l)) * \mu(R_C(3^n l) - c(3^n l), t_0 \chi(3^n l, 3^n l)) \geq 1 - \varepsilon \end{aligned}$$

Similarly, $\nu(R_c(l) - R_C(l), c) \leq \varepsilon$. Hence, it follows that $\mu(R_c(l) - R_C(l), c) = 1$ and $\nu(R_c(l) - R_C(l), c) = 0$ for all $c > 0$. Therefore, $R_c(l) = R_C(l)$ for all $l \in V$. This completes the proof of theorem. $\square$

Corollary 3.2. Let us consider a function $c : V \longrightarrow F$ such that for all $c_1 \geq 0$, $\varrho \neq -3$

$$\lim_{t \rightarrow \infty} \mu(D_c(l, m), tc_1(\|l\|^e + \|m\|^e)) = 1, \quad \lim_{t \rightarrow \infty} \nu(D_c(l, m), tc_1(\|l\|^e + \|m\|^e)) = 0$$

uniformly in VXV . Then there exists a unique multiplicative inverse cubic mapping $R_c : V \rightarrow F$ such that for $\varrho < -3$,

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27c_1 t \|l\|^e}{1 - 3^{e+3}}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27c_1 t \|l\|^e}{1 - 3^{e+3}}) = 0$$

and for $\varrho > -3$,

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27c_1 t \|l\|^e}{3^{e+3} - 1}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27c_1 t \|l\|^e}{3^{e+3} - 1}) = 0$$

uniformly in V .

Proof. Continuing with similar arguments as in Theorem 3.1, equation (3.4) becomes

$$\begin{aligned} \lim_{n \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27t}{2} \sum_{k=0}^{\infty} 27^k \varphi(3^k l, 3^k l)) &= 1, \\ \lim_{n \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27t}{2} \sum_{k=0}^{\infty} 27^k \varphi(3^k l, 3^k l)) &= 0 \end{aligned}$$

Now, Replacing $\varphi(l, m)$ by $c_1(\|l\|^e + \|m\|^e)$ we get

$$\begin{aligned} \lim_{n \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27t}{2} \sum_{k=0}^{\infty} 27^k c_1 \{\|3^k l\|^e + \|3^k l\|^e\}) &= 1, \\ \lim_{n \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27t}{2} \sum_{k=0}^{\infty} 27^k c_1 \{\|3^k l\|^e + \|3^k l\|^e\}) &= 0, \end{aligned}$$

and after solving these equations, we arrive at the desired result. $\square$

Corollary 3.3. Let $c : V \rightarrow F$ be a mapping and α, τ be two real numbers such that $\varrho = \alpha + \tau \neq -3$. Let there exists a non-negative real number δ_2 such that

$$\lim_{t \rightarrow \infty} \mu(D_c(l, m), t\delta_2(\|l\|^\alpha \|m\|^\tau)) = 1, \quad \lim_{t \rightarrow \infty} \nu(D_c(l, m), t\delta_2(\|l\|^\alpha \|m\|^\tau)) = 0$$

uniformly in VXV . Then there exists a unique multiplicative inverse cubic mapping $R_c : V \longrightarrow F$ such that for $\varrho < -3$,

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27\delta_2 t \|l\|^e}{2(1 - 3^{e+3})}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27\delta_2 t \|l\|^e}{2(1 - 3^{e+3})}) = 0$$

and for $\varrho > -3$,

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27\delta_2 t \|l\|^e}{2(3^{e+3} - 1)}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27\delta_2 t \|l\|^e}{2(3^{e+3} - 1)}) = 0$$

uniformly in V .

Proof. Assuming $\varphi(l, m) = \delta_2 |l|^\alpha |m|^\tau$ in Theorem 3.1, we achieve the required result. $\square$

Corollary 3.4. *Let δ_3 be a fixed non-negative real number and $\varrho \neq \frac{-3}{2}$ be real number. Suppose $c : V \rightarrow F$ is a mapping such that*

$$\lim_{t \rightarrow \infty} \mu(D_c(l, m), t\delta_3((\|l\|)^{2\varrho} + (\|m\|)^{2\varrho}) + (\|l\|)^{\varrho}(\|m\|)^{\varrho}) = 1$$

$$\lim_{t \rightarrow \infty} \nu(D_c(l, m), t\delta_3((\|l\|)^{2\varrho} + (\|m\|)^{2\varrho}) + (\|l\|)^{\varrho}(\|m\|)^{\varrho}) = 0$$

uniformly in VXV . Then there exists a unique multiplicative inverse cubic mapping $R_c : V \rightarrow F$ such that for $\varrho < \frac{-3}{2}$,

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27.3\delta_3 t \|l\|^{2\varrho}}{2(1 - 3^{2\varrho+3})}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27.3\delta_3 t \|l\|^{2\varrho}}{2(1 - 3^{2\varrho+3})}) = 0$$

for $\varrho > \frac{-3}{2}$,

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27.3\delta_3 t \|l\|^{2\varrho}}{2(3^{2\varrho+3} - 1)}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27.3\delta_3 t \|l\|^{2\varrho}}{2(3^{2\varrho+3} - 1)}) = 0$$

uniformly in V .

Proof. The proof follows directly by taking $\varphi(l, m) = \delta_3((\|l\|)^{2\varrho} + \|m\|^{2\varrho}) + \|l\|^{\varrho}\|m\|^{\varrho}$ in Theorem 3.1. $\square$

Corollary 3.5. *Let us consider a function $c : V \rightarrow F$ such that for all $c_2 \geq 0$,*

$$\lim_{t \rightarrow \infty} \mu(D_c(l, m), tc_2) = 1, \quad \lim_{t \rightarrow \infty} \nu(D_c(l, m), tc_2) = 0$$

uniformly in VXV . Then there exists a unique multiplicative inverse cubic mapping $R_c : V \rightarrow F$ such that

$$\lim_{t \rightarrow \infty} \mu(R_c(l) - c(l), \frac{27c_2 t}{52}) = 1, \quad \lim_{t \rightarrow \infty} \nu(R_c(l) - c(l), \frac{27c_2 t}{52}) = 0$$

uniformly in V .

Proof. The proof follows directly by taking $\varphi(l, m) = c_2$ in Theorem 3.1. $\square$

4. Conclusion

We wind up this paper with a conclusion that we have proved generalized Hyers- Ulam stability results of MICF equation involving a general control function, sum of powers of norms, product of different powers of norms and mixed product-sum of powers of norms pertinent to the results established by Gavruta and Rassias in intuitionistic fuzzy Normed spaces.

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