

SOME BITOPOLOGICAL SEPARATION AXIOMS USING Q^* - OPEN SETS

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Abstract. In this paper , we introduce pairwise Q^* - H_i - spaces , ($i = 0, 1, 2$) and pairwise Q^* - U_i - Spaces ($i = 0, 1$) in topological spaces and study its properties .

1. Introduction

Separation axioms are properties by which the topology on a space X separates points from points, points from closed sets and closed sets from each other. The various separation axioms give rise to a sequence of successively stronger requirements, which are put upon the topology of a space to separate varying types of subsets.

In 1963, Levine [12] introduced the concept of semi - open sets. Since then , a considerable number of papers discussing separation axioms, essentially by replacing open sets by semi-open sets, have appeared in the literature. For instance, Maheshwari and Prasad introduced semi- T_0 , semi- T_1 , semi- T_2 , s - normality and s - regularity as a generalization of T_0 , T_1 , T_2 , regularity and normality axioms respectively, and investigated their properties . The notion of semi-open sets was used by Maheshwari and Prasad to introduce pairwise semi- T_0 , pairwise semi- T_1 , pairwise semi- T_2 , pairwise s - regular and pairwise s -normal spaces .Moreover , s - normal (resp. semi normal) spaces were introduced and studied by Maheshwari and Prasad [14] (resp. Dorsett [9]). The notion of Q^* - open sets in a topological space was introduced by Murugalingam and Lalitha[19, 20].

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) . In this paper , we introduce pairwise Q^* - H_i - spaces , ($i = 0, 1, 2$) and pairwise Q^* - U_i - Spaces ($i = 0, 1$) in bitopological spaces and study its properties.

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2. Preliminaries

A topological space (X, τ) is a hyperconnected space if X cannot be written as the union of two proper closed sets. Moreover, for a topological space X the following conditions are equivalent: i) No two nonempty open sets are disjoint. ii) X cannot be written as the union of two proper closed sets. i.e) X is hyperconnected. iii) Every nonempty open set is dense in X . iv) The interior of every proper closed set is empty.

A subset A of a space X is said to be bi- Q^* open [23] if it is both $\tau_i - Q^*$ open and $\tau_j - Q^*$ open, where $i, j = 1, 2$ and $i \neq j$. A subset A of a bitopological spaces X is called [19] $\tau_1\tau_2 - Q^*$ - closed if i) $\tau_1 - \text{int} (A) = \phi$ and A is τ_2 - closed . ii) $\tau_1\tau_2 - Q^*$ - open if $X - A$ is $\tau_1\tau_2 - Q^*$ - closed in X . The set of all Q^* - closed sets [19] with X is a topology . It is denoted by $\tau_{Q^*} = \sigma^*$. A bitopological space X is said to be pairwise semi- H_0 [26] if for every pair of points x and y such that $x \notin \tau_i - \text{scl}\{y\}$ there exists a τ_i - semi - open set U and a τ_j - semi - open set V such that $x \in U, y \in V, U \cap V = \phi$, $i \neq j$, $i, j = 1, 2$. A bitopological space X is said to be pairwise semi- H_1 [26] if for every pair of points x and y such that $\tau_i - \text{scl}\{x\} \cap \tau_j - \text{scl}\{y\} = \phi$, there exists a τ_i - semi - open set U and a τ_j -semi - open set V such that $x \in U, y \in V, U \cap V = \phi$, $i \neq j$, $i, j = 1, 2$. A bitopological space X is said to be pairwise semi- H_2 [26] if for every τ_j -semi - closed set A and a point x such that $\tau_i - \text{scl}\{x\} \cap A = \phi$, there exists a τ_i -semi - open set U and a τ_j - semi- open set V such that $x \in U, A \subseteq V, U \cap V = \phi$, $i \neq j$, $i, j = 1, 2$. A bitopological space X is called pairwise Q^*T_0 [23] if for any pair of distinct points x, y of X , there exists a set which is either $\tau_i - Q^*$ open or $\tau_j - Q^*$ open containing one of the points but not the other , where $i, j = 1, 2$ and $i \neq j$. A bitopological space X is called pairwise Q^*T_1 [23] if for every distinct points x, y of X , there is a $\tau_i - Q^*$ open set U and a $\tau_j - Q^*$ open set V such that $x \in U, y \notin U$ and $y \in V, x \notin V$, where $i, j = 1, 2$ and $i \neq j$. A bitopological space X is called pairwise Q^*T_2 [23] or pairwise Q^* Hausdorff if given distinct points x, y of X , there is a $\tau_i - Q^*$ open set U and a $\tau_j - Q^*$ open set V such that $x \in U, y \in V, U \cap V = \phi$ where $i, j = 1, 2$ and $i \neq j$.

