Poincare Journal of Analysis & Applications
Vol. 2018 (2(II)), Special Issue (IWWFA-III, Delhi), 97-102

Using linear Legendre wavelet in a synapses neural network

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Date of Receiving : 15.04.2018
Date of Revision : 13.08.2018
Date of Acceptance : 20.12.2018

Abstract. Wavelet neural networks are a new class of networks that combine the classic neural networks and the wavelet analysis. In this study for the first time, linear Legendre wavelet is used to constract a wavelet synapses neural network in order to approximation simplefit data set. By finding weight matrix, our neural network will be trained and finally we will test our network.

1. Introduction

In the late 19th and early 20th centuries a few preliminary studies about neural networks (NN) were started. They dealt with some scientific topics like physics, psychology and the like. In the 1940s, Warren Mcculloch and Walter Pitts started to use NN as it is use today [9]. They showed that the NN can compute all sorts mathematical or logical functions [8]. Later Donald Hebb proposed a mechanism describing learning in live neurons [4]. The first usage of networks appeared in the 1950s when Frank Rosenblatt and others created perceptron networks. Rosenblatt also wrote a book on neurocomputing entitled " Principles of Neurodynamics" [10]. During the 1980s, John Hopfield became interested in NN and wrote two important papers on NN [5, 6]. At this time, by describing the concept of the usage of statistics in recurrent neural networks, research on neural networks gained new impetus, and new concepts emerged. Since approximation of continuous functions by networks is very useful in the modeling systems and due to the similarities of wavelet transforms and NN, the idea of matching them was introduced [11]. Although the applications of wavelet in small dimension is very good, wavelet neural networks can analyse problem with large dimension [12]. Some basis functions like spline, polynomial and radial are used as activation function in different kinds of NN. For the first time, Boubez and Peskin used wavelet basis in the NN [2]. They used orthonormal wavelet basis in NN and showed that network weight can be obtained directly and independently. So far, numerous articles on neural networks have

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2010 *Mathematics Subject Classification.* 65T60, 97R80, 92B20.
Key words and phrases. Legendre Wavelet , Neural Network, Synapses neural network.
It is a pleasure to thank my colleague Dr. Farshid Keynia for helpful discussions on various aspects of this paper.
Communicated by: Anton Makarov

been written and new applications are being published every day.
This research has five sections. In section 2, we mension the difinition of linear Legendre wavelets. In section 3, one can take some information about NN and wavelet neural networks (WNN). In section 4, we talk about the simplefit problem and a neural network for that problem will be trained. The result of our WNN for simplefit data set is given in section 5.

2. Legendre wavelets of degree 1

Khelat and Yousefi by definition of an MRA and some properties of it, computed linear Legendre motherwavelets [7] . Linear Legender wavelet has two one-dimensional scaling functions $\phi^0(t) = 1$ and $\phi^1(t) = \sqrt{3}(2t-1)$, and two mother wavelets

$$\psi^0(t) = \begin{cases} -\sqrt{3}(4t-1), & 0 \leq t < 1/2, \\ \sqrt{3}(4t-3), & 1/2 \leq t < 1, \end{cases}$$

$$\psi^1(t) = \begin{cases} 6t-1, & 0 \leq t < 1/2, \\ 6t-5, & 1/2 \leq t < 1. \end{cases}$$

on $[0, 1)$.
The diagram of ψ^0 and ψ^1 are shown in the figurs 1 and 2.

