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FROM H-CONVEX EXISTENCE TO ORBITAL RETRACTIONS: SETWISE PICARD PRINCIPLES IN UNIFORM SPACES

ABDELBASSET FELHI

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Abstract. We study quantitative convergence to a possibly non-singleton equilibrium set in a Hausdorff uniform space. For a saturated generating calibration, setwise descent and summable orbital drift yield finite pseudometric lengths, convergence to an individual equilibrium, and explicit tail estimates. The hypotheses actually imply that the target is exactly the fixed-point set; the orbit limit is therefore a retraction, continuous whenever the selector is continuous. A separate compact H-convex existence theorem supplies the target for a multivalued problem, without claiming that existence alone produces a quantitative selector. Residual error bounds and summable inexact evaluations yield computable certificates when their constants and error budgets are known. Two convex-valued examples exhibit exact algebraic decay and, more significantly, a continuous selector that is neither Fejér monotone nor nonexpansive, although its residual decreases geometrically. We compare the result explicitly with weakly Picard and quasi-Fejér theory and do not claim novelty for the underlying summability argument.

1. Introduction

Fixed-point theorems for locally H-convex uniform spaces primarily assert existence of equilibria of compact multimaps; Tarafdar's theorem and Park's upper-semicontinuous extension are representative [10, 19]. Existence alone neither provides a single-valued iteration nor controls its rate. Earlier uniform-space iteration results, including work relative to a fixed-point set, go back at least to Tarafdar [18]; the use of saturated generating pseudometrics and retractions also has substantial precedents [5]. We consider the separate, conditional question of what follows *once* a selector has quantitative information on its distance to the equilibrium set and on its successive displacements.

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