3. Pairwise $Q^* - H_i$ -Spaces ($i = 0, 1, 2$) and pairwise $Q^* - U_i$ - Spaces ($i = 0, 1$)

Here we introduce the bitopological analogues of $Q^* - H_i$ - spaces, ($i = 0, 1, 2$) and $Q^* - U_i$ - Spaces ($i = 0, 1$).

Definition 3.1. A bitopological space X is said to be pairwise $Q^* - H_0$ if for every pair of points x and y such that $x \notin \sigma_i^* - \text{cl}\{y\}$ there exists a $\tau_i - Q^*$ - open set U and a $\tau_j - Q^*$ - open set V such that $x \in U, y \in V, U \cap V = \phi$, $i \neq j$, $i, j = 1, 2$.

Example 3.2. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b\}\}$. Let $a, b \in X$ such that $\{a\} \notin \sigma_1^* - \text{cl}(\{b\})$, there exists a $\tau_i - Q^*$ - open set $U = \{a\}$ and a $\tau_j - Q^*$ - open set $V = \{b\}$ such that $a \in U, b \in V, U \cap V = \phi$. Therefore, X is pairwise $Q^* - H_0$.

Definition 3.3. A bitopological space X is said to be pairwise $Q^* - H_1$ if for every pair of points x and y such that $\sigma_i^* - \text{cl}\{x\} \cap \sigma_j^* - \text{cl}\{y\} = \phi$, there exists a $\tau_i - Q^*$ - open set U and a $\tau_j - Q^*$ - open set V such that $x \in U, y \in V, U \cap V = \phi$, $i \neq j$, $i, j = 1, 2$.

Example 3.4. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a, b\}, \{b, c, d\}\}$, $\tau_2 = \{\phi, X, \{c, d\}, \{b, c, d\}, \{a, b, d\}\}$. Let $a, c \in X$ such that $\sigma_i^* - \text{cl} \{a\} \cap \sigma_j^* - \text{cl} \{c\} = \{a\} \cap \{c\} = \phi$, there exists a $\tau_i - Q^*$ -open set $U = \{a, b\}$ and a $\tau_j - Q^*$ -open set $V = \{c, d\}$ such that $x \in U, y \in V, \{a, b\} \cap \{c, d\} = \phi$. Therefore, X is called pairwise $Q^* H_1$.

Definition 3.5. A bitopological space X is said to be pairwise $Q^* - H_2$ if for every $\tau_j - Q^*$ -closed set A and a point x such that $\sigma_i^* - \text{cl} \{x\} \cap A = \phi$, there exists a $\tau_i - Q^*$ -open set U and a $\tau_j - Q^*$ -open set V such that $x \in U, A \subseteq V, U \cap V = \phi, i \neq j, i, j = 1, 2$.

Example 3.6. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a, b\}, \{b, d\}\}$, $\tau_2 = \{\phi, X, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. Let $a, b \in X$ such that $\sigma_i^* - \text{cl} (\{a\}) \cap \{d\} = \{a, c\} \cap \{d\} = \phi$. Then there is a $\tau_1 - Q^*$ -open set $U = \{a, b\}$ & $\tau_2 - Q^*$ -open set $V = \{c, d\}$ such that $a \in U, \{d\} \subseteq \{c, d\}$ and $U \cap V = \phi$. Therefore, X is called pairwise $Q^* H_2$.