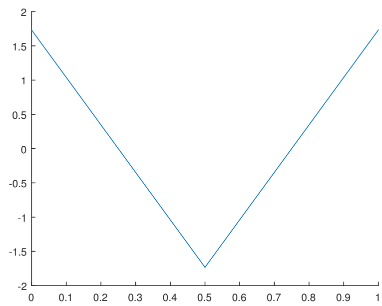

Figure 1. The diagram of ψ^0

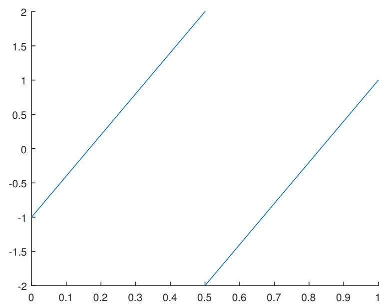

Figure 2. The diagram of ψ^1

3. Wavelet Neural Networks

The usage of orthonormal wavelets is limited to problems with less dimension. Since wavelet bases are generated by translations and dilations of mother wavelet, the number of elements of this kind of bases increasing extremely. Therefore, constructing and saving wavelet bases for large dimensional problem is expensive. By usage of inverse wavelet transform, one can reduce the number of the elements of a wavelet basis which are needed to reconstruct signals. It means that by adaptive discretization of the inverse wavelet transform, wavelet bases can be used for large dimensional problems. In adaptive discrezitation, we should find parameter c_{ij} in $f = \sum_i \sum_j c_{ij}\psi_{ij}$ as output. This is too similar training neural networks. Indeed, we can consider the above equation as an NN by a hidden layer where ψ is the activation function for the hidden layer and the activation function for output layer is linear function. There are two kinds of WNN, one is wavelet synapeses neuron model and the other is wavelet activation function neuron model [1].

3.1. Wavelet Synapses Neural Network (WSNN). For every input x_s, the output is written as

$$y_s = \sum_i \sum_j c_{ij}^s \psi_{ij}(x_s), \tag{3.1}$$

where c_{ij}^s is the weight of input x_s and ψ is our wavelet. The final output is $\hat{y} = \sum_s y_s$. The WSNN has three layers as follow:

Layer 1) In this layer which is called input layer, every ψ_{ij} effects on every inputs directly.

Layer 2) In this layer which is hidden layer, first every output of layer 1 multiplied with its weight and then linear summation of them will computed.

Layer 3) In this layer which is called output layer, all output of hidden layer are linearly summed and the final output, $\hat{y}$, will computed. It means that $\hat{y} = \sum_{k=1}^n y_k$, where n is the number of inputs applied to the network.

The figure 3, shows the WSNN model.

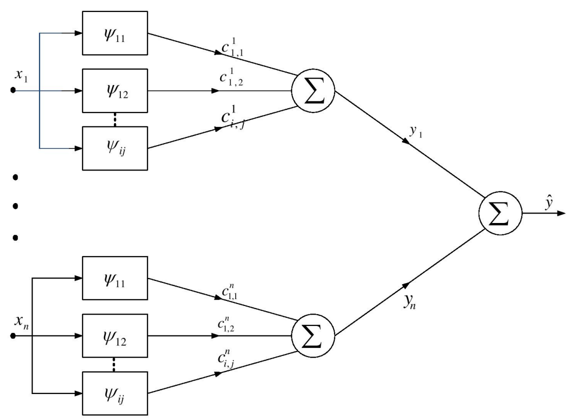

FIGURE 3. Wavelet synapses neural network

3.2. Wavelet Activation Function Neural Network (WAFNN). In this network, a wavelet basis usese as activation function. This model has three layer as follow:
Layer 1) In the input layer, every input x_s is multiplied with its related weight, c_{ij}^s.
Layer 2) In the hidden layer, at first we compute $\sum_s c_{ij}^s x_s$ and then applied to each wavelet function ψ_{ij}, i.e. $y_{ij} = \psi_{ij}(\sum_s c_{ij}^s x_s)$.
Layer 3) In the output layer, first every output of hidden layer is multiplied with weight w_{ij} and the linear summation of them consists our final output $\hat{y}$. It means that $\hat{y} = \sum_i \sum_j w_{ij} y_{ij}$.
The figure 4, shows the WAFNN model.

