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COUPLED CRITICAL PROBLEMS INVOLVING DEGENERATE GRUSHIN OPERATORS WITH VARIABLE EXPONENTS

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Abstract. We study a critical system involving the $p(x)$- and $q(x)$-Grushin operators, which combine the degeneracy of Grushin-type structures with the flexibility of variable exponents. Using variational methods in variable exponent Sobolev–Grushin spaces, we prove the existence of nontrivial weak solutions under suitable assumptions on the parameters. In addition, we investigate regularity issues and show that every weak solution is locally bounded and Hölder continuous in the interior of the domain.

1. Introduction

Degenerate elliptic operators of Grushin type arise naturally in the analysis of anisotropic and subelliptic phenomena, and they have been the subject of intensive study in the past decades. The classical p-Grushin operator is defined as

$$\Delta_{\gamma,p}u = \operatorname{div}\left(|\nabla_\gamma u|^{p-2}\nabla_\gamma u\right),$$

where $\nabla_\gamma = (\nabla_x, |x|^\gamma \nabla_y)$ incorporates the anisotropy of the underlying geometry and exhibits degeneracy along the singular set $\{x = 0\}$. This lack of uniform ellipticity introduces new analytical challenges compared with the standard p-Laplacian operator.

A number of important contributions in this direction can be found in the literature. For instance, in the work *Bifurcation and multiplicity results for critical problems involving the p-Grushin operator* (see [12]), the authors considered the critical problem

$$-\Delta_{\gamma,p}u = \lambda|u|^{p-2}u + |u|^{p_\gamma^*-2}u \quad \text{in } \Omega, \qquad u = 0 \text{ on } \partial\Omega,$$

and established the existence of weak solutions together with bifurcation and multiplicity results by means of variational techniques. Such results highlight the richness of the variational structure associated with degenerate elliptic operators of Grushin type.

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