Definition 3.7. A bitopological space X is said to be pairwise $Q^* - U_0$ if for every pair of points x and y such that $x \notin \sigma_i^* - \text{cl} \{y\}$, there exists a $\tau_i - Q^*$ -open set U and a $\tau_j - Q^*$ -open set V such that $x \in U, y \in V, \sigma_j^* - \text{cl} (U) \cap \sigma_i^* - \text{cl} (V) = \phi, i \neq j, i, j = 1, 2$.

Example 3.8. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, c, d\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Let $a, b \in X$ such that $\{a\} \notin \sigma_i^* - \text{cl} (\{b\})$. Then there is a $\tau_1 - Q^*$ -open set $U = \{a\}$ & $\tau_2 - Q^*$ -open set $V = \{b\}$ such that $\sigma_j^* - \text{cl} (\{a\}) \cap \sigma_i^* - \text{cl} (\{b\}) = \{a\} \cap \{b\} = \phi$. Therefore, X is called pairwise $Q^* U_0$.

Example 3.9. A bitopological space X is said to be pairwise $Q^* - U_1$ if for every pair of points x and y such that $\sigma_i^* - \text{cl} \{x\} \cap \sigma_j^* - \text{cl} \{y\} = \phi$, there exists a $\tau_i - Q^*$ -open set U and a $\tau_j - Q^*$ -open set V such that $x \in V, y \in U, \sigma_j^* - \text{cl} (U) \cap \sigma_i^* - \text{cl} (V) = \phi, i, j = 1, 2$.

Example 3.10. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{b, c, d\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}\}$. Let $a, b \in X$ such that $\sigma_i^* - \text{cl} \{a\} \cap \sigma_j^* - \text{cl} \{b\} = \phi$, there exists a $\tau_i - Q^*$ -open set $U = \{b, c, d\}$ and a $\tau_j - Q^*$ -open set $V = \{a\}$ such that $x \in V, y \in U, \sigma_j^* - \text{cl} (\{b, c, d\}) \cap \sigma_i^* - \text{cl} (\{a\}) = \{b, c, d\} \cap \{a\} = \phi$. Therefore, X is called pairwise $Q^* U_1$.

Theorem 3.11. Every pairwise Q^* normal space is pairwise $Q^* - H_2$.

Proof. Let X is pairwise Q^* normal space. Let $x \in X$ and let A be a $\tau_j - Q^*$ -closed set such that $\sigma_i^* - \text{cl} \{x\} \cap A = \phi$. By pairwise Q^* -normality of X , there exists a $\tau_i - Q^*$ -open set U and a $\tau_j - Q^*$ -open set V such that $\sigma_i^* - \text{cl} \{x\} \subseteq V, A \subseteq U, U \cap V = \phi$. Hence $x \in V, A \subseteq U, U \cap V = \phi$. Hence, X is pairwise $Q^* - H_2$. $\square$

Theorem 3.12. Every pairwise Q^* normal space is pairwise H_2 .

Proof. Straight forward. $\square$

Theorem 3.13. Every pairwise Q^* normal space is pairwise semi- H_2 .

Proof. Straight forward. $\square$

Theorem 3.14. *Every pairwise Q^* - H_2 space is pairwise Q^* - H_1 .*

Proof. Let X is Q^* - H_2 space . Let x and y be two distinct points of X such that $\sigma_i^* - \text{cl} \{x\} \cap \sigma_j^* - \text{cl} \{y\} = \phi$. Since X is pairwise Q^* - H_2 , therefore there exists a τ_i - Q^* - open set U and a τ_j - Q^* - open set V such that $x \in V, \sigma_j^* - \text{cl} \{y\} \subseteq U, U \cap V = \phi$. Thus $x \in V, y \in U, U \cap V = \phi$. Hence, X is pairwise Q^* - H_1 . $\square$