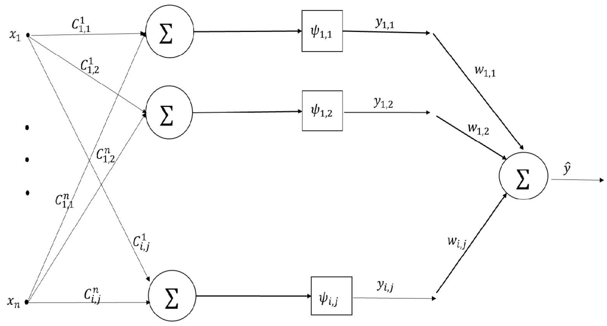

FIGURE 4. Wavelet activation function neural network

4. Simplefit Problem and WSNN

Simplefit data set is a ready example in the toolbox of matlab software. Once the NN has fit the data, it forms a generalization of the input-output relationship and can be used to generate outputs form inputs that was not trained on. This dataset can be used to demonstrate how a WSNN can be trained to estimate the relationship between two sets of data. Simplefit input and the target are a 1×94 matrix of real numbers. In this research, our WSNN is trained to estimate simplefit data set by using linear Legendre wavelet and we choose accidently 71 input for training WSNN and 23 remaining data are used to test our network. Also since we have two mother wavelets, for any one-dimensional input x_s, the output y_s is written as the follow:

$$y_s = \sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij}^s \psi_{ij}^0(x_s) + \sum_{i=1}^{p} \sum_{j=1}^{q} d_{ij}^s \psi_{ij}^1(x_s), \tag{4.2}$$

where p and q are two numbers in $\mathbb{N}$ and $2pq$ is the number of neurons. One have to know that for taking the number of neurons we consider the amount of error. Now in order to find fixed weight coffecients matrix such that by using it, all data can be approximated as the equation 4.2, we consider our simplefit WSNN as an algebric equation $Y = CX$, where Y is a 1×71 output matrix, C is an unknown cofficient matrix and X is $2pq \times 71$ matrix of aplying input to each ψ_{ij}. For example if $p = 2$ and $q = 1$ the martrices Y, C

and X are in the following form:

$$Y = \begin{pmatrix} y_1 & y_2 & \dots & y_{71} \end{pmatrix}, \; C = \begin{pmatrix} c_{11} & c_{21} & d_{11} & d_{21} \end{pmatrix}$$

and

$$X = \begin{pmatrix} \psi^0_{11}(x_1) & \psi^0_{11}(x_2) & \dots & \psi^0_{11}(x_{71}) \\ \psi^0_{21}(x_1) & \psi^0_{21}(x_2) & \dots & \psi^0_{21}(x_{71}) \\ \psi^1_{11}(x_1) & \psi^1_{11}(x_2) & \dots & \psi^1_{11}(x_{71}) \\ \psi^1_{21}(x_1) & \psi^1_{21}(x_2) & \dots & \psi^1_{21}(x_{71}). \end{pmatrix}$$

Now we have $C = Y(X^T(XX^T)^{-1})$. It means that by pseudo inverse matrix of X the matrix C will be computed. After taking C, the approximate matrix $\hat{Y}$ will be computed by the equation $\hat{Y} = CX$. By computing mean square error, we can take the error of $Y - \hat{Y}$. In the figure 5 one can see the simplefit WSNN.

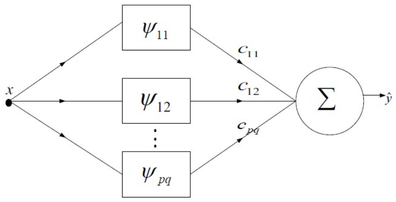

Figure 5. Simplefit wavelet synapses neural network

5. Conclusion

By using Matlab sofware, after computing C, we can take $\hat{Y}$. By using two figures 6 and 7, we can compare $\hat{Y}$ with the real output. In the figure 6, one can see the learning pattern and the figure 7, shows the result of testing of learninig. In the first step, the mean squre error is 0.0012 and in the test step, the value of error is 0.0043. So, by comparing approximated solution with real solution and by noticing the errors, we can conclude that Linear Legendre wavelet is a very suitable wavelet for designing a WSNN for simplefit data.
Note that in the two last figurs, the figures which are shown by stars show our computed output and the figures which are drawen by line show real output.

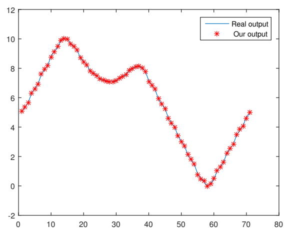

Figure 6. Learning pattern

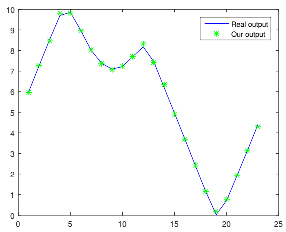

FIGURE 7. Result testing

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