Theorem 3.15. *Every pairwise Q^* - H_0 space is pairwise Q^* - H_1 .*

Proof. Let X is pairwise Q^* - H_0 space. Let x and y be two distinct points of X such that $\sigma_i^* - \text{cl} \{x\} \cap \sigma_j^* - \text{cl} \{y\} = \phi$. Hence $x \notin \sigma_j^* - \text{cl} \{y\}$. Since X is pairwise Q^* - H_2 , therefore there exists a τ_i - Q^* - open set U and a τ_j - Q^* - open set V such that $x \in V, y \in U, U \cap V = \phi$. Hence, X is pairwise Q^* - H_1 . $\square$

Definition 3.16 (22). A bitopological space X is pairwise $Q^* - R_0$ if for each $\tau_i - Q^*$ open set $G, x \in G$ implies $\sigma_j^* - \text{cl} (x) \subset G$, where $i, j = 1, 2$ and $i \neq j$.

Definition 3.17 (22). A bitopological space X is pairwise $Q^* - R_1$ if for each $x, y \in X, \sigma_i^* - \text{cl} (x) \neq \sigma_j^* - \text{cl} (y)$, there exist disjoint sets $U \in Q^*O(X, \tau_j)$ and $V \in Q^*O(X, \tau_i)$ such that $\sigma_j^* - \text{cl} (x) \subset U$ and $\sigma_i^* - \text{cl} (y) \subset V$ where $i, j = 1, 2$ and $i \neq j$.

Theorem 3.18. *Every pairwise Q^* - R_1 space is pairwise Q^* - H_0 .*

Proof. Let X is pairwise Q^* - R_1 space. Let $x \notin \sigma_i^* - \text{cl} \{y\}$. Then $\sigma_j^* - \text{cl} \{x\} \neq \sigma_i^* - \text{cl} \{y\}$. Thus there exists a τ_i - Q^* - open set U and a τ_j - Q^* - open set V such that $x \in U, y \in V, U \cap V = \phi$. Hence, X is pairwise Q^* - H_0 . $\square$

Theorem 3.19. *Every pairwise Q^* - R_1 space is pairwise Q^* - H_1 .*

Proof. Follows in view of Theorem 3.18 and 3.15. $\square$

Definition 3.20. A space X is said to be pairwise strongly Q^* - regular if for each τ_j - Q^* - closed subset A of X and $x \notin A$, there exist disjoint τ_i - Q^* - open set U and τ_j - Q^* - open set V such that $x \in U$ and $A \subseteq V$.

Theorem 3.21. *Every pairwise Q^* - H_0 space is pairwise Q^* - R_0 .*

Proof. Let X is pairwise Q^* - H_0 space. Let $x \in G \in Q^*O(\tau_i)$ and let $y \in X - G$. Then $x \notin \sigma_i^* - \text{cl} \{y\}$. Since X is pairwise Q^* - H_0 , there exists a τ_i - Q^* - open set U and a τ_j - Q^* - open set V such that $x \in U, y \in V, U \cap V = \phi$. Thus $\{x\} \cap V = \phi$ so that $y \notin \sigma_j^* - \text{cl} \{x\}$. Hence $X - G \subseteq X - \sigma_j^* - \text{cl} \{x\}$ or $\sigma_j^* - \text{cl} \{x\} \subseteq G$. Thus X is pairwise Q^* - R_0 . $\square$

Theorem 3.22. *Every pairwise strongly Q^* - regular space is pairwise Q^* - H_2 .*

Proof. Let X is pairwise strongly Q^* - regular space. Let $x \in X$ and let A be a τ_j - Q^* - closed subset of X such that $\sigma_i^* - \text{cl} \{x\} \cap A = \phi$. Then $x \notin A$. By pairwise strongly Q^* - regularity of X , there exists a τ_i - Q^* - open set U and a τ_j - Q^* - open set V such that $x \in V, A \subseteq U, U \cap V = \phi$. Hence, X is pairwise Q^* - H_2 . $\square$

Theorem 3.23. *A space is pairwise Q^* - T_2 if and only if it is pairwise Q^* - T_0 and pairwise Q^* - H_0 .*

Proof. Let X is pairwise Q^* - T_2 space. Clearly, X is Q^* - T_0 . Let $x, y \in X$ such that $x \notin \sigma_i^* - \text{cl} \{y\}$. Then $x \neq y$, since X is pairwise Q^* - T_2 , there exists a τ_i - Q^* -open set U and a τ_j - Q^* -open set V , such that $x \in U, y \in V, U \cap V = \phi$. Hence X is pairwise Q^* - H_0 . Conversely, let X is pairwise Q^* - T_0 and pairwise Q^* - H_0 . Let x, y be two distinct points of X . Since X is pairwise Q^* - T_0 , there exists a τ_i - Q^* -open set U or a τ_j - Q^* -open set V such that $x \in U, y \notin U$ or $x \notin U, y \in V$. Thus $x \notin \sigma_i^* - \text{cl} \{y\}$ or $y \notin \sigma_j^* - \text{cl} \{x\}$. Let $x \notin \sigma_i^* - \text{cl} \{y\}$. Since the space is pairwise Q^* - H_0 , there exists a τ_i - Q^* -open set P and a τ_j - Q^* -open set Q such that $x \in P, y \in Q, P \cap Q = \phi$. The result follows similarly in case $y \notin \sigma_j^* - \text{cl} \{x\}$. Hence X is pairwise Q^* - T_2 . $\square$

Theorem 3.24. *A space is pairwise Q^* - T_2 if and only if it is pairwise Q^* - T_1 and pairwise Q^* - H_1 .*

Proof. Let X is pairwise Q^* - T_2 space. Clearly, X is pairwise Q^* - T_1 . Let $x, y \in X$ such that $\sigma_i^* - \text{cl} \{x\} \cap \sigma_j^* - \text{cl} \{y\} = \phi$. Then x, y are distinct points of X so that there exists a τ_i - Q^* -open set U and a τ_j - Q^* -open set V , such that $x \in U, y \in V, U \cap V = \phi$. Hence, X is pairwise Q^* - H_1 .

Conversely, let X is pairwise Q^* - T_1 and pairwise Q^* - H_1 . Let x, y be two distinct points of X . Since X is pairwise Q^* - T_1 , therefore $\{x\}$ and $\{y\}$ are bi- Q^* -closed sets. Hence $\sigma_j^* - \text{cl} \{x\} \cap \sigma_i^* - \text{cl} \{y\} = \{x\} \cap \{y\} = \phi$. Since X is pairwise Q^* - H_1 , there exists a τ_i - Q^* -open set U and a τ_j - Q^* -open set V such that $x \in U, y \in V, U \cap V = \phi$. Hence X is pairwise Q^* - T_2 . $\square$

Theorem 3.25. *Every pairwise strongly Q^* -regular space is pairwise Q^* - U_0 .*

Proof. Let X is pairwise strongly Q^* -regular space. Let $x, y \in X$ such that $x \notin \sigma_i^* - \text{cl} \{y\}$. Since the space is pairwise strongly Q^* -regular, there exists a τ_i - Q^* -open set U and a τ_j - Q^* -open set V such that $x \in U, \sigma_i^* - \text{cl} \{y\} \subseteq V, \sigma_j^* - \text{cl} (U) \cap \sigma_i^* - \text{cl} (V) = \phi$. Hence $x \in U, y \in V, U \cap V = \phi$ and thus X is pairwise Q^* - U_0 . $\square$

Theorem 3.26. *Every pairwise Q^* - U_0 space is pairwise Q^* - H_0 .*

Proof. Let X is pairwise Q^* - U_0 space. Let $x, y \in X$ such that $x \notin \sigma_i^* - \text{cl} \{y\}$. Since X is pairwise Q^* - U_0 , there exists a τ_i - Q^* -open set U and a τ_j - Q^* -open set V such that $x \in U, y \in V, \sigma_j^* - \text{cl} (U) \cap \sigma_i^* - \text{cl} (V) = \phi$. Hence $x \in U, y \in V, U \cap V = \phi$ and thus X is pairwise Q^* - H_0 . $\square$

Theorem 3.27. *Every pairwise Q^* - U_0 space is pairwise H_0 .*

Proof. Straight forward. $\square$

Theorem 3.28. *Every pairwise Q^* - U_0 space is pairwise semi- H_0 .*

Proof. Straight forward. $\square$

Theorem 3.29. *Every pairwise Q^* - U_1 space is pairwise Q^* - H_1 .*

Proof. Let X is pairwise Q^* - U_1 space. Let $x, y \in X$ such that $\sigma_i^* - \text{cl} \{x\} \cap \sigma_j^* - \text{cl} \{y\} = \phi$. Since X is pairwise Q^* - U_1 , there exists a τ_i - Q^* -open set U and a τ_j - Q^* -open set V such that $x \in U, y \in V, \sigma_j^* - \text{cl} (U) \cap \sigma_i^* - \text{cl} (V) = \phi$. Hence $x \in V, y \in U, U \cap V = \phi$ and thus X is pairwise Q^* - H_1 . $\square$

Theorem 3.30. *Every pairwise Q^* - U_1 space is pairwise semi- H_1 .*

Proof. Straight forward. $\square$

Theorem 3.31. Every pairwise Q^* normal space is pairwise semi- U_1 .

Proof. Straight forward. $\square$

Remark 3.32. The above relation between pairwise Q^* - H_i axioms, $i \in \{0, 1, 2\}$ and pairwise Q^* - U_i axioms, $i \in \{0, 1, 2\}$ can be summarized in the following diagram:

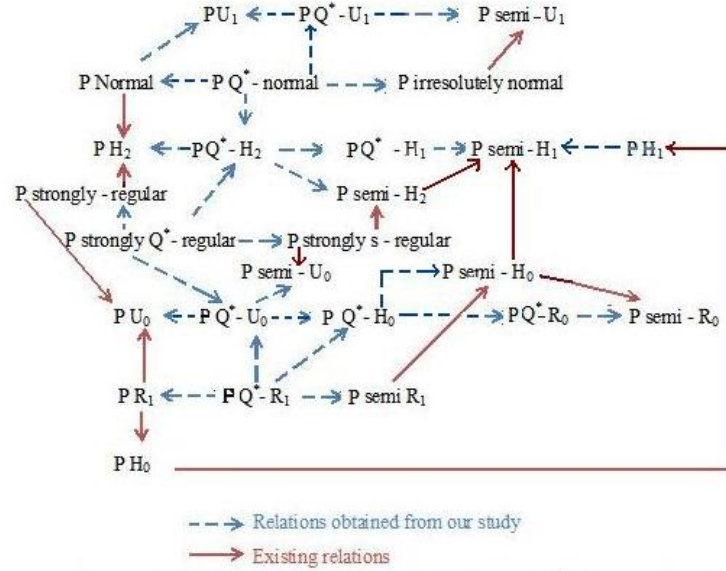


Fig 3.1 Relations between existing $P - H_0$ and $P Q^* - H_0$

Pairwise $Q^*-T_2 \Rightarrow$ Pairwise Q^*-T_0 + Pairwise Q^*-H_0
 Pairwise $Q^*-T_2 \Rightarrow$ Pairwise Q^*-T_1 + Pairwise Q^*-H_1
 Pairwise semi- $T_2 \Rightarrow$ Pairwise semi- T_0 + Pairwise semi- H_0
 Pairwise semi- $T_2 \Rightarrow$ Pairwise semi- T_1 + Pairwise semi- H_1
 Pairwise semi- $T_2 \Rightarrow$ Pairwise Q^*-T_0 + Pairwise Q^*-H_0
 Pairwise semi- $T_2 \Rightarrow$ Pairwise Q^*-T_1 + Pairwise Q^*-H_1

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